

The impact of monetary policy normalisation on secured money markets

3rd ChaMP Workstream 1 Workshop

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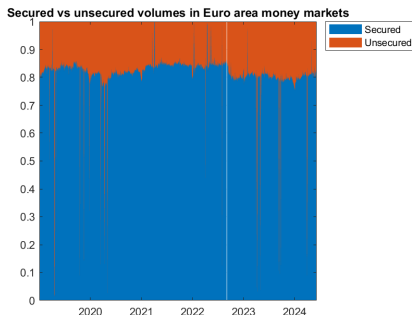
21 June 2024

Overview

- 1 Introduction and motivation
- 2 Repo markets: a balance between the supply and demand of funding and collateral
- 3 Recent developments in repo markets: the post-pandemic and monetary policy normalisation period
- 4 Data and methodology to assess the factors explaining repo rates
- 5 Results
- 6 Conclusions
- 7 Annex

Introduction

- Money markets constitute a key element for financial markets and **monetary policy transmission**
- Since the GFC, **secured money markets (repo)** have been the main source of funding and collateral exchange
- The participation of non-bank financial intermediaries (**NBFIs**) has been increasing, possibly affecting market functioning and money market developments
- The main **objective (funding vs collateral)** for participating in repo markets has changed along time



Introduction: Literature review

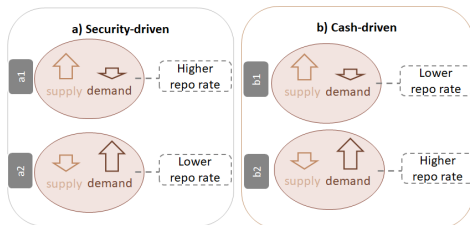
- 1 Literature that assess the **impact of unconventional monetary policy (UMP) on money markets**
Carrera de Souza and Hudepohl (2022), Arrata et al. (2020), Brand et al. (2019)
- 2 Previous studies on the **interrelations between sovereign debt markets and repo**: determinants of *repo specialness*, the demand for certain bonds (e.g. cheapest-to-deliver, benchmark or short positions) is also transmitted to repo markets
 - *Arrata et al. (2020)*: relevance of short positions in increasing demand for collateral
 - *Nagel (2016)*: increase in monetary policy rate expectations drive up the demand of "money-like-assets" and hedging for additional rate hikes
 - *Dufour and Skinner (2020)*: study determinants of repo specialness
 - *Jappelli et al. (2024)*: when there is excess of collateral demand, the competence for getting the bond, reduces rates

Our work is close to these two strands of literature, combining the **effects of UMP on asset scarcity and interrelations on sovereign debt markets**. We analyse **higher collateral demand** in a context of **monetary policy normalisation**.

Repo markets: a balance between the supply and demand of funding and collateral

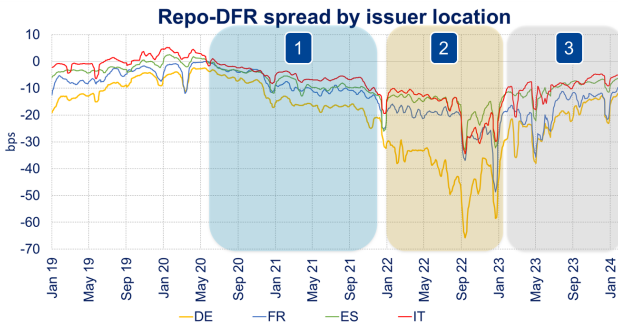
Two primary usages driving repo transactions:

- **Cash-driven transactions:** the lender accepts any collateral included in a basket of securities (*General Collateral*). Price determined by supply/demand of cash.
- **Security-driven transactions:** the motivation for a repo is the preference for a specific collateral. Price is based on the demand and supply of such security.



Recent developments in repo markets

1. Post-Covid extension of UMP
2. Shift in MP rate expectations: increasing rates. No significant change in asset purchases holdings
3. Shift in MP rate expectations: decreasing rates. Decrease in ECB balance sheet (TLTRO+ending asset purchases)



Dataset: granular repo rate transactions

Data: repo transactions

- Individual transactions from January 2019 - June 2023 (recently updated until february 2024) from MMSR
- Information on interest rate, volume, counterparty, collateral and type of transaction (borrowing or lending)
- Transactions with one-day maturity: O/N, T/N, S/N, which represents around 80% of daily volume
- Transactions backed by government bonds from Spain, Germany, France and Italy (around 90% volume).

