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by Christian Ewerhart [2],
Nuno Cassola [3]
and Natacha Valla [4]

In 2005 all ECB publications will feature a motif taken from the €50 banknote.

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Abstract. The fixed rate tender is one of the main procedural formats relied upon by central banks in their implementation of monetary policy. This fact stands in a somewhat puzzling contrast to the prevalent view in the theoretical literature that the procedure, by fixing interest rate and quantity at the same time, does not allow a strategic equilibrium. We show that an equilibrium exists under general conditions even if bidders expect true demand to exceed supply on average. The outcome is typically inefficient. It is argued that the fixed rate tender, in comparison to other tender formats, may be an appropriate instrument for central bank liquidity management when market conditions are sufficiently calm.

JEL classification codes: D44, E52

Keywords: Fixed rate tenders, rationing, equilibrium, inefficiency.

Non-technical summary

The fixed rate tender is one of the main mechanisms used by central banks in their implementation of monetary policy. The Eurosystem, for instance, relied on fixed rate tenders to provide the banking system with cash reserves in its regular open market operations from January 1999 through June 2000. The Bank of England, the Swiss National Bank, the Bundesbank as well as many other central banks have been using fixed rate tenders for many years. More recently, fixed rate tenders have been employed by the Eurosystem also in a number of so-called fine-tuning operations.[3]

Given their pervasive use in the practice of central banking, it is striking that the existing theoretical literature has mostly rejected proportional rationing schemes such as the fixed rate tender on the grounds that a strategic equilibrium may not be feasible (see Bénassy [6], Nautz and Oechssler [22], and Ehrhart [13, 14]). But indeed, when the benchmark rate lies sufficiently above the tender rate, as it may happen, e.g., in the expectation of increasing interest rates, then any marginal increase in the allotment creates a strictly positive profit margin, making it optimal to submit excessively large bids. With this logic being followed by all participants in the tender, there cannot be an equilibrium.

In this paper, we show that a Bayesian equilibrium nevertheless exists in the fixed rate tender even if unconstrained bidders *know* that demand will exceed supply. For a liquidity providing operation, for example, the intuition is that a bidder with a given demand, who is uncertain about the allotment quota, faces a trade-off between obtaining "too little" and "too much" liquidity. Having formed rational expectations about the allotment quota and the uncertainty thereof, each bidder scales her bid optimally to balance this trade-off. It turns out that the fixed rate tender allows a Bayesian equilibrium under quite general conditions provided that market conditions are not too extreme.

We also show that the equilibrium allocation resulting from fixed rate tenders is typically inefficient, so that secondary market trading should be observable after the tender. On the other hand, it is likely that the inefficiency is small when the uncertainty about the allotment quota is limited, which should be the case under "normal" market conditions.

[3] In a fixed rate tender, an interest rate is announced before the auction. Then bidders submit quantity bids, and bids are prorated if total demand exceeds the total amount supplied by the central bank.

Why have fixed rate tenders performed less well in more recent times? As the main explanations for overbidding, the literature has stressed so far interest rate expectations, a potentially tight allotment policy, adaptive behaviour, and the fear of being squeezed in the last tender of a reserve maintenance period. Our analysis suggests two further explanations why fixed rate tenders may have become less successful. Firstly, secondary markets, including markets for collateral have become increasingly sophisticated and efficient. The spread between effective bid and ask quotes may have tightened when compared to, e.g., the situation in the German money market before January 1999. This makes it more likely that either bid quotes lie above the tender rate, causing excessive overbidding and inducing market participants to follow an adaptive disequilibrium behaviour, or ask quotes lie below the tender rate, causing underbidding and an insufficient performance of central bank liquidity management.

Another potential factor suggested by the present analysis is that with aggregate information about liquidity becoming available for market participants, uncertainty about aggregate demand may be significantly reduced. However, this uncertainty is identified as one of the critical conditions for an equilibrium to exist. After all, when there is certainty that demand exceeds supply, there cannot be an equilibrium. Thus, from the perspective of the central bank, this would suggest a case for less transparency about liquidity conditions in the market when the fixed rate tender is employed.

Given that fixed rate tenders allow the central bank to provide a very clear signal about the current monetary policy stance, we conclude that these tenders can indeed be an appropriate instrument for the implementation of monetary policy, provided that market conditions are not too extreme.

1. Introduction

The fixed rate tender is one of the main mechanisms used by central banks in their implementation of monetary policy.[4] The Eurosystem, for instance, relied on fixed rate tenders to provide the banking system with cash reserves in its regular open market operations from January 1999 through June 2000. The Bank of England, the Swiss National Bank, the Bundesbank as well as many other central banks have been using fixed rate tenders for many years. More recently, fixed rate tenders have been employed by the Eurosystem also in a number of so-called fine-tuning operations.[5]

Given their pervasive use in the practice of central banking, it is striking that the existing theoretical literature has mostly rejected proportional rationing schemes such as the fixed rate tender on the grounds that a strategic equilibrium may not be feasible (see Bénassy [6], Nautz and Oechssler [22], and Ehrhart [13, 14]). But indeed, when the market benchmark lies sufficiently above the tender rate, as it may happen, e.g., in the expectation of increasing interest rates, then any marginal increase in the allotment creates a strictly positive profit margin, making it optimal to submit excessively large bids. With this logic being followed by all participants in the tender, there cannot be an equilibrium.[6]

In this paper, we show that a Bayesian equilibrium nevertheless exists in the fixed rate tender even if unconstrained bidders *know* that demand will exceed supply. For a liquidity providing operation, for example, the intuition is that a bidder with a given demand, who is uncertain about the allotment quota, faces a trade-off between obtaining "too little" and "too much" liquidity. Having formed rational expectations about the allotment quota and the uncertainty thereof, each bidder scales her bid optimally to balance this trade-off. It turns out that the fixed rate tender allows a Bayesian equilibrium under quite general conditions provided that market conditions are not too extreme.

[4] According to Bindseil [8], "(n)early all central banks have sometimes used fixed rate tenders and in fact it even seems that the majority of central banks currently prefer it to pure auctions." Other main types of procedures used by central banks are the variable rate tender with either uniform or discriminatory pricing rule, which have been studied first in the context of oil leases by Wilson [29].

[5] In a fixed rate tender, an interest rate is announced before the auction. Then bidders submit quantity bids, and bids are prorated if total demand exceeds the total amount supplied by the central bank.

[6] Drazen [12] provides an insightful survey of traditional disequilibrium theory. Academic interest in quantity rationing has been renewed in particular by the conference on central bank operations hosted by the Deutsche Bundesbank in Frankfurt in September 2000 (see von Hagen [28]).

The two main conditions that will be imposed on market demand in order to ensure existence may look familiar to the central banker with a background in operational issues. The first requirement, very weak in our view, will be that the central bank's forecasting of the true demand underlying the tender is necessarily imperfect. Thus, there may be a small probability that the supply of reserves exceeds the actual liquidity deficit in the market to a non-marginal extent.

