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REGIME-SWITCHING GLOBAL VECTOR AUTOREGRESSIVE MODELS

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Abstract

The purpose of the paper is to develop a Regime-Switching Global Vector Autoregressive (RS-GVAR) model. The RS-GVAR model allows for recurring or non-recurring structural changes in all or a subset of countries. It can be used to generate regime-dependent impulse response functions which are conditional upon a *regime-constellation* across countries. Coupling the RS and the GVAR methodology improves out-of-sample forecast accuracy significantly in an application to real GDP, price inflation, and stock prices.

Keywords: Global macroeconometric modeling, nonlinear modeling, regime switching, forecasting and simulation

JEL classification: C32, E17, G20

Non-technical summary

In the course of the recent 2009-2011 worldwide financial crisis, the notion of *nonlinearity* has gained ever more prominence. For instance, a view that has increasingly spread is that expansionary monetary policy would have much less potential to induce price inflation at times of subdued real activity compared to normal times, rationalizing relatively stronger conventional or even unconventional expansionary monetary policy measures at times of crisis. The argument rests on the assumption that during recessionary phases, aggregate output levels rest far below potential so that additional demand which expansionary policy aims to spur can be satisfied indeed by higher production and therefore be passed through to prices to a lesser extent.

The aim of the paper is to devise a method that can help substantiate such general ideas about nonlinear, regime-dependent dynamics in a global model framework. The paper, to that end, takes the global vector autoregressive model methodology as a basis and moves a step toward allowing for nonlinear dynamics: Country models will be allowed to be governed by local regime processes which determine the dynamics within as well as interdependencies across countries. Global dynamics will become dependent on an assumed *regime-constellation* across countries.

Besides discussing how the econometric model is set up, estimated and solved, the latter for it to be useful for forecasting and impulse response analysis, an empirical application to GDP, price inflation, and stock prices serves to highlight that the out-of-sample forecast performance of the GVAR with regime-switching can improve relative to an otherwise identically structured GVAR without switching. Moreover, shock simulations suggest that for instance an otherwise identical positive shock to real activity in the US (despite spreading widely to affect real activity across countries) would be inducing higher price inflation at times of a strong growth cross-country constellation, as opposed to rather muted global price responses in a weak growth environment.

1 Introduction

As an econometric approach to modeling the increasing economic interdependencies across countries, the Global Vector Autoregressive (GVAR) model methodology has gained widespread interest in recent years [see e.g. [11], [43], [44], [20],[8], [9]]. Interlinkages between countries can be modeled directly by combining, traditionally via trade-weights, a set of country-specific VARs that contain weighted foreign variable vectors. This approach allows modeling simultaneously a large number of countries, accommodating as well a broad set of economic variables in one model which, if modeled in an otherwise unrestricted conventional VAR be unfeasible to be estimated due to a too high number of parameters. Recent GVAR applications include e.g. [17] who study how credit supply shocks propagate internationally, [24] who integrate Contingent Claims Analysis (CCA)-based indicators into a GVAR for sovereigns, banks and the corporate sector, and [25] who demonstrate how a GVAR can be set up for multiple cross-sections.

As has been noted by other authors, e.g. [42], structural breaks can occur in different ways, for instance with regard to autoregressive dynamics, trends, or cointegrating relationships. In general, a Markov-switching approach to autoregressive modeling [see [27], [28], [29], [37], and references therein] has the advantage that one can accommodate structural changes across regimes, both with respect to autoregressive dynamics and the covariance structure of shocks. Regime changes may either be due to one-time events (e.g. a severe financial crisis) or as well be recurring (consider e.g. regular business cycle movements).

Regime-switching models have meanwhile become increasingly popular in the field of empirical macroeconomic research, including applications to GDP, inflation, interest rates, equity returns and volatility, and to examine the role of regime-dependent determinants as well as effects of monetary policy.[1] Also in other areas of research, such as meteorology and speech recognition, regime-switching models

[1] Empirical applications of regime-switching models for GDP can be found, inter alia, in [27], [3], [45], [41], and [35]. Applications to inflation and interest rates include e.g. [18], [21], and [1]. Papers that investigate whether monetary policy is itself regime-dependent or causing regime-dependent output responses include [22], [33], [48], [39], and [49].

have been found to be useful model devices.[2] Notable methodological advances since the initial contribution of [27] can be found in an extension of the Markov-switching model with endogenous transition probabilities in [13] and [19], with an empirical application of that methodology presented e.g. in [2]; a model in which instead of switching probabilities the regime process itself is endogenous was discussed in [36]. Moreover, regime-switching has been applied in dynamic factor model settings ([7]) and been combined with Mixed Data Sampling (MIDAS) techniques in [26].[3]

The focus of the paper will first lie in outlining how the econometric model specification for an RS-GVAR would look like, on the specificities that arise with respect to estimation, and eventually on providing an application. An out-of-sample forecast simulation will aim to emphasize the potential of the RS-GVAR to improve forecast accuracy relative to otherwise identically structured conventional GVARs without regime switching.

2 The RS-GVAR model

2.1 Local regime-switching models

We assume that the global model comprises $N+1$ countries that are indexed by $i = 0, 1, 2, ..., N$.

A set of country-specific observed endogenous variables are collected in a $k_i \times 1$ vector $\mathbf{y}_{it}$ which is related to a number of autoregressive lags up to P and a $k_i^* \times 1$ vector of foreign variables $\mathbf{y}_{it}^*$ that enters the model time-contemporaneously and with a number of lags up to Q, that is,

[2] See e.g. [52] (meteorology) and [47] (speech recognition).

[3] In parallel to Markov-switching models, two other forms of regime-switching models have been developed: 'Smooth transition' models in [23] and 'threshold' autoregressive models in [51] and [46]. In the latter, regime switches are triggered by observed variables, with the trigger level being endogenous (i.e. unobserved and therefore to be estimated along with the model coefficients).

$$\mathbf{y}_{it} = \mathbf{a}_{i,0,s_{it}} + \mathbf{a}_{i,1,s_{it}} t + \sum_{p=1}^{P} \mathbf{\Phi}_{i,p,s_{it}} \mathbf{y}_{i,t-p} + \sum_{q=0}^{Q} \mathbf{\Lambda}_{i,q,s_{it}} \mathbf{y}^{*}_{i,t-q} + \mathbf{\Psi}_{s_{it}} \mathbf{d}_t + \epsilon_{it} \tag{1}$$

where $\mathbf{a}_{i,0,s_{it}}$, $\mathbf{a}_{i,1,s_{it}}$, $\mathbf{\Phi}_{i,p,s_{it}}$, $\mathbf{\Lambda}_{i,q,s_{it}}$, and $\mathbf{\Psi}_{s_{it}}$ are coefficient matrices of size $k_i \times 1$, $k_i \times 1$, $k_i \times k_i$, $k_i \times k_i^*$, and $d_i \times 1$ respectively. The vector $\mathbf{d}_t$ may contain exogenous variables that are common to all cross-section items. We assume that $\epsilon_{it} \sim i.i.d.N\left(\mathbf{0}, \mathbf{\Sigma}_{ii,s_{it}}\right)$.

The subscript s_{it} attached to all coefficient matrices and the covariance matrix $\mathbf{\Sigma}_{ii,s_{it}}$ signals that they be allowed to depend on the regime s prevailing in country i at time t, where s_{it} is for the time being assumed to be the outcome of an unobserved R-state Markov chain that is by assumption independent of $\epsilon_{it'}$ for all t and t'. The s_{it} can assume integer values between $1, ..., R_i$, where R_i is the number of regimes that one allows country i's dynamics to switch between.