Data: other

- Supply of collateral: ECB footprint
- Demand of collateral: monetary policy uncertainty, flight-to-quality and monetary policy rate expectations
- Liquidity/credit risk

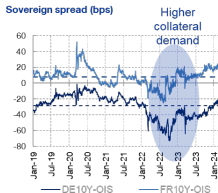
Conjunctural factors driving demand of collateral

Short positions in sovereign markets



Expected path of policy rates

Demand for short-term safe assets and flight-to-quality



Increase in monetary policy uncertainty



Factors driving supply of collateral

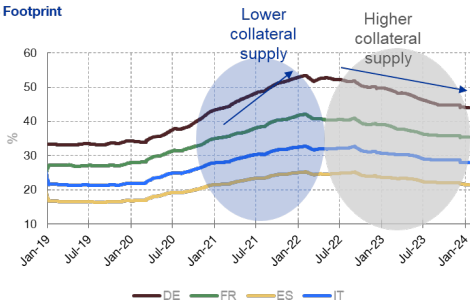
ECB footprint

Eurosystem holdings over free

$$\text{float}_{j,t} = \frac{PSPP_{j,t} + PEPP_{j,t}}{\text{Free float}_{j,t}},$$

where Free-float = Outstanding amount - Eurosystem holdings - Pledged collateral + SLF balance

ECB Footprint



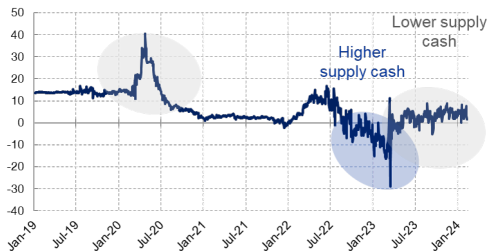
Other factors increasing the SUPPLY

- Increase SL limit
- TLTRO repayments
- Increasing gov.debt. issuances

Euribor-OIS (credit and liquidity risk)

- Euribor 3 months (reference for interbank rates) - OIS (risk-free rate)
- Commonly used as a proxy for credit/liquidity risk in the interbank market
- Should be more relevant during “cash-driven” periods
- Interesting indicator for periods of financial stress
- Negative liquidity premium observed in 2022, related to excess liquidity and preference for short tenors

Euribor-OIS 3M (bps)



Econometric specification

Linear mixed-effects model, with separate regressions for **each country (DE,FR,IT,ES)**

$$\begin{aligned} \text{Repo} - \text{DFR}_{i,t,j} = & \beta_{\text{Slope}} \cdot \text{Slope}_t + \beta_{\text{SMOVE}} \cdot \text{SMOVE}_t + \beta_{\text{Sovereign}} \cdot \text{Sovereign}_{t,j} \\ & + \beta_{\text{Euribor}} \cdot \text{EuriborOIS}_t + \beta_{\text{Eurosystemholdings}} \cdot \text{Eurosystemholdings}_{t,j} + \\ & \alpha_{\text{collateral}_i} + \alpha_{\text{counterparty}_i} + \alpha_{\text{time}} \\ & + \text{Quarter and year end controls} + \text{Type transaction}_{i,t,j} + \epsilon_{i,t,j} \end{aligned} \quad (1)$$

where subindex i refers to each transaction, t denotes day of the transaction, and j the country. $\alpha_{\text{collateral}}$, $\alpha_{\text{counterparty}}$ and α_{time} account for collateral, counterparty and time differences.

Dummies for year and quarter-end for each specific year are included as well as type of transaction (borrowing/lending) identifier.

Useful approach to identify:

- **Cross-time changes in demand for collateral:** e.g., higher interest rate expectations will motivate short positions
- **Cross-collateral heterogeneity:** within same period of time, some collaterals (specials) will be more demanded
- **Cross-counterparty heterogeneity:** within same period of time, repo rates differ across NBFIs, banks...

Which sectors contribute more to a higher collateral demand?