The other requirement will be that an individual counterparty (i.e., a commercial bank participating in the tender) expects aggregate true demand for liquidity to be on average not much higher than supply. In our view, this assumption is very plausible in markets for interbank liquidity, in which many transactions just transfer money from one bank to another, keeping the aggregate liquidity position of the banking system unaffected. Transactions that affect the liquidity position of the banking system (i.e., autonomous factors) can be forecasted as an aggregate in a more or less reliable way by the central bank, which adjusts supply accordingly. As a consequence, counterparties can expect supply to cover, on average, a significant fraction of the aggregate demand.

When these two conditions are satisfied, the fixed rate tender allows a Bayesian equilibrium in which excess demand translates in a straightforward way into larger and more variable bids and consequently, into smaller and less predictable allotment quotas. Even though a market price cannot be discovered with a fixed rate tender, still some implicit form of information aggregation appears to take place. Indeed, while higher bids indicate a larger gap between demand and supply, the uncertainty about the extent of overbidding replaces the rationing function usually assigned to the market price.

It will not surprise the reader that the equilibrium allocation resulting from fixed rate tenders is typically inefficient. Typically means here under the mild condition that true demand exceeds supply with strictly positive probability. The inefficiency arises because the individual bidder, as a consequence of individual rationality, must obtain "too much" liquidity with strictly positive probability. But this is inefficient!

We are not first in discussing bidding behaviour in fixed rate tenders. In a seminal contribution, Ayuso and Repullo [3] explain the overbidding observed in the Eurosystem over the period January 1999 through June 2000 as a consequence of an asymmetric objective function for the central bank. As market rates go up in response to a tight allotment policy, bidders submit increasingly higher bids to work against the rationing. However, with bids reaching excessively high levels, the submission of a bid that exceeds the given stock of collateral incurs the risk of being penalised by the central bank, so that an equilibrium can be obtained. Ayuso and Repullo's model differs from our model by its focus on the case of excessive overbidding, which implies a cost even for the announcement of bids.

Bindseil [8] provides an excellent survey of the experience with fixed rate tenders by modern central banks, stressing in particular the case of the Eurosystem. He also analyses the aggregate behaviour of a banking system facing a cost of bidding that depends on the total bid.[7] Välimäki [27] assumes that a bank may have to pay a two-part penalty consisting of a rate on missing collateral and a fixed amount for non-compliance. He then studies the decision of an individual bank to bid optimally against a given probability distribution of aggregate bids submitted by the other banks.

Ehrhart [14] extends and refines the Bénassy-Nautz-Oechssler non-existence result in several directions, analysing in particular the case of repeated interaction. Most relevantly for the present analysis, the paper also contains a numerical example of an equilibrium with uncertainty about supply which entails a similar strategic reasoning on the part of the individual bidder as suggested by the present analysis. It will be noted, however, that in Ehrhart's approach, the basic uncertainty about the allotment quota is caused by uncertainty about supply, while in our model this uncertainty is caused by uncertainty about the demand of other bidders.[8]

The remainder of the paper is structured as follows. Section 2 introduces the basic model. Section 3 discusses existence of equilibrium in fixed rate tenders. Efficiency is treated in Section 4. Section 5 presents two tractable examples. Section 6 concludes. The Appendix contains formal proofs of Theorems 1 and 2, as well as some technical material used in the discussion of the examples.

[7] See also Bindseil [7].

[8] The rationing game is also discussed in the literature on supply chain management, where it arises in a natural way when several independent retailers send their orders to a common supplier. Lee, Padmanabhan, and Whang [17] describe an equilibrium in a model with exogenous cost functions and perfect information.

2. The model

A central bank wishes to distribute a given quantity of a perfectly divisible good, which is normalised to one for notational convenience. There are $n \geq 2$ counterparties in the market. The preferences of an individual counterparty i may depend on a type parameter θ_i, which is assumed to be observable only by counterparty i. It is common knowledge, however, that the θ_i are drawn ex ante from a set $\Theta_i = [0; \overline{\theta}_i]$ for some $\overline{\theta}_i \in (0; 1)$, according to a joint probability distribution μ, which is assumed to possess a strictly positive density on the product set $\Theta = \Theta_1 \times ... \times \Theta_n$. A counterparty i of type θ_i maximizes a utility function

$$U_i(q_i, t_i, \theta_i) = \int_0^{q_i} v_i(x_i, \theta_i) dx_i - t_i,$$

where q_i is the quantity obtained, $v_i(x_i, \theta_i)$ is the marginal valuation of bidder i with type θ_i at quantity q_i, and t_i is the transfer paid by bidder i. We will assume that $v_i(x_i, \theta_i)$ is continuously differentiable on $\mathbb{R}_+ \times \Theta_i$, where $\partial v_i / \partial q_i < 0$.[9] It will be noted without difficulty that the framework is one of private values (each bidder knows his valuation function), in which values are not necessarily independent, and in which bidders may be heterogeneous ex ante.

The counterparties participate in a fixed rate tender. The central bank announces that the good will be sold at a price p_0. The working of the mechanism is then as follows. First, counterparties submit nonnegative bids $b_i(\theta_i) \geq 0$. The total of incoming bids amounts to

$$b(\theta_1, ..., \theta_n) = \sum_{i=1}^{n} b_i(\theta_i).$$

Proportional rationing is applied when aggregate demand exceeds supply. Thus, if $b(\theta_1, ..., \theta_n) \leq 1$, then counterparty i obtains a quantity

$$q_i(\theta_1, ..., \theta_n) = b_i(\theta_i)$$

equal to the submitted bid. However, if the total of incoming bids exceeds the supply of one unit, i.e., if $b(\theta_1, ..., \theta_n) > 1$, then bids are prorated, and counterparty i obtains

$$q_i(\theta_1, ..., \theta_n) = \frac{b_i(\theta_i)}{b(\theta_1, ..., \theta_n)}.$$

[9] In the case of a liquidity providing operation of the Eurosystem, decreasing marginal valuations may result from various factors. First, opportunity costs of collateral vis-á-vis the central bank may be increasing. Second, the eligibility criteria imposed on interbank collateral may differ from the criteria imposed on central bank collateral. Finally, a commercial bank may attach a premium to interbank lending, either in terms of perceived risks or in terms of a regulatory opportunity cost (see Bindseil, Weller, and Wuertz [9]).

The gross transfer paid by counterparty i to the central bank is in both cases given by

$$t_i(\theta_1, ..., \theta_n) = p_0 q_i(\theta_1, ..., \theta_n).$$

Given these rules of the rationing game, it is clear that the bidders' marginal valuations must satisfy a number of restrictions to make the problem interesting.