There are transition probabilities $p_{i,lm}$ that govern the evolution of the local regimes. They signal how likely it is that country i switches from regime l to m in two consecutive periods t and $t+1$, conditional on that i's dynamics were in regime l at time t. That is,

$$P_i \left\{ s_{i,t+1} = m | s_{it} = l \right\} = p_{i,lm} \tag{2}$$

2.2 Regime-conditional densities and inference about regimes

Let $\mathcal{Y}_{it} = \left(\mathbf{y}'_{it}, \mathbf{y}'_{i,t-1}, ..., \mathbf{y}'^{*}_{it}, \mathbf{y}'^{*}_{i,t-1}, ... \right)'$ be a vector comprising the observations up to time t. If the model is to contain a constant or any further exogenous variables, they will be included in $\mathcal{Y}_{it}$. The density of $\mathbf{y}_{it}$ conditional on the regime $s_{it} = m$ prevailing, the data $\mathcal{Y}_{it}$ and the local models' parameter space α is

$$f_i \left(\mathbf{y}_{it} | s_{it} = m, \mathcal{Y}_{i,t-1}; \alpha \right) \tag{3}$$

For all $i = 0, 1, ..., N$, we construct a vector η_{it} that comprises R_i densities, that is,

$$\eta_{it} = \begin{bmatrix} f_i\left(\mathbf{y}_{it}|s_{it} = 1, \mathcal{Y}_{i,t-1}; \alpha\right) \\ f_i\left(\mathbf{y}_{it}|s_{it} = 2, \mathcal{Y}_{i,t-1}; \alpha\right) \\ \ldots \\ f_i\left(\mathbf{y}_{it}|s_{it} = R_i, \mathcal{Y}_{i,t-1}; \alpha\right) \end{bmatrix} \tag{4}$$

Since the ϵ_{it} are multivariate Normal by assumption, the conditional density will have the following format.

$$f_i\left(\mathbf{y}_{it}|s_{it} = m, \mathcal{Y}_{it}; \alpha\right) = \frac{1}{(2\pi)^{k_i/2}\left|\mathbf{\Sigma}_{ii,s_{it}}\right|^{1/2}} \exp\left(-\frac{1}{2}\epsilon_{it}'\mathbf{\Sigma}_{ii,s_{it}}^{-1}\epsilon_{it}\right) \tag{5}$$

where $|\mathbf{\Sigma}_{ii,s_{it}}|$ is the determinant of the country-specific covariance matrix. The ϵ_{it} come from equation (1). They are dependent only on the current regime s_{it}.

$$\epsilon_{it} = \mathbf{y}_{it} - \mathbf{a}_{i,0,s_{it}} - \mathbf{a}_{i,1,s_{it}}t - \sum_{p=1}^{P}\mathbf{\Phi}_{i,p,s_{it}}\mathbf{y}_{i,t-p} - \sum_{q=0}^{Q}\mathbf{\Lambda}_{i,q,s_{it}}\mathbf{y}_{i,t-q}^{*} - \mathbf{\Psi}_{s_{it}}\mathbf{d}_t \tag{6}$$

Combining the latter two equations and summing over the sample gives us the likelihood $\mathcal{L}_i$ for the observed data for all i.

$$\mathcal{L}_i\left(\alpha\right) = \sum_{t=1}^{T}\log f_i\left(\mathbf{y}_{it}|\mathcal{Y}_{it}; \alpha\right) \tag{7}$$

where $f_i\left(\mathbf{y}_{it}|\mathcal{Y}_{it}; \alpha\right) = \mathbf{1}'\left(\widehat{\xi}_{i,t|t-1} \odot \eta_{it}\right)$, i.e. we integrate out the dependence on the regime by weighing regime dependent densities with one-step ahead filtered probabilities.

We let $P\left\{s_{it} = m|\mathcal{Y}_{it}, \mathbf{\Theta}\right\}$ be the probability that country i is in regime m at time t, which is dependent on the data $\mathcal{Y}_{it}$ observed until t and on knowledge of the model's parameters $\mathbf{\Theta}$ comprising the α and the local transition probabilities. We collect these conditional probabilities in an $R_i \times 1$ vector $\widehat{\xi}_{i,t|t}$ that we refer to as the *filtered* probabilities.

Along with the $\widehat{\xi}_{i,t|t}$ we shall also construct one-step ahead predictions of these probabilities which are denoted as $\widehat{\xi}_{i,t+1|t}$. The unobserved regimes, probabilities respectively, can then be inferred by iterating on the following two equations.

$$\widehat{\xi}_{i,t+1|t} = \mathbf{P}_i \cdot \widehat{\xi}_{i,t|t} \tag{8}$$

$$\widehat{\xi}_{i,t|t} = \frac{\widehat{\xi}_{i,t-1|t} \odot \eta_{i,t}}{\mathbf{1}' \left(\widehat{\xi}_{i,t-1|t} \odot \eta_{i,t} \right)} \tag{9}$$

where $\odot$ denotes element-wise multiplication and the η_{it} were defined in equation (4).

A final step entails computing *smoothed* local regime probabilities. Unlike filtered probabilities, smoothed probabilities at time t are not based on information only until t but the whole sample until time T which renders them smoother than their filtered counterparts. We employ the algorithm proposed by [34] to estimate the smooth probabilities. It entails another iterative procedure:

$$\widehat{\xi}_{i,t|T} = \widehat{\xi}_{i,t|t} \odot \left[\mathbf{P}_i' \left(\widehat{\xi}_{i,t+1|T} \oslash \widehat{\xi}_{i,t+1|t} \right) \right] \tag{10}$$

with $\oslash$ denoting element-wise division. This time the iteration goes backward in time. The starting value $\xi_{iT|T}$ can be set to the filtered probabilities $\widehat{\xi}_{it|t}$ at $t = T$, i.e. to the end of the in-sample period.

In order to estimate the local models we employ the Expectation-Maximization (EM) algorithm. For details we refer to [12].

2.3 Regime-constellation-dependent global solution of the model

For solving the global model, we define a country-specific $(k_i + k_i^*) \times 1$ vector $\mathbf{z}_{it}$ as follows.

$$\mathbf{z}_{it} = \begin{bmatrix} \mathbf{y}_{it} \\ \mathbf{y}_{it}^* \end{bmatrix} \tag{11}$$

The country models in equation (1) can then be reformulated.

$$\mathbf{A}_{i,0,s_{it}} \mathbf{z}_{it} = \mathbf{a}_{i,0,s_{it}} + \mathbf{a}_{i,1,s_{it}} t + \mathbf{A}_{i,1,s_{it}} \mathbf{z}_{i,t-1} + ... + \mathbf{A}_{i,P,s_{it}} \mathbf{z}_{i,t-P} + \epsilon_{it} \tag{12}$$

where it is assumed for ease of notation in the following that $P = Q$ and the global exogenous variable vector $\mathbf{d}_t$ be empty. The $\mathbf{A}_{i,p,s_{it}}$ coefficient matrices are of size $k_i \times (k_i + k_i^*)$ and have the following form.

$$\begin{aligned} \mathbf{A}_{i,0,s_{it}} &= (\mathbf{I}_{k_i}, -\mathbf{\Lambda}_{i,0,s_{it}}) \\ \mathbf{A}_{i,1,s_{it}} &= (\mathbf{\Phi}_{i,1,s_{it}}, \mathbf{\Lambda}_{i,1,s_{it}}) \\ &\dots \\ \mathbf{A}_{i,P,s_{it}} &= (\mathbf{\Phi}_{i,P,s_{it}}, \mathbf{\Lambda}_{i,P,s_{it}}) \end{aligned} \tag{13}$$

The endogenous variables across countries are stacked in one global vector $\mathbf{y}_t$ which is of size $k \times 1$ where $k = \sum_{i=1}^{N} k_i$. Here, the local variable vectors $\mathbf{z}_{it}$ will have to mapped to the global endogenous variable vector $\mathbf{y}_t$ which is accomplished via $(k_i \times k_i^*) \times k$ link matrices $\mathbf{W}_i$. With $\mathbf{z}_{it} = \mathbf{W}_i \mathbf{y}_t$ at hand one can rewrite the model once more.