- *Nguyen et al., 2023*: MP transmission works better when transactions are done primarily by banks.
- *Jappelli et al., 2024*: arbitrageurs investors (i.e. NBFIs) borrow the overpriced and more demanded bonds to sell it short.

Need to account for **different effects of short-positioning demand across counterparties** (Banks, NBFIs, CCP)

$$\begin{aligned} \text{Repo} - \text{DFR}_{i,t,j} = & \beta_{\text{Slope}} * \text{Counterparty}_{\text{sector}} \cdot \text{Slope}_t + \beta_{\text{SMOVE}} \cdot \text{SMOVE}_t + \\ & \beta_{\text{Sovereign}} \cdot \text{Sovereign}_{t,j} + \beta_{\text{Euribor}} \cdot \text{EuriborOIS}_t + \\ & \beta_{\text{Eurosystemholdings}} \cdot \text{Eurosystemholdings}_{t,j} + \\ & \alpha_{\text{collateral}} + \alpha_{\text{Reportingagent}} + \alpha_{\text{time}} + \text{Quarter and year end controls} + \epsilon_{i,t,j} \end{aligned} \quad (2)$$

That way, we can identify **heterogeneities across sectors**.

Exploring heterogeneities across collaterals: *on-the-run specialness*

Is the effect of short positions different for *on-the-run* bonds?

- *Dufour and Skinner, 2020*: consider time-varying bond characteristics determining collateral specialness.
- *d'Amico and Pancost, 2022*: bonds *on-the-run* have a higher price than others (motivated by higher demand)

Need to account for different effects across *on-the-run* and *off-the-run* bonds

$$\begin{aligned} \text{Repo} - \text{DFR}_{i,t,j} = & \beta_{\text{Slope}} * \text{ontherun}_{i,t,j} \cdot \text{Slope}_t + \beta_{\text{Slope}} * \text{NBFI}_{\text{sector}} \cdot \text{Slope}_t + \\ & \beta_{\text{SMOVE}} \cdot \text{SMOVE}_t + \beta_{\text{Sovereign}} \cdot \text{Sovereign}_{t,j} + \beta_{\text{Euribor}} \cdot \text{EuriborOIS}_t + \\ & \beta_{\text{Eurosistemholdings}} \cdot \text{Eurosistemholdings}_{t,j} + \\ & \alpha_{\text{Reportingagent}} + \alpha_{\text{time}} + \text{Quarter and year end controls} + \epsilon_{i,t,j} \end{aligned} \quad (3)$$

That way, we can identify **time changing effects of higher policy rate expectations and heterogeneities across bonds.**

German collateral

The downward pressure on repo-DFR spread driven by short positions is mainly working through **NBFI** and, at a lesser extent, using *on-the-run bonds*

<i>dependent variable: repo-DFR spread</i>	(I)	(II)	(III)	(IV)	(V)	(VI)
Slope	-	-4.07*** (0.00)	-4.25*** (0.00)	-4.49*** (0.00)	-	-
SMOVE	-	-0.17*** (0.00)	-0.17*** (0.00)	-0.16*** (0.00)	-0.16*** (0.00)	-0.18*** (0.00)
Sov.spread	-	0.31*** (0.00)	0.22*** (0.00)	0.23*** (0.00)	0.23*** (0.00)	0.23*** (0.00)
EuriborOIS	-	0.55*** (0.00)	0.20*** (0.00)	0.25*** (0.00)	0.26*** (0.00)	0.27*** (0.00)
APP	-0.53*** (0.00)	-0.52*** (0.00)	-0.56*** (0.00)	-0.49*** (0.00)	-0.48*** (0.00)	-0.51*** (0.00)
NBFI vs bank	-	-	-	-	-4.39*** (0.00)	-5.80*** (0.00)
CCP vs bank	-	-	-	-	1.66*** (0.00)	0.92*** (0.00)
Slope bank	-	-	-	-	-3.16*** (0.00)	-3.74*** (0.00)
Slope NBFI	-	-	-	-	-4.41*** (0.00)	-3.98*** (0.00)
Slope CCP	-	-	-	-	-4.55*** (0.00)	-3.75*** (0.00)
Slope on-the-run	-	-	-	-	-	-1.56*** (0.00)
Adjusted Rsquared	42%	56%	62%	68%	68%	64%
Observations	1,411,529	1,411,529	1,411,529	1,411,529	1,411,529	1,411,529
<i>Quarter and year end controls</i>	yes	yes	yes	yes	yes	yes
<i>Time effects</i>	no	no	no	yes	yes	yes
<i>Collateral FE</i>	no	no	yes	yes	yes	no
<i>Reporting agent location FE</i>	no	no	yes	yes	yes	yes