Specifically, we will assume that $v_i(0, \theta_i) > p_0$ for all counterparties i and for all types $\theta_i > 0$. Without this assumption, type θ_i of counterparty i has a dominant strategy of not participating. Similarly, we assume that $v_i(q_i, \theta_i) < p_0$ for q_i sufficiently large. Without this assumption, the decision problem for the individual counterparty may not be well-defined. As $v_i(q_i, \theta_i)$ is strictly decreasing in q_i, there is a well-defined quantity q_i such that $v_i(q_i, \theta_i) = p_0$. We will refer to this quantity as the *demand* of type θ_i. To ease the exposition, it will be imposed that for any given $q_i \geq 0$, there is at most one type with demand $\theta_i = q_i$. We may then rename the types without loss of generality, so that $v_i(\theta_i, \theta_i) = p_0$. Following from this convention, type and demand of a counterparty are two words with the same meaning.[10]

As discussed in the Introduction, the existence of an equilibrium may not be guaranteed in fixed rate tenders when aggregate demand is both strong and deterministic. This well-known result generalises in a straightforward way to a set-up with incomplete information about demand.[11] Nautz and Oechssler [22] argue convincingly that adaptive behaviour may replace rational behaviour when the circumstances of the tender exclude the possibility of equilibrium behaviour. Having pointed out that the non-existence result stands in a somewhat puzzling contrast to the widespread use of the fixed rate tender by central banks, we will show now that an equilibrium may indeed exist when market demand is "close to balanced".

3. Existence

The approach followed in the proof of the existence theorem is to focus on an equilibrium candidate in which there is an upper bound on the extent of overbidding. Specifically, we will assume that bidder i of type θ_i is considering

[10] The results of this paper do not appear to depend on this assumption.
[11] If needed, a formal statement and proof can be obtained from the authors.

submitting a bid $b_i = b_i(\theta_i)$, assuming that the bids of the other counterparties $j \neq i$ satisfy

$$b_j(\theta_j) \leq \alpha\theta_j \tag{1}$$

for some given overbidding factor $\alpha > 1$. Then, provided that bids will be prorated, bidder i will receive a share of

$$q_i(\theta) = \frac{b_i}{b_i + \sum_{j\neq i} b_j(\theta_j)} \geq \frac{b_i}{b_i + \alpha\sum_{j\neq i}\theta_j}.$$

As marginal valuations are strictly declining, the inequality allows to put an upper bound on the "marginal loss" that an individual counterparty must accept in the case when she obtains too little liquidity. Under certain assumptions discussed below, this type of argument allows to derive that also bidder i does not exaggerate her true demand by a factor of more than α, i.e.,

$$b_i(\theta_i) \leq \alpha\theta_i. \tag{2}$$

In this case, the extent of overbidding finds a finite limit, leading to the existence theorem stated below.

The result relies on two main assumptions. The first assumption is that there is enough uncertainty about the true demand of the other bidders. Formally, let $\Theta_{-i} = \Theta_1 \times ... \times \Theta_{i-1} \times \Theta_{i+1} \times ... \times \Theta_n$. For $\theta_i \in \Theta_i$ and $q \in (0;1)$ denote by

$$\Theta^q_{-i}(\theta_i) = \{\theta_{-i} \in \Theta_{-i} | \sum_{j=1}^{n} \theta_j \leq q\}$$

the set of all type vectors θ_{-i} so that aggregate demand is less than q. We say that *forecasting is imperfect* if there is a $q \in (0;1)$ and an $\varepsilon > 0$ such that

$$\int_{\Theta^q_{-i}(\theta_i)} \sum_{j\neq i} \theta_j d\mu(\theta_{-i}|\theta_i) \geq \varepsilon \tag{3}$$

for each counterparty i and for each type θ_i. This condition says intuitively that conditional on aggregate demand being low, the expected demand of the other counterparties never becomes negligible, uniformly over counterparty i's demand.

Such an assumption is not implausible in a central bank context. It is also an intuitive condition for an equilibrium to exist. After all, in the absence of uncertainty about true demand, each counterparty could perfectly predict the bids submitted by the other bidders. An equilibrium can then exist only when there is common knowledge that supply is ample enough to satisfy demand. The uncertainty about the true demand of the other bidders implies an uncertainty of the individual bidder as to the extent to which his strategic bid will be rationed. With decreasing marginal valuations, this implies a cost to excessive overbidding.

The second main assumption is that on average, true demand must not exceed supply by too much. Formally, denote by

$$\widehat{\Theta}_{-i}(\theta_i) = \Theta_{-i} \setminus \Theta_{-i}^1(\theta_i) = \{\theta_{-i} \in \Theta_{-i} | \sum_{j=1}^{n} \theta_j > 1\}$$

the set of all type vectors θ_{-i} such that aggregate demand exceeds supply. We say that *demand is balanced* if there is a sufficiently small $\delta > 0$ such that

$$\int_{\widehat{\Theta}_{-i}(\theta_i)} (\sum_{j=1}^{n} \theta_j - 1) d\mu(\theta_{-i}|\theta_i) \leq \delta$$

for each i and for each θ_i. It will become clear that this condition allows for the interesting case that expected demand is known to be higher than supply. Such a situation is feasible, e.g., if the forecasting of autonomous liquidity factors is subject to a misspecification, or if the central bank submits consistently too little liquidity, as suggested by the analysis of Ayuso and Repullo [3]. Whatever the precise interpretation, the relative generality allowed by the second assumption should in any case add a significant degree of robustness to prior existence results that relied on the assumption that supply exceeds demand with probability one.

Theorem 1. *Assume that forecasting is imperfect and that demand is balanced. Then the fixed rate tender allows a Bayesian equilibrium, which is possibly in mixed strategies.*

A formal proof can be found in the Appendix. Theorem 1 offers a rationale for the use of fixed rate tenders with proportional quantity rationing in the practice of contemporaneous central banking. In fact, the balancedness assumption suggests why we observe the fixed rate procedure especially in central bank liquidity management. After all, when the central bank aims at neutralising liquidity fluctuations between the banking sector and the remaining part of the economy, then the demand structure in the banking sector is captured in a rather intuitive way by the balancedness criterion.[12]

[12] Theorem 1 may also shed light on the fact that the fixed rate tender format has not been used by the Federal Reserve System. Given that the Fed faces a seasonal demand for reserves and implements an explicit interest rate target, the balanced demand assumption is unlikely to be satisfied.