$$\mathbf{A}_{i,0,s_{it}} \mathbf{W}_i \mathbf{y}_t = \mathbf{a}_{i,0,s_{it}} + \mathbf{a}_{i,1,s_{it}} t + \mathbf{A}_{i,1,s_{it}} \mathbf{W}_i \mathbf{y}_{t-1} + ... + \mathbf{A}_{i,P,s_{it}} \mathbf{W}_i \mathbf{y}_{t-P} + \epsilon_{it} \tag{14}$$

Now, we move from country-specific models to the global model by stacking the former in one global system. That is,

$$\mathbf{G}_{0,S_t}\mathbf{y}_t = \mathbf{a}_{0,S_t} + \mathbf{a}_{1,S_t}t + \mathbf{G}_{1,S_t}\mathbf{y}_{t-1} + ... + \mathbf{G}_{P,S_t}\mathbf{y}_{t-P} + \epsilon_t \tag{15}$$

where $S_t = \{s_{1,t}, s_{2,t}, ..., s_{N,t}\}$ is the regime-constellation across countries that is assumed while forming the $k \times k$ matrices $\mathbf{G}_{0,S_t}$. They have the following format.

$$(\mathbf{G}_{0,S_t}, ..., \mathbf{G}_{P,S_t}) = \begin{pmatrix} \mathbf{A}_{0,1,S_t}\mathbf{W}_1 & & \mathbf{A}_{P,1,S_t}\mathbf{W}_1 \\ \mathbf{A}_{0,2,S_t}\mathbf{W}_2 & , ..., & \mathbf{A}_{P,2,S_t}\mathbf{W}_2 \\ ... & & ... \\ \mathbf{A}_{0,N,S_t}\mathbf{W}_N & & \mathbf{A}_{P,N,S_t}\mathbf{W}_N \end{pmatrix} \tag{16}$$

The S_t having a subscript t points to the fact that the global solution of the model and therefore its dynamic properties vary over time.

Finally, we obtain a reduced form of the global model by pre-multiplying the system with the inverse of $\mathbf{G}_{0,S_t}$.

$$\mathbf{y}_t = \mathbf{G}_{0,S_t}^{-1}\mathbf{a}_{0,S_t} + \mathbf{G}_{0,S_t}^{-1}\mathbf{a}_{1,S_t}t + \mathbf{G}_{0,S_t}^{-1}\mathbf{G}_{1,S_t}\mathbf{y}_{t-1} + ... + \mathbf{G}_{0,S_t}^{-1}\mathbf{G}_{P,S_t}\mathbf{y}_{t-P} + \mathbf{G}_{0,S_t}^{-1}\epsilon_t \tag{17}$$

Solving the global model, as mentioned above, is dependent on an assumption as to a regime-constellation, where the S_t would be the regimes, regime probabilities respectively, as inferred throughout the sample period. More generally, we can define an $R_i \times (N + 1)$ matrix Ξ that indicates the desired regimes for the $N+1$ countries. Its columns shall each sum to one and be denoted as Ξ_i.

Unlike the smooth regime probabilities $\widehat{\xi}_{i,t|T}$, the Ξ has no time script since it is used to request one particular regime-constellation Ξ at a time for solving the model. For setting the desired regimes, one can follow either of the following two approaches:

1. An arbitrary regime-constellation can be chosen, that is, each country i's dynamics are set to the desired regime R_i with weight one. The Ξ would therefore contain only zeros and ones.

2. The regimes can be set according to the estimated constellation at selected points in time, thus the rows of Ξ would be set equal to the inferred regime probabilities, that is, $\Xi_i = \widehat{\xi}_{i,t|T}$.

A combination of the two approaches is of course also conceivable, where for some countries an assumption would entail a prescription to one specific regime and for others a mixture of regimes via inferred or hypothesized probabilities.

In either setting, the Ξ is used to compute a weighted average of the local models' parameter space, denoted by tildes in the equation that follows.

$$\widetilde{\mathbf{A}}_{i,p} = \sum_{r=1}^{R_i} \mathbf{A}_{i,p,s_i=r} \cdot \Xi_i(r) \tag{18}$$

where $\Xi_i(r)$ is the r-th row of Ξ_i. The same weighting applies to intercepts $\mathbf{a}_{i,0,s_{it}}$ and time trend coefficients $\mathbf{a}_{i,1,s_{it}}$.

Since the solution of the model is regime-constellation dependent, impulse responses and forecasts from the global model will be so, too. Forecasting from the model entails an additional step which is to generate predicted state probabilities that are obtained by multiplying the inferred smooth regime probabilities $\widehat{\xi}_{i,t|t}$ at the forecast origin with the estimated transition matrix $\mathbf{P}_i$ raised to the power of h, the forecast horizon.

$$E\left(\widehat{\xi}_{i,t+h|t}\right) = \mathbf{P}_i^h \cdot \widehat{\xi}_{i,t|t} \tag{19}$$

Since the predicted regime-probabilities vary along the horizon (until they approach their ergodic, long-run mean), the global system needs to be solved repeatedly along the horizon, i.e. h times, to produce a further one-step ahead iterative forecast at each step.

Once the ergodic state probabilities have been approached with sufficient precision, one can stop re-solving the RS-GVAR and keep the global parameter space constant.

2.4 Estimating the global covariance matrix

Local covariance matrix estimates based on the local model equations' residuals, if allowed to be distinct across regimes, would be computed as follows.

$$\mathbf{\Sigma}_{ii,s_i=m} = \frac{\sum_T \epsilon_{it}\epsilon_{it}^T \sqrt{P\left\{s_{it}=m|\mathcal{Y}_{it},\mathbf{\Theta}\right\}}}{\sum_T P\left\{s_{it}=m|\mathcal{Y}_{it},\mathbf{\Theta}\right\}} \tag{20}$$

where the residuals ϵ_{it} are to be understood as generated conditional on regime m's dynamics from local model i. The local smooth regime probabilities from the rows corresponding to regime m of $\widehat{\xi}_{i,t|t}$ are employed as an estimate for $P\left\{s_{it}=j|\mathcal{Y}_{it},\mathbf{\Theta}\right\}$ in case that a concrete regime was requested. To generalize the notation for the case that a mixture of regimes was requested via Ξ_i, we would write

$$\mathbf{\Sigma}_{ii,\Xi_i} = \frac{\sum_T \epsilon_{it}\epsilon_{it}^T \sqrt{\widehat{\xi}'_{i,t|t}\Xi_i}}{\sum_T \widehat{\xi}'_{i,t|t}\Xi_i} \tag{21}$$

For obtaining an estimate of the global covariance matrix $\mathbf{\Sigma}$, we propose to estimate the compartments of the matrix, meaning the $k_i \times k_i$ blocks referring to the local k_i^2 variances and covariances for each local model on the diagonal and the additional $k_i \times k_i$ blocks of covariances between any pair of countries, individually. Local covariance matrix estimates are obtained by employing the formula in equation (20), using the local regime probabilities.

For a pair of countries i and j, i.e. for the off-diagonal blocks $\mathbf{\Sigma}_{ij}$ of the global covariance matrix, an estimate of the *joint* probabilities of the pair having been in the requested regime-constellation has to be provided which is obtained by measuring how often the pair has been in the requested regime-constellation throughout the sample period. An intermediate estimate $\mathbf{\Sigma}^*_{ij}$ would be computed as follows.

$$\mathbf{\Sigma}^*_{ij,s_i=m,s_j=l} = \frac{\sum_T \epsilon_{it}\epsilon_{it}^T \sqrt{P\left\{s_{it}=m|\mathcal{Y}_{it},\mathbf{\Theta}\right\}P\left\{s_{jt}=l|\mathcal{Y}_{it},\mathbf{\Theta}\right\}}}{\sum_T P\left\{s_{it}=m|\mathcal{Y}_{it},\mathbf{\Theta}\right\}P\left\{s_{jt}=l|\mathcal{Y}_{it},\mathbf{\Theta}\right\}} \tag{22}$$

Only $\mathbf{\Sigma}^*_{ij}$'s upper right $k_i \times k_i$ part, i.e. the covariances, will then be used to fill the respective compartment in the global matrix $\mathbf{\Sigma}$.