*P-values in parenthesis: Significant levels: *p < 0.1; **p < 0.05; ***p < 0.01. CCP refers to non bilateral transactions.*

French collateral

<i>dependent variable: repo-DFR spread</i>	(I)	(II)	(III)	(IV)	(V)	(VI)
Slope	-	-2.93*** (0.00)	-3.07*** (0.00)	-3.39*** (0.00)	-	-
SMOVE	-	-0.11*** (0.00)	-0.12*** (0.00)	-0.11*** (0.00)	-0.11*** (0.00)	-0.12*** (0.00)
Sov.spread	-	-0.03*** (0.00)	-0.13*** (0.00)	-0.11*** (0.00)	-0.11*** (0.00)	-0.12*** (0.00)
EuriborOIS	-	0.29*** (0.00)	0.15*** (0.00)	0.24*** (0.00)	0.24*** (0.00)	0.24*** (0.00)
APP	-0.69*** (0.00)	-0.24*** (0.00)	-0.58*** (0.00)	-0.46*** (0.00)	-0.46*** (0.00)	-0.56*** (0.00)
NBFI vs bank	-	-	-	-	-4.80*** (0.00)	-2.22*** (0.00)
CCP vs bank	-	-	-	-	0.43*** (0.00)	2.38*** (0.00)
Slope bank	-	-	-	-	-0.03 (0.93)	-0.16 (0.64)
Slope NBFI	-	-	-	-	-3.43*** (0.00)	-3.39*** (0.00)
Slope CCP	-	-	-	-	-3.56*** (0.00)	-3.36*** (0.00)
Slope on-the-run	-	-	-	-	-	-1.06*** (0.00)
Adjusted Rsquared	35%	41%	50%	55%	55%	48%
Observations	1,189,501	1,189,501	1,189,501	1,189,501	1,189,501	1,189,501
<i>Quarter and year end controls</i>	yes	yes	yes	yes	yes	yes
<i>Time effects</i>	no	no	no	yes	yes	yes
<i>Collateral FE</i>	no	no	yes	yes	yes	no
<i>Reporting agent location FE</i>	no	no	yes	yes	yes	yes

*P-values in parenthesis: Significant levels: *p < 0.1; **p < 0.05; ***p < 0.01. CCP refers to non bilateral transactions.*

Spanish collateral

<i>dependent variable: repo-DFR spread</i>	(I)	(II)	(III)	(IV)	(V)	(VI)
Slope	-	-2.51*** (0.00)	-2.05*** (0.00)	-2.08*** (0.00)	-	-
SMOVE	-	-0.03*** (0.00)	-0.03*** (0.00)	-0.04*** (0.00)	-0.04*** (0.00)	-0.04*** (0.00)
EuriborOIS	-	0.24*** (0.00)	0.13*** (0.00)	0.16*** (0.00)	0.16*** (0.00)	0.15*** (0.00)
APP	-0.53*** (0.00)	-0.11*** (0.00)	-0.73*** (0.00)	-0.68*** (0.00)	-0.68*** (0.00)	-0.89*** (0.00)
NBFI vs bank	-	-	-	-	-3.09*** (0.00)	-2.61*** (0.00)
CCP vs bank	-	-	-	-	1.64*** (0.00)	0.51*** (0.00)
Slope bank	-	-	-	-	-1.29*** (0.00)	-1.73*** (0.00)
Slope NBFI	-	-	-	-	-0.47** (0.03)	-2.48** (0.03)
Slope CCP	-	-	-	-	-3.18*** (0.00)	-2.48*** (0.00)
Slope on-the-run	-	-	-	-	-	-0.35*** (0.00)
Adjusted Rsquared	43%	49%	53%	57%	58%	54%
Observations	1,164,961	1,164,961	1,164,961	1,164,961	1,164,961	1,164,961
<i>Quarter and year end controls</i>	yes	yes	yes	yes	yes	yes
<i>Time effects</i>	no	no	no	yes	yes	yes
<i>Collateral FE</i>	no	no	yes	yes	yes	no
<i>Reporting agent location FE</i>	no	no	yes	yes	yes	yes