4. Efficiency

It is generally perceived that rationing generates inefficient allocations. Indeed, when both price and quantity are held fixed at the same time, there is no obvious mechanics by which demand and supply should be matched.[13] As the previous section has shown, this general argument is somewhat qualified in the presence of incomplete information. When demand is not deterministic, the scarcity of supply in relation to market demand will be reflected by the extent of overbidding, and therefore in the more pronounced trade-off between obtaining too much and too little of the good. Thus, even though the tender price does not increase in response to stronger demand, so does the expected variability of the allotment quota, and therefore also the cost of overstating demand in the bid. Why then do we obtain an inefficient allocation? The point is that in equilibrium, the individual bidder has to be uncertain about the resulting allocation. With strictly positive probability, the allotment will be larger than desired. As will be argued below, this drives the inefficiency.

The definition of allocative efficiency is repeated here for the convenience of the reader. An ex-post allocation $q = (q_1, ..., q_n)$ is *feasible* if $q_i \geq 0$ for all i and $\sum_{i=1}^n q_i \leq 1$. Denote by

$$W(q, \theta) = \sum_{i=1}^{n} \{ \int_0^{q_i} v_i(x_i, \theta_i) dx_i - p_0 q_i \}$$

the welfare associated with an ex-post allocation q in a state θ. The reader will note that no positive welfare is associated with any fraction of the good potentially left with the auctioneer, e.g., following an episode of insufficient demand. A feasible ex-post allocation q is *efficient* if it maximises the welfare functional under the feasibility constraint. The inefficiency of the fixed rate tender can now be stated without further assumptions as follows.

Theorem 2. *Assume that* $\sum_{i=1}^n \theta_i > 1$ *with strictly positive probability. Then any Bayesian equilibrium of the fixed-price tender is ex-post inefficient.*

The proof is in the Appendix. The Theorem says that the outcome of the fixed rate tender is typically inefficient. Trading in the secondary markets in response to allotment decisions should therefore be observable. The intuitive reason for

[13] Bindseil [8] suitably compares the problems created by extreme forms of overbidding with the inefficiencies arising from queuing.

the inefficiency is reflected in the trade-offs that underlie the preparation of the bid. An efficient allocation never allocates too much of the good to an individual bidder. However, as explained in the previous section, this feature is inconsistent with the individual profit maximisation of the individual bidder in the relevant scenario where demand exceeds supply with positive probability. In equilibrium, the individual counterparty must be uncertain about whether the allotment will be higher or lower than her demand at the tender rate. For example, if the counterparty knew with certainty that the allotment will exceed her demand, then she would downsize the bid correspondingly. Similarly, if the counterparty knew with certainty that the allotment will be lower than her demand, then the bid should be increased. This simple argument shows that the equilibrium allocation in a fixed rate tender will always be inefficient unless it is obvious that rationing does not occur.

In our view, the inefficiency identified in Theorem 2 does not make fixed rate tenders an inappropriate instrument for central bank liquidity management. While the procedure definitely leads to inefficient outcomes, the extent of these inefficiencies may be small when the uncertainty about the allotment quota is limited, as should be the case under "normal" market conditions. Moreover, the extent of the inefficiency may be smaller than the inefficiencies arising from alternative auction formats, such as the variable rate tender with either uniform or discriminatory pricing. After all, both the uniform and the discriminatory pricing rules are known to cause differential incentives for bid shading and thereby an inefficiency. Moreover, this inefficiency may be significant if the population of bidders is either small or, as in the case of the Eurosystem, markedly heterogeneous.[14] It should also be noted that the main criticism from the market side about the use of fixed rate tenders during the episode of extreme overbidding in the Eurosystem seemingly has been that the unequal situation regarding eligible collateral across countries of the euro area implied "unjust" advantages for some counterparties. We will discuss further advantages and disadvantages of fixed rate tenders in the conclusion.

5. Examples

Mainly for illustrative purposes, this section develops two simple set-ups in which Bayesian equilibrium strategies can be computed in an explicit fashion.

[14] See Ausubel and Cramton [1], Back and Zender [4], Engelbrecht-Wiggans and Kahn [15, 16], and Swinkels [25, 26].

The first set-up can be interpreted as a situation in which marginal valuations are derived from the possibility for counterparties to trade in a secondary market in the presence of non-trivial transaction costs. The second set-up entails the somewhat unexpected feature that an equilibrium can be obtained even when there is common knowledge among the bidders that bids will be prorated.

Bid-ask spreads. There are $n \geq 2$ counterparties. For every counterparty i, there are two types $\underline{\theta}_i < \overline{\theta}_i$. Moreover, $0 < \underline{\theta}_i < 1/n$ for $i = 1, ..., n$. To obtain an equilibrium it must be imposed that for all bidders i, the conditional probability $\pi(\overline{\theta}_{-i}|\overline{\theta}_i)$ that all types are high given that counterparty i is of the high type, is assumed to be neither too small nor too large.[15] Marginal valuations are given by

$$v(q_i, \theta_i) = \begin{cases} p_a^i & \text{if } q_i \leq \theta_i \\ p_b^i & \text{if } q_i > \theta_i, \end{cases}$$

for prices $p_a^i > p_0^i = p_0 + \varepsilon_i > p_b^i$. The constant $\varepsilon_i \geq 0$ can be interpreted as idiosyncratic transaction costs, and can be set to zero. In the context of open market operations, the assumptions on the valuations may be interpreted in the sense that an individual bank faces a strictly positive spread between lending and deposit rates, with ε_i representing roughly the individual opportunity costs of collateral.

Consider an equilibrium candidate in which only the high types overbid, and in which rationing occurs only when all counterparties are of the high type, i.e.,

$$b_i(\underline{\theta}_i) + \sum_{j \neq i} b_j(\overline{\theta}_j) < 1 < b_i(\overline{\theta}_i) + \sum_{j \neq i} b_j(\overline{\theta}_j) \tag{4}$$

for all i. Under these conditions, only high types bother to overbid. This feature of the example allows deriving an explicit expression for equilibrium demand. Specifically,

$$\sum_{i=1}^{n} b_i(\overline{\theta}_i) = \frac{n-1}{\sum_{i=1}^{n} \vartheta_i} \tag{5}$$

is the aggregate demand of high types, where

$$\vartheta_i = \frac{1 - \pi(\overline{\theta}_{-i}|\overline{\theta}_i)}{\pi(\overline{\theta}_{-i}|\overline{\theta}_i)} \frac{p_0^i - p_b^i}{p_a^i - p_0^i}. \tag{6}$$

Equation (5) implies a straightforward comparative statics with respect to the parameters characterising the bidding environment of any counterparty i. E.g.,

[15] The reader is referred to the Appendix for the precise form of the assumptions needed to sustain the equilibrium.

when counterparty i finds it increasingly difficult to find access to market funding due to a lowered credit rating, i.e., when p_a^i increases, then funding through central bank operations becomes increasingly attractive for i and aggregate demand goes up. Similarly, when counterparty i, maybe in response to a higher perceived uncertainty about overall financial stability, assigns a higher cost to deposits, e.g., by reducing risk limits for unsecured loans extended in the interbank market, then p_b^i increases, excess liquidity becomes undesirable, and consequently aggregate demand decreases. If transaction costs ε_i increase for some i, demand declines as well. Finally, if an individual counterparty's demand is stronger positively correlated with aggregate demand, i.e., if $\pi(\overline{\theta}_{-i}|\overline{\theta}_i)$ increases ceteris paribus, then demand will increase due to the higher expected rationing.