Again, if a mixture of regimes was to be considered, we would generalize the notation as follows.

$$\mathbf{\Sigma}^*_{ij,\Xi_i,\Xi_j} = \frac{\sum_T \epsilon_{it}\epsilon_{it}^T \sqrt{(\widehat{\xi}'_{i,t|t}\Xi_i)(\widehat{\xi}'_{j,t|t}\Xi_j)}}{\sum_T (\widehat{\xi}'_{i,t|t}\Xi_i)(\widehat{\xi}'_{j,t|t}\Xi_j)} \tag{23}$$

The rationale for estimating the global matrix' blocks sequentially is that a pair-wise set of covariances shall depend on the probability of that very pair to prevail in the requested regime-constellation. An alternative - to compute the joint probability of the overall constellation across all $N+1$ countries to prevail and use that to estimate the global matrix in one go - would not be meaningful, since the covariance estimates for any pair should not depend on the assumed regimes for any third country.[4]

3 Empirical application

3.1 Data and model structuring

The model is set up for a sample of 18 countries and three endogenous variables: Real GDP, personal consumption expenditure prices, and stock price indices, data for which was retrieved from OECD databases and Bloomberg / Datastream, respectively. All variables are modeled in quarter-on-quarter (QoQ) logarithmic differences to render them stationary at conventional levels of significance (at most 10%). To

[4]While by construction the global matrix $\mathbf{\Sigma}$ is symmetric and has non-zero variances on its diagonal, it may not in all cases be positive semi-definite. Inspired by a method proposed by [32], we implement an algorithm that adjusts $\mathbf{\Sigma}$ such that it will be positive semi-definite in such cases, i.e. subject to the constraint that its eigenvalues and diagonal elements are all non-negative, while at the same time closest to the original $\mathbf{\Sigma}$ (using an L2-norm), applied to the upper triangular part of the matrix to guarantee that it remains symmetric. The adjusted $\mathbf{\Sigma}^{adj}$ can then be employed to conduct stochastic simulations of the global model. The experience in different empirical settings has so far suggested that an adjustment is necessary in only very few cases, and if it was needed then the corrections were very small in terms of the magnitude of the adjustment.

the quarterly changes in personal consumption expenditure prices we from now on refer to as 'inflation'. The quarterly data sample covers the period from 1996Q1-2011Q4 (64 observations). For an overview of countries and variables, including basic summary statistics, see Table 1.

We employ a specification search for structuring the local models (yet based on a GVAR without regime-switching) that chooses the lag numbers for autoregressive and foreign variable vectors, P and Q, as well as whether or not to include a linear trend, optimally according to the Bayesian Information Criterion (BIC). For each local model, estimates for all conceivable combinations of a trend being/not being present and between zero and a specified maximum number (set to two) for P and Q were generated. The specification resulting in the minimum BIC was chosen.

The weight matrix for constructing foreign variable vectors in all country models is based on IMF Direction-of-Trade Statistics (DOTS) data for bilateral exports and imports as of 2006. That point in time has been chosen to guarantee that the out-of-sample forecast simulation for the sample 2008Q-2011Q4 would have been feasible retrospectively in real time. Results presented in the following are not very sensitive to that choice and remain robust when using trade weight matrices based on other years (or averages of matrices from different years).

3.2 Regime-inference

The inference of regimes at country level is based on year-on-year (YoY) rates of change in real GDPs, that is, it is accomplished by referring to only a subset of the model variables and moreover using a different transformation compared to the GDP in the core of the model where QoQ log differences are applied. The reason for following that approach was twofold: 1) YoY rates of change of GDP are more persistent and transition of their means between regimes we see as a more natural choice for our interpretation of 'growth regimes'. It also better resembles the official business cycle dates that statistical authorities in some countries set, such as NBER recession dates for the US. 2) For later assessing the global model dynamics conditional on assumed regime-constellations, it is easier to interpret and label the regimes per country, compared to the case where the joint dynamics of real activity, inflation, and stock markets would be allowed to jointly determine the local regime processes. Generally, it remains a matter of what the empirical analysis aims to address when choosing the variables that determine the regime processes. One could consider taking subsets or the entirety of model variables or other off-model variables to that end.[5]

The number of regimes has been formally tested for all countries using the method proposed by [30]. The test results suggest that three regimes (alternative hypothesis) should be preferred to a two-regime setting (Null hypothesis) for all countries, with p-values following the [30] method for the Null against the alternative being virtually zero for all countries. Indeed, when operating with only two regimes, one of the two regimes captures solely the deep recession of 2007-2009, with the earlier recession periods being lumped together with expansion periods to one regime.[6]

The resulting regime probabilities based on YoY changes of GDPs and three regimes are summarized in Figure 1. They show that the third regime in most

[5] We also estimated a global model based on regimes that were inferred from QoQ log differences of GDP but found that the subsequent impulse responses (as presented later in the paper) differed to a much lesser extent conditional on different regime-constellation assumptions. Our interpretation of this finding is that changes in regime let us more adequately identify changes in model dynamics (coefficients) when defining growth regimes based on a more persistent measure of growth in real activity. Put differently, we see it as in indication of only persistent changes in regime to imply persistent changes in dynamics, whereas abrupt, yet transient QoQ changes might not immediately cause dynamics to change. To properly capture the rather persistent YoY rates of change and thereby adequately infer the regimes, four autoregressive lags have been allowed per country. The models were estimated in 'Hamilton-type', i.e. the means and residual variances were allowed to switch regimes while autoregressive coefficients were assumed to be equal across regimes. In the literature, there is no clear consensus (nor explicit treatment) as to whether QoQ or YoY rates of quarterly GDP should be taken as a reference to infer growth regimes. Original Hamilton-type regime-switching models take QoQ rates as a reference and model a transition in their mean. Applications with transformations other than QoQ, involving also threshold autoregressive models, can be found e.g. in [14], [50],[5], [31] and [2].

[6] Applying a more recent technique proposed by [6] for choosing the number of lags confirms that three regimes should be preferred to two regimes. When allowing for four regimes, the estimation was no longer feasible for a number of countries because the fourth state was inferred to prevail for only too few periods, thus resulting in almost perfect in-sample fit in that regime, respectively causing the likelihood in that regime to diverge to plus infinity.

countries captures the deep recession following the 2007-2009 financial crisis period.[7]

Table 2 and Figure 2 summarize/visualize the regime-conditional averages of the model variables across countries. They are averages weighted via the estimated regime probabilities shown in Figure 1. In accordance with the inferred regime probabilities based on YoY rates of change of GDPs, the three regimes will be labeled as 'expansion', 'medium growth', and 'recession' respectively. For YoY rates of GDP the three regimes imply cross-country median rates of [3.8,1.6,-3.2]%. Median annualized inflation rates equal about [2.0,1.9,0.7]%, i.e. there is a clear tendency for prices to rise at much smaller rates (or for selected countries to fall on average) during recession periods. For stock prices, annualized rates of change level around [12.8,-2.2,-3.5]%. The estimates suggest that stock markets tended to fall/crash at times of medium growth, and then to further deteriorate during recession regimes.

3.3 Structure of the global model

The model structure is summarized in Table 3 where it can be seen that the chosen lag numbers for autoregressive and foreign variable vectors are rather asymmetric across countries. For the majority of countries the inclusion of only time-contemporaneous foreign variable vectors suffices, where for six countries a first lag thereof was deemed relevant. Residuals for all model equations have been subjected to tests for remaining serial correlation. Durbin-Watson statistics confirm they are sufficiently free of serial correlation (Table 3).