*P-values in parenthesis: Significant levels: *p < 0.1; **p < 0.05; ***p < 0.01. CCP refers to non bilateral transactions.*

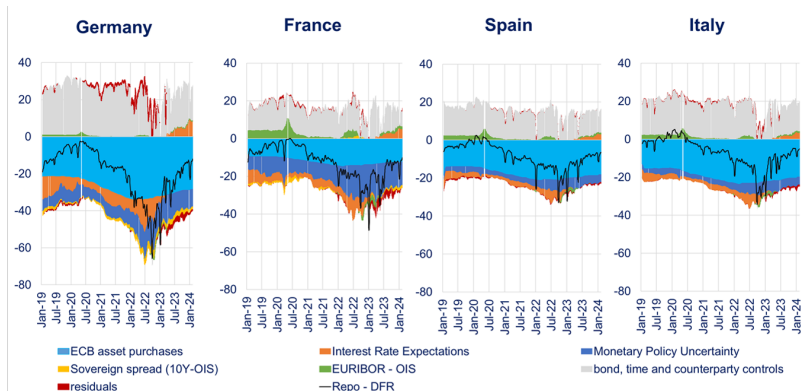
Annex: only bilateral trades

Italian Collateral

<i>dependent variable: repo-DFR spread</i>	(I)	(II)	(III)	(IV)	(V)	(VI)
Slope	-	-1.49*** (0.00)	-1.53*** (0.00)	-1.52*** (0.00)	-	-
SMOVE	-	-0.04*** (0.00)	-0.04*** (0.00)	-0.04*** (0.00)	-0.04*** (0.00)	-0.05*** (0.00)
EuriborOIS	-	0.25*** (0.00)	0.16*** (0.00)	0.19*** (0.00)	0.20*** (0.00)	0.17*** (0.00)
APP	-0.68*** (0.00)	-0.39*** (0.00)	-0.61*** (0.00)	-0.55*** (0.00)	-0.55*** (0.00)	-0.61*** (0.00)
NBFI vs bank	-	-	-	-	-11.13*** (0.00)	-10.85*** (0.00)
CCP vs bank	-	-	-	-	-1.47*** (0.00)	-1.08*** (0.00)
Slope bank	-	-	-	-	-3.33*** (0.00)	-2.93*** (0.00)
Slope NBFI	-	-	-	-	-1.22*** (0.00)	-1.00*** (0.00)
Slope CCP	-	-	-	-	-3.47** (0.03)	-3.00** (0.03)
Slope on-the-run	-	-	-	-	-	-0.54** (0.03)
Adjusted Rsquared	22%	29%	40%	44%	45%	36%
Observations	2,221,427	2,221,427	2,221,427	2,221,427	2,221,427	2,221,427
<i>Quarter and year end controls</i>	yes	yes	yes	yes	yes	yes
<i>Time effects</i>	no	no	no	yes	yes	yes
<i>Collateral FE</i>	no	no	yes	yes	yes	no
<i>Reporting agent location FE</i>	no	no	yes	yes	yes	yes

*P-values in parenthesis: Significant levels: *p < 0.1; **p < 0.05; ***p < 0.01. CCP refers to non bilateral transactions.*