Always-rationing equilibria. Our second example illustrates the possibility that an equilibrium may exist even if the probability of rationing is one. There are two bidders. Marginal valuations for $q_i < 1$ are given by

$$v_i(q_i,\theta_i) = p_0 + \frac{\theta_i - q_i}{q_i^2(1-q_i)}. \qquad (7)$$

Types are independently distributed on the interval

$$\Theta_i = [\frac{1}{\mu+1}; \frac{\mu}{\mu+1}]$$

for some constant $\mu > 1$, according to the triangular density

$$g(\theta_i) = \frac{2(\mu+1)(1-\theta_i)}{\mu-1}.$$

With these specifications, there is an equilibrium in which type θ_i submits a bid

$$b_i(\theta_i) = \mu\frac{\theta_i}{1-\theta_i}.$$

In this particular case, marginal valuations fall quickly, and the probability of high types, who overbid more excessively, is comparably low, which allows an equilibrium. The details of the derivation can be found in the Appendix.

6. Concluding remarks

Fixed rate tenders are one of the main procedures by which central banks may seek control of liquidity conditions in the interbank market for overnight deposits. The use of this tender format has recently come under criticism in response to an episode of increasing and ultimately excessive overbidding in the euro area during the initial phase of Stage III of EMU. Specifically, it had been argued in the literature that tenders with a posted price are inconsistent with equilibrium behaviour on the part of counterparties participating in the tender, and that the fixed rate tender format is consequently not an appropriate instrument for the implementation of monetary policy.

In the formal analysis, we have shown that fixed rate tenders may indeed allow equilibrium behaviour provided that counterparties possess private information and market conditions are sufficiently calm. Specifically, we have offered a simple model in which bidders with quantity demand face an uncertainty about the allotment quota, giving rise to a trade-off between obtaining "too little" and "too much" liquidity. Thus, in our model, the limiting effect on bids is caused by the fact that with a certain probability, demand by the other bidders will be weak, and the allotment may turn much larger than needed.[16]

In addition, we showed in the framework of the model that, as a consequence of the demand uncertainty necessary to sustain equilibrium behaviour, the outcome of the fixed rate tender is typically inefficient. But we also argued that these inefficiencies may be small under "normal" market conditions, and would therefore play a subordinate role for the regular implementation of monetary policy.

It is clear that a single model cannot capture the full list of pros and cons that ultimately determine the central bank's choice of a specific procedural format. Other factors influencing this decision may include, but are not limited to, the extent to which a tender signals the current stance of monetary policy, the extent to which quantitative objectives can be implemented, the principle of equal treatment vis-à-vis individual counterparties, as well as informational efficiency. The experience in the Eurosystem suggests that, among the various procedures in use, fixed rate tenders perform optimally with respect to the signaling function, and maybe less optimally with respect to some of the other objectives that may be pursued with an individual operation. E.g., during the period of excessive overbidding, counterparties with limited access to eligible collateral may have been at a disadvantage compared to other bidders. Moreover, in recent

[16] Seller discretion and uncertainty about supply, especially towards the upside, should further stabilise bidding behaviour. Related arguments have been made in a more auction theoretic context (see Lengwiler [18], Back and Zender [5], LiCalzi and Pavan [19], Damianov [10], and McAdams [20]).

uses of fixed rate tenders for liquidity absorbing operations, the total of bids has on some occasions not reached the benchmark allotment.

Why have fixed rate tenders performed less well in more recent times? As the main explanations for overbidding, the literature has stressed so far interest rate expectations, a potentially tight allotment policy, adaptive behaviour, and the fear of being squeezed in the last tender of a reserve maintenance period.[17] Our analysis suggests two further explanations why fixed rate tenders may have become less successful. Firstly, secondary markets, including markets for collateral have become increasingly sophisticated and efficient. The spread between effective bid and ask quotes may have tightened when compared to, e.g., the situation in the German money market before January 1999. This makes it more likely that either bid quotes lie above the tender rate, causing excessive overbidding and inducing market participants to follow an adaptive disequilibrium behaviour, or ask quotes lie below the tender rate, causing underbidding and an insufficient performance of central bank liquidity management.[18]

Another potential factor suggested by the present analysis is that with forecasting becoming increasingly precise, and more and more information about aggregate liquidity conditions being provided to the market, the uncertainty about aggregate demand may be significantly reduced. However, this uncertainty has been identified as one of the critical conditions for an equilibrium to exist. Thus, from the perspective of the central bank, this would suggest a case for less transparency about liquidity conditions in the market when the fixed rate tender is employed.

While the fixed rate tender may be inappropriate under special circumstances as identified by previous research, the results obtained in this paper suggest that when market conditions are "normal", the procedure may indeed work quite smoothly. As a consequence, given that the signalling function may occasionally dominate the other objectives, we conclude that fixed rate tenders, at least under sufficiently calm market conditions, can indeed be an appropriate instrument for the implementation of monetary policy.

[17] See Ayuso and Repullo [2], Nautz and Oechssler [23], and Nyborg and Strebulaev [24]. An additional role may have played the fact that the Bundesbank [11] still required bids to be collateralised.

[18] The break-down of an equilibrium in a mechanism with quantity rationing under a more efficient secondary market has been conjectured already in Bénassy's (1977) work on neo-Keynesian price rigidities.

Appendix

Proof of Theorem 1. Consider a profile of measurable bidding strategies $\{b_i(.)\}_{i=1,\ldots,n}$ in the fixed rate tender. It is clear that a type $\theta_i = 0$ cannot gain from submitting a strictly positive bid. On the other hand, no type θ_i rationally submits a bid $b_i < \theta_i$. We may therefore assume in the sequel that $\theta_i > 0$ and that $b_i > 0$ without making additional arguments. The expected utility of a bidder i of type θ_i is given by

$$\Pi_i(b_i, \theta_i) = \int_{\Theta_{-i}} \int_0^{q_i(\theta_i, \theta_{-i})} \{v_i(x_i, \theta_i)dx_i - t_i(\theta_i, \theta_{-i})\} d\mu(\theta_{-i}|\theta_i),$$

where b_i is the bid. Write

$$b_{-i}(\theta_{-i}) = \sum_{j \neq i} b_j(\theta_j) \tag{8}$$

for the aggregate bid of bidders $j \neq i$, and

$$\widehat{q}_i(b_i, b_{-i}) = \begin{cases} b_i & \text{if} \quad b_i + b_{-i} \leq 1 \\ \dfrac{b_i}{b_i + b_{-i}} & \text{if} \quad b_i + b_{-i} > 1 \end{cases}$$