Recursive eigenvalues of the global model based on historically inferred regime probabilities are visualized in Figure 3. 'Continuous regimes' means that the global model was solved at each point in time by employing the inferred regime probabilities for weighing the local parameter spaces as described in the previous section. 'Discrete regimes' means that regime probabilities were first rounded, that is, to assume that the regime with the highest probability in each quarter was to receive weight one to then derive the global solution of the model in each quarter.

[7]Transition matrix estimates for across markets are not reported since they are not very central to the discussion. They are available from the authors on request.

In between 60%-62.5% of the sample period, the global model was stable. It appears that in particular during the periods when the majority of countries moved toward recession regimes (around 2001 and during 2007-2009) the global model dynamics went into explosive territory. The historical maximum moduli of eigenvalues would approach 197 and 13, respectively based on the rounded (discrete) and exact (continuous) regime probabilities.

3.4 Simulating shock scenarios

To reveal the global model's inner dynamics, shock scenarios will in the following be presented which are based on regime-constellations as of 2006Q1 and 2011Q4.[8] The corresponding regime probabilities could be read from Figure 1, but for convenience are plotted again in bar chart format in Figure 4 for just these two points in time.

As of the pre-crisis standpoint in 2006Q1, the majority of countries is inferred to prevail in the expansion regime, with some exceptions such as Belgium where respectively about 55% and 45% probability are assigned to strong expansion and medium positive growth. For Luxembourg, Norway and Portugal we see close to 100% probability attached to the medium positive growth regime.

As of the post-crisis viewpoint in 2011Q4, the majority of countries is inferred to have moved back to medium growth, with only Portugal yet being in recession following a double-dip of its GDP growth.

The first example simulation entails a positive 1 STD shock to US GDP growth that amounts to +0.75 percentage points (pp) to QoQ log differences. Figures 5-7 show the responses of GDP, inflation, and stock markets up to a 24-quarter horizon. Responses are expressed in cumulative logarithmic differences.

The responses of GDPs (Figure 5) suggest that real activity appears more reac-

[8] The impulse response analysis presented here considers shocks to model variables, conditional on regime assumptions, similar in spirit to the methodology presented in [16]. Alternatives have been considered in [38], where the regime status at the outset of a simulation horizon would be allowed to converge back to the ergodic steady state regime. The author also considers deriving the responses to exogenous shifts in regime.

tive during the expansion period than under the post-crisis, weak growth environment. For all countries, with the exception of only Canada, a positive and significant response has been simulated under the strong growth regime-constellation. When assuming the weak growth constellation, responses are smaller in magnitude and generally insignificant, except for the US itself where the shock originated.

With regard to inflation (Figure 6), there is a tendency for rates to increase significantly under the strong growth scenario, with up to 1% cumulative change in prices in the US itself, and e.g. 1.3% in Germany, 3.8% in Ireland, and 4.5% in Italy. When setting the weak growth regime constellation instead, price responses remain muted, with respect to magnitude and significance, for the majority of countries. Exceptions are Austria, Italy, France, and the Netherlands, where prices even fall somewhat on impact of the expansionary shock in the US.

Stock markets (Figure 7) are more reactive under the weak growth compared to the strong growth constellation. In either case, they react positively and significantly, but more so under the recession regimes, where mean responses very consistently across countries display a hump within the first 5-10 quarters, while the mean under strong growth regimes cumulates rather steadily with constant slope. Finally, much higher uncertainty surrounds the simulated responses under the weak growth constellation, a feature that is reflective of the model residuals' variance and covariance structure during the weak growth regime constellation.

Overall, in particular when considering the responses of prices, the positive shock to US activity (if for example thought of as being a result of some successful expansionary monetary policy measure conduced by the FED), would suggest that inflation is generally much less responsive at times of low activity compared to times of high activity.[9] Theories that would substantiate such empirical regularity are e.g. the capacity constraint model[10] which implies that prices become more responsive to a marginal increase in aggregate demand the closer firms come to their capacity constraint. At the extreme, when firms would not be able to increase

[9]Contemplating about the role and effect of monetary policy is arguably general here because i) There is no monetary policy variable included in the model, and ii) even if it was, we operate with generalized impulse responses and thus do not aim to identify shocks.

[10]See e.g. [40].

production any further, they would compensate the additional demand solely by passing it through to prices. This mechanism implies that a Phillips curve would not be linear but convex.[11]

The second example scenario entails a negative shock to prices in Germany, with QoQ inflation falling by -0.35pp on impact. Resulting responses of GDP, inflation, and stock prices are shown in Figures 8-10.

GDP responses (Figure 8) suggest that real activity appears more reactive during the strong growth regime-constellation at the pre-crisis standpoint. GDPs fall by up to 1% cumulative over the 6-year horizon, with the US itself contracting by approximately -0.3%. Responses are significant for all countries but Canada and Finland. Under the weak growth regime setting, cumulative mean responses are negative, but are significant only right on impact. At longer horizons, cumulative responses are not distinguishable from zero.

In terms of price responses (Figure 9), the estimates suggest that inflation is more dependent at times of a weak growth constellation across countries, when all countries follow the negative shock to Germany, except for Denmark, the UK and Norway. Conditional on the strong growth constellation, prices appear rather flat and cumulative responses insignificant. Italy responds in a way that is maybe counterintuitive, where its price response is significantly positive as a result of the initial negative price shock in Germany.

Finally, stock markets (Figure 10) would under the strong growth assumption fall significantly (therefore in tandem with GDPs), while cumulative responses are positive and borderline significant under the weak growth environment. Thus, while a fall in prices would not have the potential (according to the estimates) to stimulate real activity significantly, it would bring confidence to the stock markets.

3.5 Out-of-sample forecasting

We assess the out-of-sample forecast performance of four model schemes that are run in parallel, under otherwise identical conditions in particular with respect to

[11] For further research about nonlinearities in the Phillips curve see also [15] and [4].

the initial in-sample calibration period: 1) AR models for all variables/countries individually, 2) VAR models per country, 3) the GVAR model, 4) the RS-GVAR model.

All models, including the (V)AR benchmarks, are structured via the BIC and estimated based on the sample from 1996Q1-2007Q1. As mentioned before, the weight matrix for constructing the foreign-variable vectors is based on trade data as of 2006. Upon estimation, the models are then calibrated and used to produce a set of 1- to 4-quarter ahead forecasts for the period from 2008Q1-2011Q4 (16 observations). The first three intermediate forecasts for within 2007 (Q2-Q4) were neglected to let the evaluation be based on a common test-sample, with the same underlying number of 1- to 4-quarter ahead predictions.

Unlike for the GVAR, which is solved only once and then calibrated based on the parameter estimates as of 2007Q1, the RS-GVAR is solved repeatedly over the test-period based on the recursively re-evaluated regime-probabilities. The underlying regime-conditional coefficient matrices are not re-estimated in order to guarantee a fair treatment of the GVAR, the RS-GVAR and all (V)AR benchmarks. For the RS-GVAR, whenever moving a step forward in the recursive out-of-sample test period, the additionally observed YoY rates of change in GDPs are used to infer a new set of regime probabilities across countries to then weigh the parameter spaces and re-solve the global model. Overall, the approach emulates a forecast process that would have been feasible therefore to be conducted in real-time.[12]

Evaluation results are collected in four groups: GVAR vs VAR performance (Figures 11-13), RS-GVAR vs GVAR (Figures 14-16), GVAR vs AR (Figures 17-19), and RS-GVAR vs AR (Figures 20-22). RMSE ratios are in all cases accompanied by a [10] test statistic to signal whether a gain in performance was significant from a statistical viewpoint (the colors of the bars reflect whether a 1%, 5%, or 10% threshold probability was reached; see footnotes to the figures for details).[13]

[12] We abstain however from taking data revisions into account, which in particular for GDP may somewhat influence the evaluation. Only in that sense, the out-of-sample forecasting exercise as conducted here could not be replicated truly in real-time.