Repo-DFR decomposition

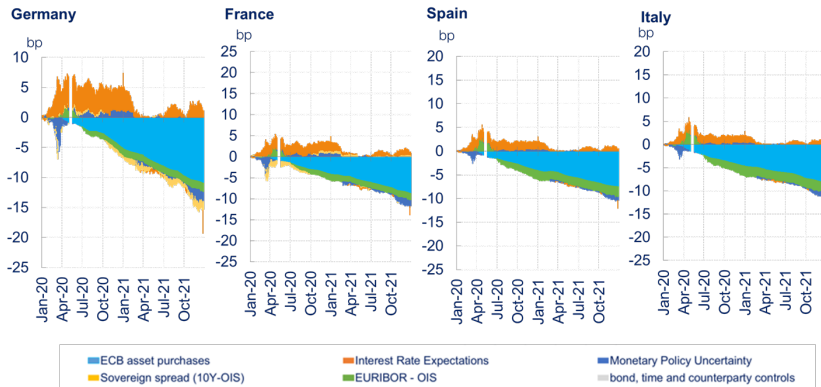


Source: MMSR and own computations. 10 days moving average repo rates computed as a weighted average of one-day maturity transactions. Last observation: 9 february 2024. Bond and counterparty controls can be understood as the transaction-specific intercept, such that contribution of those effects to daily repo spreads are computed as the volume-weighted average of transaction-level intercepts. Similarly, daily residuals are volume-weighted average of transactional-level residuals.

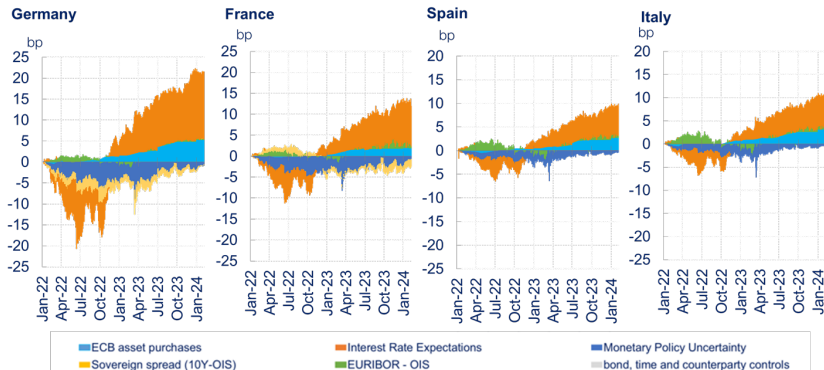
Controls refer to: $\alpha_{collateral} + \alpha_{counterparty} + \alpha_{time} + Quarter\ and\ year\ end\ controls + Type\ transaction_{t,i,j} + \epsilon_{i,t,j}$

Annex: show estimates

Cumulative changes of factors contributing to repo-DFR spread: post-COVID period (2020-21)



Cumulative changes of factors contributing to repo-DFR spread: MP normalisation period (2022-24)



Conclusions

- We provide an assessment of main factors driving changes in repo-DFR spread during COVID and since the start of MP normalisation.
- ECB footprint (assessed through Eurosystem holdings over free float) played a relevant role in explaining collateral scarcity and repo-DFR spread levels, but...
- ...other sources of collateral demand arise in 2022, amid MP normalisation.
- We present an empirical assessment of such **conjunctural factors**: rising (and later declining) **MP rate expectations**, **MP uncertainty**, **sovereign spread (flight-to-quality)**, while controlling for **structural factors** (ECB footprint), **funding pressures** (EURIBOR-OIS) and...
- ...we show that **higher demand** for collateral in repo markets to enter into **short positions** was **stronger for on-the-run bonds**.
- We use transactional data to account for differences across counterparties and collateral, which improve our estimates and have been used to **explore the transmission of monetary policy through non-bank financial intermediaries**.
- Indeed, the effect of short positions on repo spread widening is **stronger** for German and French collateral when **traded by NBFIs**. Additionally, for all collaterals, **repo-DFR spread** (for an average transaction) is around **5 to 10 bps bigger** when the counterparty is a **NBFI**.
- In the last year, ECB balance sheet reduction, MP rates repricing, and gov. bonds long positions were the main factors driving repo rates up.

Conclusions: future work

- Enrich time variant characteristics of bonds, incorporating market data to show prices in the cash market.
- Using data on effective short positions.
- Using additional data on the "final" counterparties in case of centrally-cleared transactions.

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- Nagel, S. (2016). The liquidity premium of near-money assets. *The Quarterly Journal of Economics*, 131:1927–1971.

Thanks for your attention!