for the rationing rule. Then, by simple substitution,

$$\Pi_i(b_i, \theta_i) = \int_0^{\infty} \int_0^{\widehat{q}_i(b_i, b_{-i})} \{v_i(x_i, \theta_i)dx_i - p_0 \widehat{q}_i(b_i, b_{-i})\} dF_{\theta_i}(b_{-i}), \tag{9}$$

where $F_{\theta_i}(.)$ is the cumulative distribution function of the random variable b_{-i} defined by (8). Let $Z(b_i) = \{1 - b_i\}$ denote the zero set where the map $b_i \rightarrow \widehat{q}_i(b_i, b_{-i})$ is not differentiable. As $\widehat{q}_i(b_i, b_{-i})$ is a continuous function of b_{-i}, and the point set $Z(b_i)$ varies in a differentiable way with b_i, one may apply Leibnitz' rule to obtain

$$\frac{\partial \Pi_i}{\partial b_i}(b_i, \theta_i) = \int_{[0;\infty) \setminus Z(b_i)} \{v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i) - p_0\} \frac{\partial \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) dF_{\theta_i}(b_{-i}). \tag{10}$$

Decomposing the right-hand side of (10) according to whether the counterparty ends up with "too little" or "too much" liquidity yields the first-order condition

$$\underbrace{\int_{[0;b^0_{-i}) \setminus Z(b_i)} \{p_0 - v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i)\} \frac{\partial \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) dF_{\theta_i}(b_{-i})}_{\text{"too much"}}$$

$$= \underbrace{\int_{(b^0_{-i};\infty)} \{v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i) - p_0\} \frac{\partial \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) dF_{\theta_i}(b_{-i})}_{\text{"too little"}}, \tag{11}$$

where

$$b_{-i}^0 = b_i \frac{1-\theta_i}{\theta_i}$$

is the aggregate bid of the other bidders that implies an allotment of $q_i = \theta_i > 0$ to counterparty i. We will now assume that (1) is satisfied for all counterparties $j \neq i$. We claim that *for $b_i > \alpha\theta_i$, the left-hand side (LHS) of the first-order condition (11) exceeds the right-hand side (RHS).*

RHS. The function $\partial v_i/\partial q_i$ is continuous on the closed and bounded set

$$\Omega_i = \{(q_i, \theta_i)|0 \leq q_i \leq \theta_i \text{ and } \theta_i \in \Theta_i\},$$

so that, by Weierstrass' theorem, there is a constant $\lambda > 0$ such that

$$\frac{\partial v_i}{\partial q_i}(q_i, \theta_i) \geq -\lambda \tag{12}$$

for all $(q_i, \theta_i) \in \Omega_i$. Figure 1 illustrates the intuitive meaning of the constant λ. As a consequence of (12),

$$v_i(q_i, \theta_i) - p_0 = v_i(q_i, \theta_i) - v_i(\theta_i, \theta_i) \leq \lambda(\theta_i - q_i) \tag{13}$$

for any $(q_i, \theta_i) \in \Omega_i$. Substituting q_i by $\widehat{q}_i(b_i, b_{-i})$ in (13) yields

$$v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i) - p_0 \leq \lambda(\theta_i - \widehat{q}_i(b_i, b_{-i})) \tag{14}$$

for all $b_{-i} \geq b_{-i}^0$. Since the right-hand side of (14) is concave in b_{-i},

$$\theta_i - \widehat{q}_i(b_i, b_{-i}) \leq (b_{-i} - b_{-i}^0) \left.\frac{\partial}{\partial b_{-i}}\right|_{b_{-i}=b_{-i}^0} \{\theta_i - \widehat{q}_i(b_i, b_{-i})\}$$
$$= (b_{-i} - b_{-i}^0)\frac{\theta_i^2}{b_i}.$$

Thus, for all $b_{-i} \geq b_{-i}^0$,

$$0 \leq v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i) - p_0 \leq \frac{\lambda\theta_i^2}{b_i}(b_{-i} - b_{-i}^0).$$

Moreover, again for $b_{-i} \geq b_{-i}^0$,

$$\frac{\partial\widehat{q}_i}{\partial b_i}(b_i, b_{-i}) = \frac{b_{-i}}{(b_i + b_{-i})^2} \leq \frac{1}{b_i + b_{-i}} \leq \frac{1}{b_i + b_{-i}^0} = \frac{\theta_i}{b_i}.$$

Thus, using $b_i > \alpha\theta_i$, one finds

$$
\begin{aligned}
\text{RHS} &= \int_{b^0_{-i}}^{\infty} \{v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i) - p_0\} \frac{\partial \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) dF_{\theta_i}(b_{-i}) \\
&\leq \frac{\lambda\theta_i^3}{b_i^2} \int_{b^0_{-i}}^{\infty} (b_{-i} - b_i \frac{1-\theta_i}{\theta_i}) dF_{\theta_i}(b_{-i}) \\
&\leq \frac{\lambda\theta_i^3}{b_i^2} \int_{b_{-i}(\theta_{-i}) \geq b^0_{-i}} (\alpha \sum_{j \neq i} \theta_j - b_i \frac{1-\theta_i}{\theta_i}) d\mu(\theta_{-i}|\theta_i) \\
&\leq \frac{\alpha\lambda\theta_i^3}{b_i^2} \int_{b_{-i}(\theta_{-i}) \geq b^0_{-i}} (\sum_{j=1}^{n} \theta_j - 1) d\mu(\theta_{-i}|\theta_i) \\
&= \frac{\alpha\lambda\theta_i^3}{b_i^2} \int_{b_{-i}(\theta_{-i}) \geq b^0_{-i}} \max\{0; \sum_{j=1}^{n} \theta_j - 1\} d\mu(\theta_{-i}|\theta_i) \\
&\leq \frac{\alpha\lambda\theta_i^3}{b_i^2} E_{\theta_i}[\max\{0; \sum_{j=1}^{n} \theta_j - 1\}] \\
&\leq \frac{\alpha\lambda\theta_i^3\delta}{b_i^2},
\end{aligned}
$$

where we have used that demand is balanced.