[13] Arguably, the out-of-sample period is rather short (16 observations). By nature of the test, however, it does take automatic account of that fact, in the sense that the Null of equal predictive accuracy for an otherwise constant difference in performance of two competing models gets harder

As regards the GVAR relative to VAR model performance, for all variables at all horizons, the global model dimension proves useful as it increases point forecast precision to a significant extent. For GDP (Figure 11), all ratios are smaller one, indicating up to 30% improvement at the shortest horizon for the Netherlands. The mean ratio across countries and horizons equals 0.9, indicating a 10% gain in performance on average.

For price inflation (Figure 12), gains are less pronounced in magnitude, but significant in many cases; The maximum gain can be seen for Belgium with about 13% improvement over the VAR at the 1-quarter horizon. For countries such as Sweden, Germany, the UK, France, Finland, and a few others, the GVAR and VAR perform rather equally well with ratios surrounding one. Indeed, the average gain across countries and horizons amounts to approximately 0%.

With regard to stock prices (Figure 13), performance gains are visible, with ratios approaching 0.54 e.g. for Portugal. On average, the gain in performance equals 27% across countries and horizons.

Turning to the RS-GVAR's performance, the precision of GDP forecasts at the shortest horizon increases by up to 25% (for Luxembourg) compared to the benchmark GVAR with otherwise identical structure. For half of the countries, the ratio at the 1-quarter horizon is less than one, with the average being therefore close to one. At longer horizons, however, the gain in performance becomes more pronounced. Ratios approach 0.75 for Norway's GDP at the 1-year horizon. For seven countries, the gain compared to the GVAR's performance is significant at conventional levels.

For price inflation (Figure 15), advantages of the RS-GVAR over the GVAR can be seen for between one and seven countries, with ratios approaching 0.79 for Sweden at the 3-quarter horizon. Compared to GDP, gains appear less significant in magnitudes as well as from a statistical viewpoint.

Concerning stock markets (Figure 16), only little improvement can be found when opposing the RS-GVAR to the GVAR. At the 4-quarter horizon, where gains, i.e. ratios smaller one, can be seen for six out of 18 countries, the average across all 18 countries would equal 1.2, suggesting a 20% loss in performance.

to reject the less out-of-sample observations are employed to conduct the test.

With respect to the comparison of GVAR vs the AR models' performance in forecasting GDP (Figure 17), notable gains for the majority countries have been measured. Ratios approach 0.64 for Luxembourg, Denmark, and Ireland at the 1-quarter horizon. The 4-quarter ahead results suggest a balanced improvement of up to 18% for Luxembourg and 8% on average across the 18 countries.

Price inflation forecasts (Figure 18) are significantly more precise from the GVAR compared to the AR, though at smaller magnitudes with regard to ratios. On average across countries and horizons, the ratio to the ARs equals 1.02, i.e. suggests that the GVAR cannot outperform the AR on average in the cross-section.

The GVAR performance compared to the AR benchmark for stock prices (Figure 19) would, as for the VAR benchmark, suggest notable gains. On average across countries and the four horizons, the RMSE ratio equals 0.73, with the maximum for instance for Portugal at the 1-quarter horizon equalling 0.54. All gains are measured to be significant from a statistical viewpoint.

Moving, finally, to the RS-GVAR comparison to AR benchmark forecasts, for GDP (Figure 20) we can see significant gains in performance for the majority of countries. The average ratio across countries and horizons suggests an approximate 10% improvement to the benchmark, with the maximum reaching 35% for Norway at the 4-quarter horizon. Again one can see a tendency for the relative performance to increase with the horizon.

The accuracy of inflation forecasts (Figure 21) from the RS-GVAR improves for between five to eight countries significantly compared to the AR. An average across countries and horizons, however, suggests that RS-GVAR and ARs perform rather equally well (mean ratio equal 1.04).

When judging on statistical grounds, for stock prices (Figure 22), the RS-GVAR generates more precise point forecasts indeed for up to 17 of 18 countries. At the 2- to 4-quarter horizons, at least half and then the majority of ratios fall below one. For the 4-quarter forecasts, the improvement amounts to 10% on average across the 18 countries.

4 Conclusions

The purpose of the paper was to develop a regime-switching global vector autoregressive model which allows the countries' dynamics to depend on *a priori* unobserved regimes and thereby let the global solution of the model be conditional on a *regime-constellation* across countries. An application served to demonstrate the use of the RS-GVAR methodology for regime-conditional scenario simulation and forecasting.

Regime-switching at country-level and therefore the derived solution for global dynamics has been found to be relevant in the application that has been presented. Impulse responses to otherwise identical shock scenarios have been found to depend on the assumed regime-constellation. For instance, a positive shock to real activity in the US would (besides generally spreading widely across the other countries) induce less pressure on prices at times of low activity as opposed to times of strong growth. It is an empirical regularity that supports the implications of theories that assign a role to how close production is to its capacity constraint and therefore imply a convex shape of the Phillips curve.

For the application presented in the paper, the out-of-sample performance of the RS-GVAR has been found to be superior to an otherwise identically structured GVAR without regime-switching. Performance gains could be observed in particular for real GDP (less so for inflation), with a tendency for the gains to become more pronounced for longer horizons (up to four quarters). In view of the rather short test period (16 quarters), the out-of-sample evaluation results should, however, be seen with caution and rather as indicative.

A methodological extension to the model in which Markov-type regimes across countries were modeled as independent would be to take explicit account of cross-country dependence at the unobserved regime level. A variant of the conventional Markov chain assumption as employed in the paper could for example look as follows:

$$P_i \left\{ s_{i,t+1} = m | s_{it} = l, s_{it}^* = k \right\} = p_{i,lmk}$$

where the probability that country i prevails in regime m at $t+1$ would not only depend on its own lagged regime status but also on the regime prevailing in the rest of the world s_{it}^* (using e.g. the same weights that are used in the GVAR core to here compute a weighted regime probability). In case that lagged global dependencies are present, the extension might further improve out-of-sample forecast accuracy of the predicted regimes or as well of the observed endogenous model variables.

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Table 1: Overview of countries/variables and basic summary statistics