Annex: only with bilateral trades (Germany and France)

- The impact of **liquidity/credit risk (EuriborOIS)** is bigger in the case of **bilateral** transactions: intuitive as centrally cleared transactions imply lower risks.
- Still the impact of **short positions** is bigger for **NBFIs and on-the-run bonds**

<i>dependent variable: repo-DFR spread</i>	German collateral		French collateral	
	all	only bilateral	all	only bilateral
SMOVE	-0.18*** (0.00)	-0.22*** (0.00)	-0.12*** (0.00)	-0.22*** (0.00)
Sov.spread	0.23*** (0.00)	0.27*** (0.00)	-0.12*** (0.00)	-0.11*** (0.00)
EuriborOIS	0.27*** (0.00)	0.34*** (0.00)	0.24*** (0.00)	0.50*** (0.00)
APP	-0.51*** (0.00)	-0.48*** (0.00)	-0.56*** (0.00)	-0.23*** (0.00)
NBFI vs bank	-5.80*** (0.00)	-4.57*** (0.00)	-2.22*** (0.00)	-0.69*** (0.00)
Slope bank	-3.74*** (0.00)	-3.95*** (0.00)	0.16 (0.64)	0.90 (0.10)
Slope NBFI	-3.98*** (0.00)	-4.43*** (0.00)	-3.39*** (0.00)	-2.64*** (0.00)
Slope on-the-run	-1.56*** (0.00)	-0.76*** (0.00)	-1.06*** (0.00)	-0.94*** (0.00)
Adjusted R-squared	64%	56%	48%	24%
Observations	1,411,529	253,618	1,189,501	200,148
<i>Quarter and year-end controls</i>	yes	yes	yes	yes
<i>Time effects</i>	yes	yes	yes	yes
<i>Collateral FE</i>	no	no	no	no
<i>Reporting agent location FE</i>	yes	yes	yes	yes

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Annex: only with bilateral trades (Spain and Italy)

- The impact of **liquidity/credit risk (EuriborOIS)** is bigger in the case of **bilateral** transactions: intuitive as centrally cleared transactions imply lower risks.
- No additional effect of short-positions for *on-the-run* bonds.

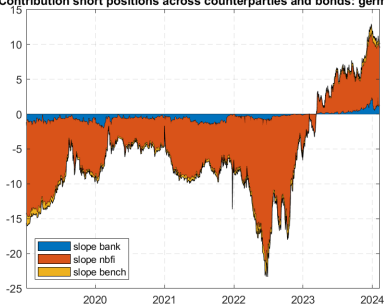
<i>dependent variable: repo-DFR spread</i>	Spanish collateral		Italian collateral	
	all	only bilateral	all	only bilateral
SMOVE	-0.04*** (0.00)	-0.06*** (0.00)	-0.05*** (0.00)	-0.07*** (0.00)
EuriborOIS	0.15*** (0.00)	0.22*** (0.00)	0.17*** (0.00)	0.32*** (0.00)
APP	-0.89*** (0.00)	-0.53*** (0.00)	-0.61*** (0.00)	-0.44*** (0.00)
NBFI vs bank	-2.61*** (0.00)	-2.45*** (0.00)	-10.85*** (0.00)	-10.61*** (0.00)
Slope bank	-1.73*** (0.00)	-1.99*** (0.00)	-2.93*** (0.64)	-4.47*** (0.10)
Slope NBFI	-2.48*** (0.00)	-1.27*** (0.00)	-1.00*** (0.00)	-1.86*** (0.00)
Slope on-the-run	-0.35*** (0.00)	-0.04 (0.27)	-0.54*** (0.11)	0.45 (0.00)
Adjusted R-squared	54%	44%	36%	22%
Observations	1,164,961	521,259	2,221,427	418,525
<i>Quarter and year-end controls</i>	yes	yes	yes	yes
<i>Time effects</i>	yes	yes	yes	yes
<i>Collateral FE</i>	no	no	no	no
<i>Reporting agent location FE</i>	yes	yes	yes	yes

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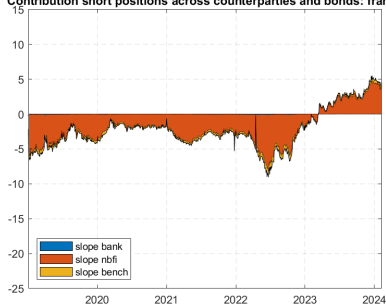
Contribution of short positions by counterparty