LHS. If the imperfect forecasting condition (3) is satisfied for some q and some ε, then it is also satisfied for any $q' > q$, and the same ε. Without loss of generality, one may therefore assume that

$$q > \max\{\overline{\theta}_1, ..., \overline{\theta}_n\}$$

and

$$q > \frac{1}{\alpha}. \tag{15}$$

As the function $\partial v_i / \partial q_i$ is continuous and strictly negative on the compact set

$$\Omega'_i = \{(q_i, \theta_i) | \theta_i \leq q_i \leq \frac{\theta_i}{q} \text{ and } \theta_i \in \Theta_i\},$$

there is a constant $\beta > 0$, independent of i and θ_i, such that

$$\frac{\partial v_i}{\partial q_i}(q_i, \theta_i) \leq -\beta$$

for all $(q_i, \theta_i) \in \Omega'_i$, as suggested by Figure 1. Let

$$b^q_{-i} = b^q_{-i}(b_i) = b_i \frac{q - \theta_i}{\theta_i} > \max\{0; 1 - b_i\}$$

denote the aggregate bid by bidders $j \neq i$ such that the allotment for bidder i is θ_i / q. Then, as marginal valuations are decreasing, and because $\widehat{q}_i(b_i, b_{-i})$ is nonincreasing in b_{-i}, one obtains for $b_{-i} \leq b^q_{-i}$ that

$$\begin{aligned} p_0 - v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i) &\geq p_0 - v_i(\widehat{q}_i(b_i, b^q_{-i}), \theta_i) \\ &= v_i(\theta_i, \theta_i) - v_i(\frac{\theta_i}{q}, \theta_i) \\ &\geq \frac{1-q}{q}\theta_i \beta \end{aligned}$$

See Figure 2 for an illustration. Moreover, for $b_{-i} \leq b^q_{-i}$,

$$\frac{b_{-i}}{(b_i + b_{-i})^2} = (\frac{b_i}{b_i + b_{-i}})^2 \frac{b_{-i}}{b_i^2} \geq (\frac{\theta_i}{b_i q})^2 b_{-i}.$$

Hence

$$\frac{\partial \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) \geq (\frac{\theta_i}{b_i q})^2 b_{-i}. \tag{16}$$

for $1 - b_i < b_{-i} \leq b^q_{-i}$. But inequality (16) is also satisfied when $b_{-i} \leq 1 - b_i$ provided that (15) holds, because in this case

$$\frac{\partial \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) = 1 \geq \frac{b_{-i}}{(\alpha q)^2} \geq (\frac{\theta_i}{b_i q})^2 b_{-i}.$$

Thus,

$$\begin{aligned} \text{LHS} &= \int_{[0;b^0_{-i}]\setminus Z(b_i)} \{p_0 - v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i)\} \frac{\partial \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) dF_{\theta_i}(b_{-i}) \\ &\geq \int_{[0;b^q_{-i}]\setminus Z(b_i)} \{p_0 - v_i(\widehat{q}_i(b_i, b_{-i}), \theta_i)\} \frac{\partial \widehat{q}_i}{\partial b_i}(b_i, b_{-i}) dF_{\theta_i}(b_{-i}) \\ &\geq \frac{(1-q)\theta_i^3 \beta}{q^3 b_i^2} \int_0^{b^q_{-i}} b_{-i} dF_{\theta_i}(b_{-i}) \end{aligned}$$

But it is straightforward to check that

$$\text{if } \sum_{j=1}^{n} \theta_j \leq q \text{ then } b_{-i}(\theta_{-i}) \leq b^q_{-i}.$$

Thus, because forecasting is imperfect,

$$\begin{aligned} \text{LHS} &\geq \frac{(1-q)\theta_i^3 \beta}{q^3 b_i^2} \int_{\Theta^q_{-i}(\theta_i)} b_{-i}(\theta_{-i}) d\mu(\theta_{-i}|\theta_i) \\ &\geq \frac{(1-q)\theta_i^3 \beta}{q^3 b_i^2} \int_{\Theta^q_{-i}(\theta_i)} \sum_{j \neq i} \theta_j d\mu(\theta_{-i}|\theta_i) \\ &\geq \frac{(1-q)\theta_i^3 \beta}{q^3 b_i^2} \varepsilon. \end{aligned}$$

For

$$\frac{1}{q} \leq \alpha \leq \frac{(1-q)\varepsilon\beta}{\lambda \delta q^3},$$

which can be satisfied for some α if δ is not too large, this implies that (10) is negative for all $b_i > \alpha\theta_i$. As (10) is strictly positive for type $\theta_i > 0$ and bid $b_i = 0$, bidder i with type θ_i will bid at most $\alpha\theta_i$. Thus, the existence problem of the fixed rate tender is reduced to the problem of finding an equilibrium of the Bayesian game in which each bidder i of type $\theta_i \in \Theta_i$ chooses a multiplier $\alpha_i^*(\theta_i) \in [1; \alpha]$ corresponding to a bid $b_i^*(\theta_i) = \alpha_i^*(\theta_i)\theta_i$. The assertion of the Theorem follows then from a standard existence result for Bayesian games with compact strategy sets and continuous utility functions (see Milgrom and Weber [21], Theorem 1 in combination with Proposition 3). □

Proof of Theorem 2. Consider a Bayesian equilibrium $\{b_i^*(.)\}_{i=1,...,n}$. Ignoring the zero set on which the rationing rule is only continuous, but not differentiable, the necessary first-order condition for bidder i of type θ_i reads

$$\int_{\Theta_{-i}} \{v_i(\widehat{q}_i(b_i^*(\theta_i), b_{-i}^*(\theta_{-i})), \theta_i) - p_0\} \frac{\partial \widehat{q}_i}{\partial b_i}(b_i^*(\theta_i), b_{-i}^*(\theta_{-i})) d\mu(\theta_{-i}|\theta_i) = 0.$$

Integrating over Θ_i yields

$$\int_{\Theta} \{v_i(\widehat{q}_i(b_i^*(\theta_i), b_{-i}^*(\theta_{-i})), \theta_i) - p_0\} \frac{\partial \widehat{q}_i}{\partial b_i}(b_i^*(\theta_i), b_{-i}^*(\theta_{-i})) d\mu(\theta) = 0. \qquad (17)$$

An ex-post allocation $q^* = (q_1^*, ..., q_n^*)$ that is efficient in state $\theta = (\theta_1, ..., \theta_n)$ satisfies $v_i(q_i^*, \theta_i) \geq p_0$. Moreover, efficiency implies $v_i(q_i^*, \theta_i) > p_0$ for all i whenever $\sum_{i=1}^n \theta_i > 1$. As $\partial\widehat{q}_i / \partial b_i > 0$, this contradicts (17). □

Lemma A.1 *There is an equilibrium in the first example of Section 5 ("Bid-ask spreads") in which bids are given by*

$$\overline{b}_i = (n-1) \frac{\vartheta_i + \sum_{j \neq i} \{\vartheta_j - \vartheta_i\}}{(\sum_{j=1}^n \vartheta_j)^2} \qquad (18)$$

for the high types of counterparty i.