		Real GDP YoY [YER4]				Real GDP QoQ [YER]				Expenditure prices QoQ [CED]				Stock prices QoQ [STOX]			
Country	Alias	Mean	STD	Min	Max	Mean	STD	Min	Max	Mean	STD	Min	Max	Mean	STD	Min	Max
Austria	AT	2.11	1.98	-5.43	4.72	0.52	0.67	-1.80	1.58	0.43	0.25	-0.12	0.95	0.89	12.12	-44.14	23.45
Belgium	BE	1.85	1.75	-4.18	5.19	0.45	0.63	-1.99	1.56	0.48	0.53	-0.93	1.87	0.90	10.94	-39.97	21.02
Canada	CA	2.58	1.97	-3.75	5.76	0.65	0.65	-2.02	1.65	0.40	0.34	-0.85	1.09	1.45	9.60	-27.17	19.00
Switzerland	CH	1.78	1.67	-3.22	4.26	0.45	0.59	-1.56	1.64	0.18	0.26	-0.72	0.97	0.92	10.11	-31.36	21.76
Germany	DE	1.33	2.29	-7.02	4.85	0.33	0.88	-4.04	1.90	0.31	0.31	-0.79	1.10	0.90	12.32	-37.89	25.13
Denmark	DK	1.30	2.54	-8.65	4.42	0.31	1.24	-2.52	3.83	0.48	0.41	-0.42	1.42	2.03	11.11	-34.89	24.14
Finland	FI	2.75	3.48	-10.42	7.11	0.68	1.34	-6.50	3.28	0.46	0.80	-1.42	2.97	1.77	15.68	-35.77	53.62
France	FR	1.64	1.61	-4.05	4.24	0.41	0.52	-1.56	1.33	0.36	0.34	-0.55	1.16	1.16	11.90	-30.66	24.73
Ireland	IE	4.39	5.01	-8.71	13.60	1.04	2.10	-3.78	6.82	0.58	0.81	-3.02	2.25	0.41	13.08	-41.32	28.09
Italy	IT	0.84	2.05	-6.65	4.06	0.19	0.71	-3.23	1.39	0.58	0.32	-0.62	1.12	0.41	12.25	-27.50	37.77
Luxembourg	LU	3.62	3.71	-8.56	10.81	0.83	1.85	-4.56	7.55	0.52	0.65	-1.27	2.37	1.68	11.95	-40.97	25.22
Netherlands	NL	2.16	2.12	-4.57	5.26	0.51	0.71	-2.24	1.91	0.51	0.44	-0.82	1.76	0.61	12.46	-36.70	20.06
Norway	NO	2.16	2.02	-2.30	6.98	0.53	1.13	-2.03	3.42	0.49	0.86	-1.52	3.18	2.27	13.39	-37.45	23.75
New Zealand	NZ	2.62	1.82	-2.19	5.67	0.66	0.80	-1.13	2.78	0.50	0.39	-0.30	1.52	-0.04	7.16	-18.41	20.25
Portugal	PT	1.60	2.31	-4.20	5.45	0.36	0.89	-2.36	2.20	0.62	0.52	-1.65	2.03	0.69	12.05	-28.44	34.23
Sweden	SE	2.62	2.75	-6.58	7.51	0.64	1.01	-3.96	2.31	0.35	0.44	-0.55	1.63	1.88	13.17	-30.49	36.24
United Kingdom	UK	2.21	2.43	-7.03	5.09	0.54	0.73	-2.33	1.41	0.55	0.48	-0.59	1.74	0.79	8.57	-22.63	19.87
United States	US	2.38	2.18	-5.16	5.18	0.59	0.72	-2.32	1.93	0.52	0.39	-1.43	1.12	1.25	9.46	-26.20	20.25

Note: Means, Min and Max expressed in percent. STD in percentage points.

Table 2: Regime-conditional means

Variable	R	AT	BE	CA	CH	DE	DK	FI	FR	IE	IT	LU	NL	NO	NZ	PT	SE	UK	US	Mean
YER4	1	3.51	3.29	4.01	2.86	3.22	3.08	4.45	2.89	6.67	2.70	7.29	3.78	4.60	3.86	4.15	4.19	3.31	3.73	3.98
	2	1.70	1.61	1.94	0.66	0.82	0.78	1.85	1.33	-0.20	0.82	2.94	1.29	1.78	1.94	1.46	1.78	1.50	1.68	1.43
	3	-3.17	-1.69	-2.25	-1.41	-4.53	-5.01	-7.58	-1.99	-7.43	-4.13	-5.05	-3.16	-1.55	-0.93	-1.79	-6.08	-4.39	-2.89	-3.61
YER	1	0.86	0.79	0.96	0.67	0.78	0.83	1.12	0.71	1.66	0.62	1.76	0.95	1.05	0.95	1.06	1.04	0.81	0.93	3.90
	2	0.36	0.38	0.54	0.26	0.22	0.08	0.39	0.32	-0.37	0.18	0.77	0.25	0.43	0.41	0.27	0.41	0.27	0.38	1.23
	3	-0.55	-0.30	-0.43	-0.32	-0.88	-1.11	-1.81	-0.41	-1.80	-0.98	-1.16	-0.62	-0.36	0.07	-0.45	-1.55	-0.90	-0.59	-3.15
CED	1	0.47	0.64	0.43	0.24	0.35	0.50	0.49	0.36	0.84	0.64	0.62	0.59	0.64	0.44	0.71	0.28	0.45	0.52	2.05
	2	0.43	0.51	0.42	0.19	0.32	0.49	0.41	0.43	0.20	0.61	0.54	0.52	0.44	0.55	0.72	0.44	0.88	0.59	1.93
	3	0.22	-0.18	0.07	-0.13	0.05	0.36	0.39	-0.02	-1.02	0.08	0.17	-0.02	0.60	0.64	0.26	0.40	0.48	0.20	0.57
STOX	1	0.26	3.37	2.14	0.82	1.44	5.84	4.17	3.44	2.23	4.61	2.88	3.25	6.34	1.58	4.60	3.15	0.82	3.42	12.08
	2	1.68	-0.56	1.00	1.50	0.71	-1.59	-3.26	-0.56	-6.39	-0.67	0.97	-2.68	1.62	-0.52	-1.19	-0.25	1.22	-1.29	-2.28
	3	-1.49	-0.11	-0.87	-0.09	-1.69	0.04	0.46	-2.07	-3.12	-3.83	2.20	1.85	-3.60	-5.76	-0.85	3.67	-0.71	-2.21	-4.04

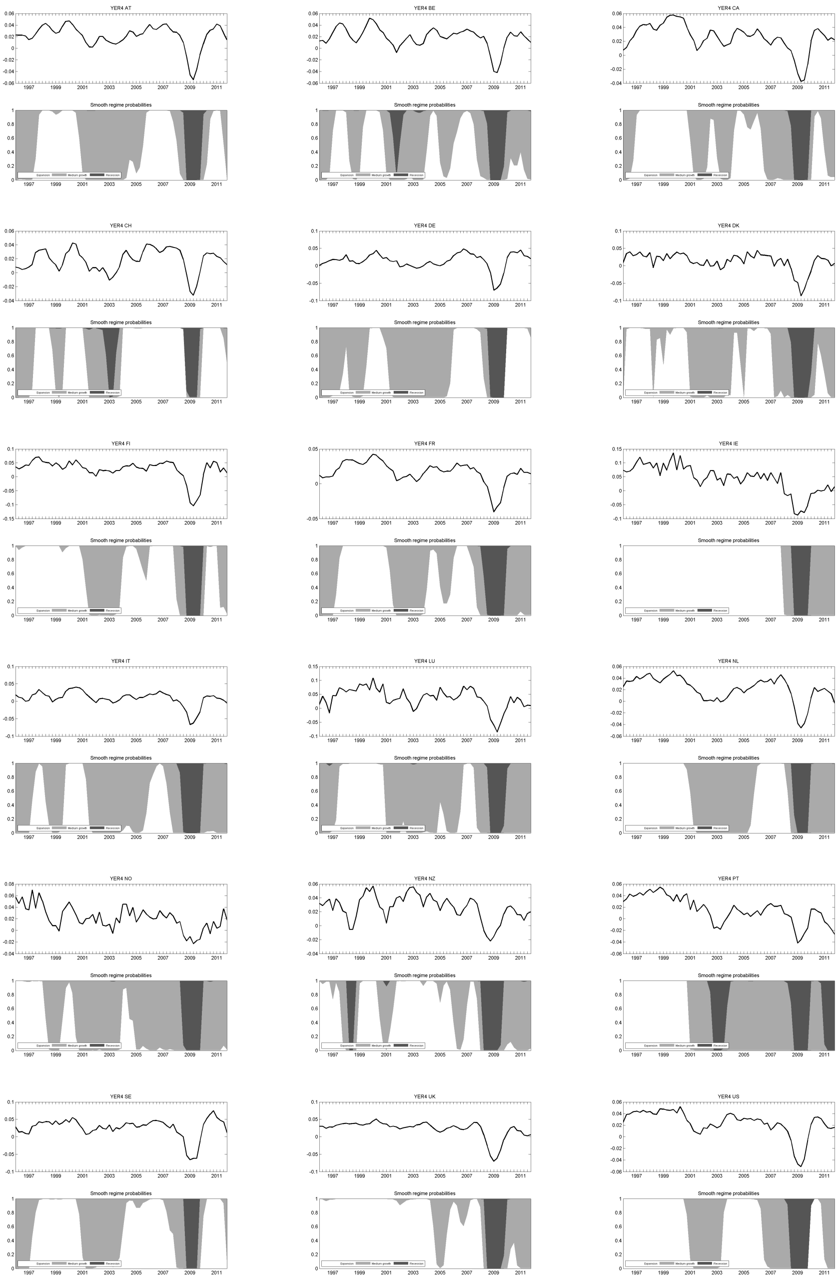
Note: The reported regime-conditional mean estimates are weighted averages of the respective model variables, where the inferred regime probabilities are used as weights. All expressed in percent.