- Short positions effect on repo-DFR spread is mainly observed through the participation of NBFIs, who borrow collateral
- This is observed by: i) higher proportion of NBFIs trades, ii) higher effect of short positions for NBFIs

Contribution short positions across counterparties and bonds: germany



Contribution short positions across counterparties and bonds: france

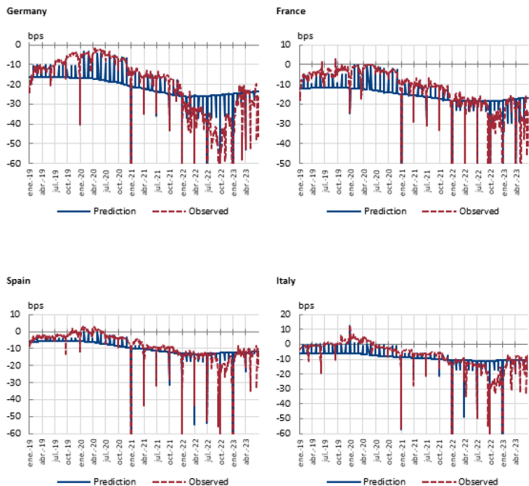


Note: We exclude centrally cleared transactions

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Estimated repo-DFR using different models

Figure: Estimated repo-DFR spread assessing only ECB footprint

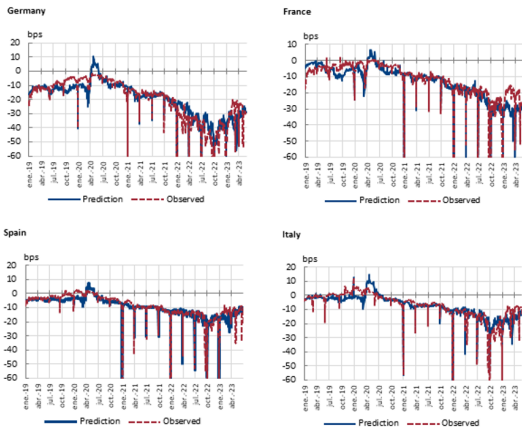


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Estimated repo-DFR using different models

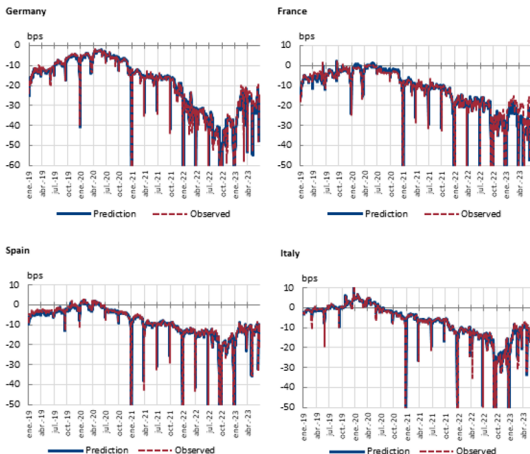
Figure: Estimated repo-DFR spread including MP normalisation



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Estimated repo-DFR using different models

Figure: Estimated repo-DFR spread including MP normalisation, collateral and counterparty effects



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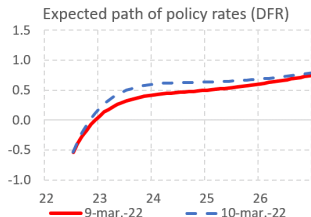
Shift in the monetary policy stance

- Significant increase in both slope and level of the expected path of policy rates during 2022
- Turning point in 2023: historically high level of MP reached but expected to decline in the future

Figure: Level and slope of the expected path of policy rates



Shift in the monetary policy stance: NS model



We apply a Kalman filter for estimating time-varying parameters of the yield curve (following Nelson-Siegel model) where y_τ is the zero-coupon yield with τ days to maturity, β_1 is the long-term level, β_2 is the slope and β_3 is the curvature.

$$y_\tau = \beta_1 + \beta_2 \cdot \frac{(1 - e^{-\lambda\tau})}{\lambda\tau} + \beta_3 \cdot \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau} \right)$$

(4)