Proof. Under the assumptions made, counterparty i's problem is given by

$$\begin{aligned} \overline{b}_i = \; & \arg\max_{b_i \geq 0} \{(1 - \pi(\overline{\theta}_{-i}|\overline{\theta}_i)) \{-b_i p_0^i + (b_i - \overline{\theta}_i) p_b^i\} \\ & + \pi(\overline{\theta}_{-i}|\overline{\theta}_i) \{-\frac{b_i}{b_i + \overline{b}_{-i}} p_0^i - (\overline{\theta}_i - \frac{b_i}{b_i + \overline{b}_{-i}}) p_a^i\}, \end{aligned} \qquad (19)$$

where $\overline{b}_{-i} = \sum_{j \neq i} \overline{b}_j$. The first-order condition for a high type of counterparty i reads

$$(1 - \pi(\overline{\theta}_{-i}|\overline{\theta}_i))(p_0^i - p_b^i) = \pi(\overline{\theta}_{-i}|\overline{\theta}_i) \frac{\overline{b}_{-i}}{(b_i + \overline{b}_{-i})^2} (p_a^i - p_0^i).$$

Rearranging gives

$$\frac{\overline{b}_{-i}}{\overline{b}_i + \overline{b}_{-i}} = \vartheta_i(\overline{b}_i + \overline{b}_{-i}). \tag{20}$$

Adding (20) up over $i = 1, ..., n$ and rearranging yields (5). The bid of the high type of counterparty i can be rewritten as

$$\overline{b}_i = (\overline{b}_i + \overline{b}_{-i})(1 - \frac{\overline{b}_{-i}}{\overline{b}_i + \overline{b}_{-i}}).$$

Using (5) and (20) yields

$$\begin{aligned}\overline{b}_i &= \frac{n-1}{\sum_{j=1}^n \vartheta_j}\left\{1 - \vartheta_i(\overline{b}_i + \overline{b}_{-i})\right\} \\ &= \frac{n-1}{\sum_{j=1}^n \vartheta_j}\left\{1 - \vartheta_i \frac{n-1}{\sum_{j=1}^n \vartheta_j}\right\} \\ &= \frac{n-1}{\left\{\sum_{j=1}^n \vartheta_j\right\}^2}\left\{\sum_{j=1}^n \vartheta_j - (n-1)\vartheta_i\right\},\end{aligned}$$

which proves (18). It remains to be shown that the two inequalities in (4) are satisfied. By (20), the first inequality in (4) is equivalent to

$$\underline{\theta}_i + \vartheta_i(\overline{b}_i + \overline{b}_{-i})^2 < 1.$$

Applying (5), and rearranging yields

$$\sum_{j=1}^n \vartheta_j > \frac{(n-1)^2}{1 - \underline{\theta}_i} \frac{\vartheta_i}{\sum_{j=1}^n \vartheta_j} \tag{21}$$

for $i = 1, ..., n$. On the other hand, using (5), the second inequality in (4) is equivalent to

$$\sum_{j=1}^n \vartheta_j < n - 1. \tag{22}$$

To find some solution for these two inequalities, restrict probabilities $\pi(\overline{\theta}_{-i}|\overline{\theta}_i)$ for the moment such that

$$\vartheta = \vartheta_1 = \vartheta_2 = ... = \vartheta_n. \tag{23}$$

Then (21) and (22) can be summarized as

$$\frac{(n-1)^2}{n^2(1 - \underline{\theta}_i)} < \vartheta < \frac{n-1}{n},$$

for $i = 1, ..., n$. A solution to these inequalities can be determined by appropriate choices for the probabilities $\pi(\overline{\theta}_{-i}|\overline{\theta}_i)$. We can then drop restriction (23) again and find open sets X_i for the probabilities $\pi(\overline{\theta}_{-i}|\overline{\theta}_i)$, for $i = 1, .., n$, so that (21) and (22) are fulfilled provided that $\pi(\overline{\theta}_{-i}|\overline{\theta}_i) \in X_i$. This proves the assertion. □

Lemma A.2. *There is an equilibrium in the second example of Section 5 ("Always rationing equilibria") in which bids are given by* $b_i(\theta_i) = \mu\theta_i/(1-\theta_i)$.

Proof. Assume that counterparty j follows the equilibrium strategy, i.e., $b_j(\theta_j) = \mu\theta_j/(1-\theta_j)$ for all θ_j. Then, clearly, $b_j(\theta_j) \geq 1$ for all types $\theta_j \geq 1/(1+\mu)$, so that rationing occurs with probability one. Hence

$$q_i(b_i, b_j(\theta_j)) = \frac{b_i}{b_i + b_j(\theta_j)} = \frac{(1-\theta_j)b_i}{(1-\theta_j)b_i + \theta_j\mu}. \tag{24}$$

Denote the cdf belonging to $g(\theta_i)$ by $G(\theta_i)$. It is straightforward to check that the problem of counterparty i is concave, and that the first-order condition reads

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta_i - q_i(b_i, b_j(\theta_j))}{q_i(b_i, b_j(\theta_j))} dG(\theta_j) = 0. \tag{25}$$

Plugging (24) into (25), and subsequently using

$$\int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta_j}{1-\theta_j} dG(\theta_j) = 1$$

yields the assertion. □

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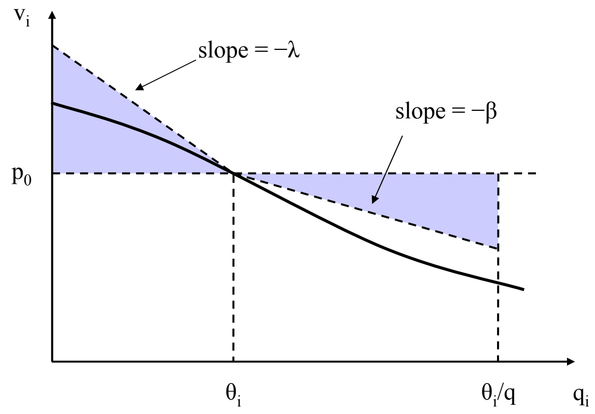

Figure 1

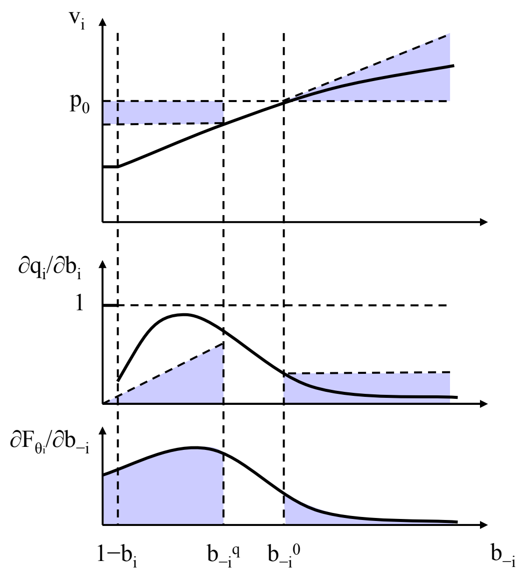

Figure 2

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