Table 3: (RS-)GVAR model structure

Country	AR lags	FVV lags	Durbin-Watson statistics		
			YER	CED	STOX
AT	1	1	1.82	1.78	2.05
BE	0	0	1.77	2.05	1.89
CA	0	1	1.96	1.91	1.75
CH	0	0	1.89	1.77	2.12
DE	1	1	1.88	1.93	2.13
DK	1	0	2.32	2.11	2.19
FI	1	1	1.82	1.99	2.16
FR	1	1	2.07	2.17	2.08
IE	1	0	2.24	2.14	1.97
IT	1	0	1.91	1.86	1.82
LU	1	0	1.98	2.19	2.03
NL	0	1	1.93	1.89	2.17
NO	1	0	2.17	2.01	2.18
NZ	0	0	2.02	1.68	1.71
PT	0	0	2.37	2.32	2.48
SE	0	0	2.04	2.45	1.81
UK	1	0	1.82	1.99	2.59
US	0	0	2.31	1.92	1.76

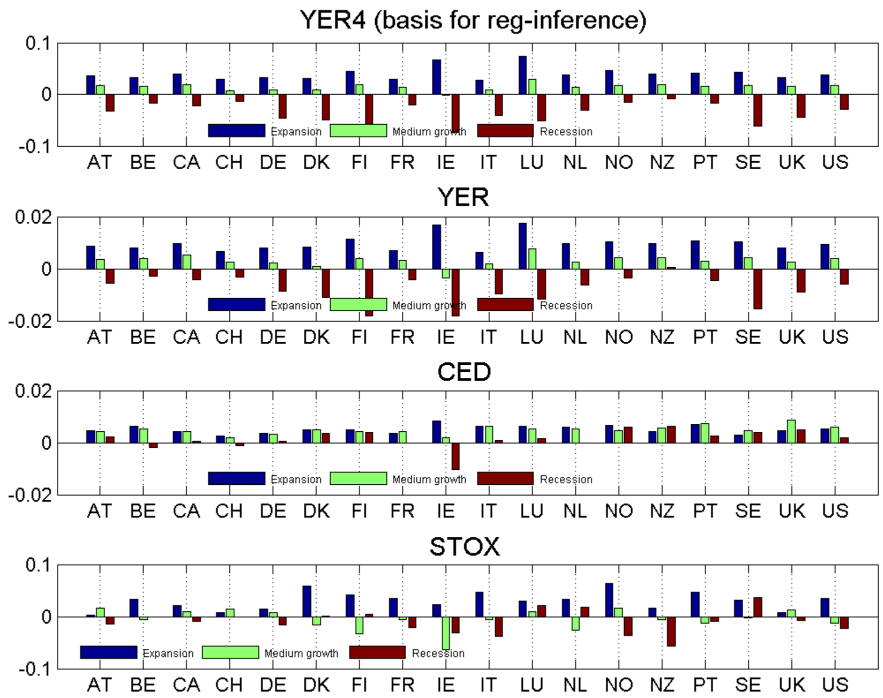
Note: Durbin-Watson statistics are based on the respective local models' residuals for the sample from 1996Q2 - 2011Q4.

Figure 1: Inferred smooth regime probabilities across countries



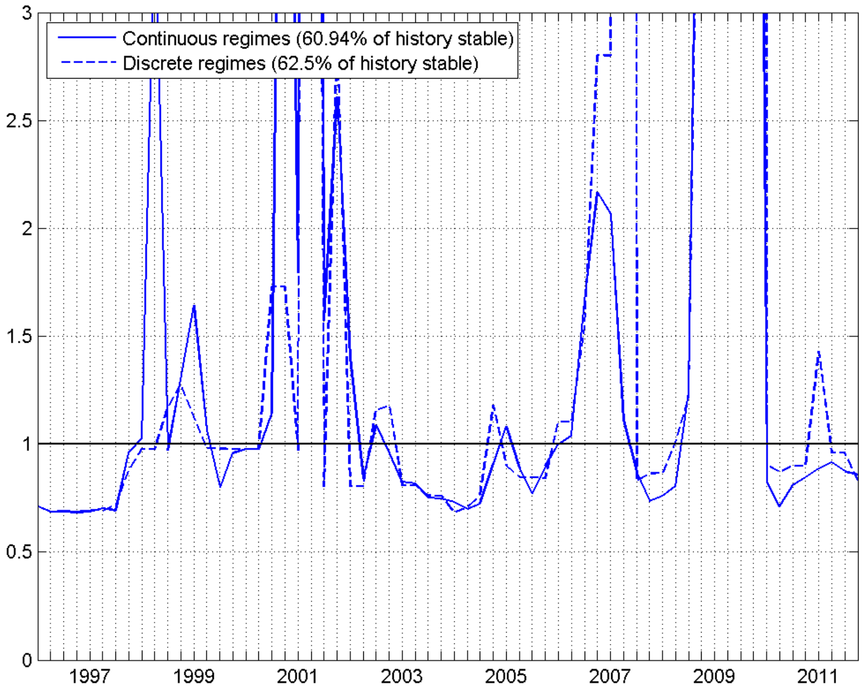
Note: Year-on-year rates of change of real GDPs are plotted along with smooth regime probabilities over the period from 1996Q1-2011Q4.

Figure 2: Regime-conditional means of model variables



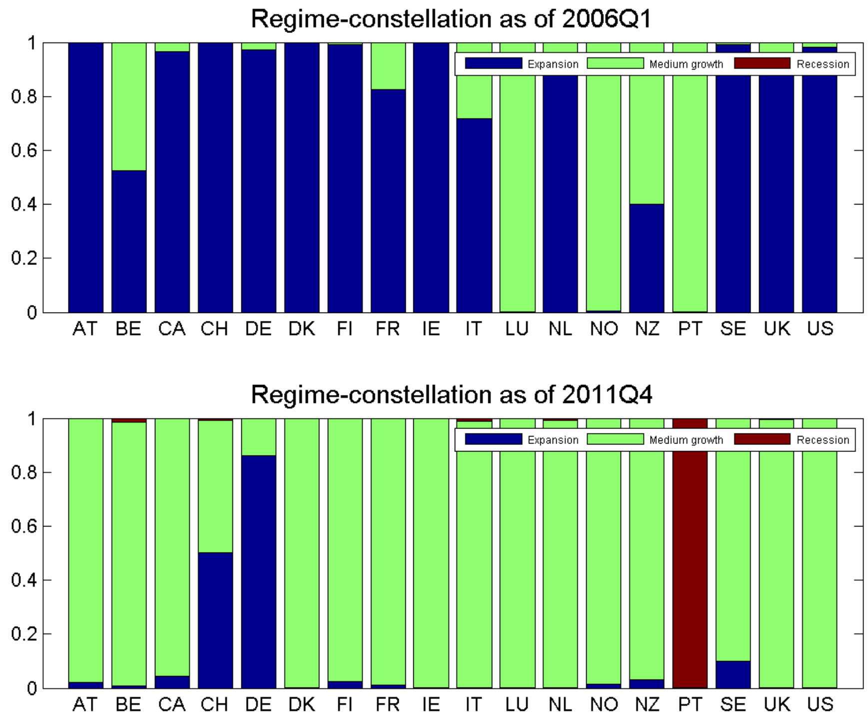
Note: The regime-conditional mean estimates are weighted averages of the respective model variables, where the inferred regime probabilities (see Figure 1) are used as weights.

Figure 3: Recursive maximum of modulus of eigenvalues of the RS-GVAR



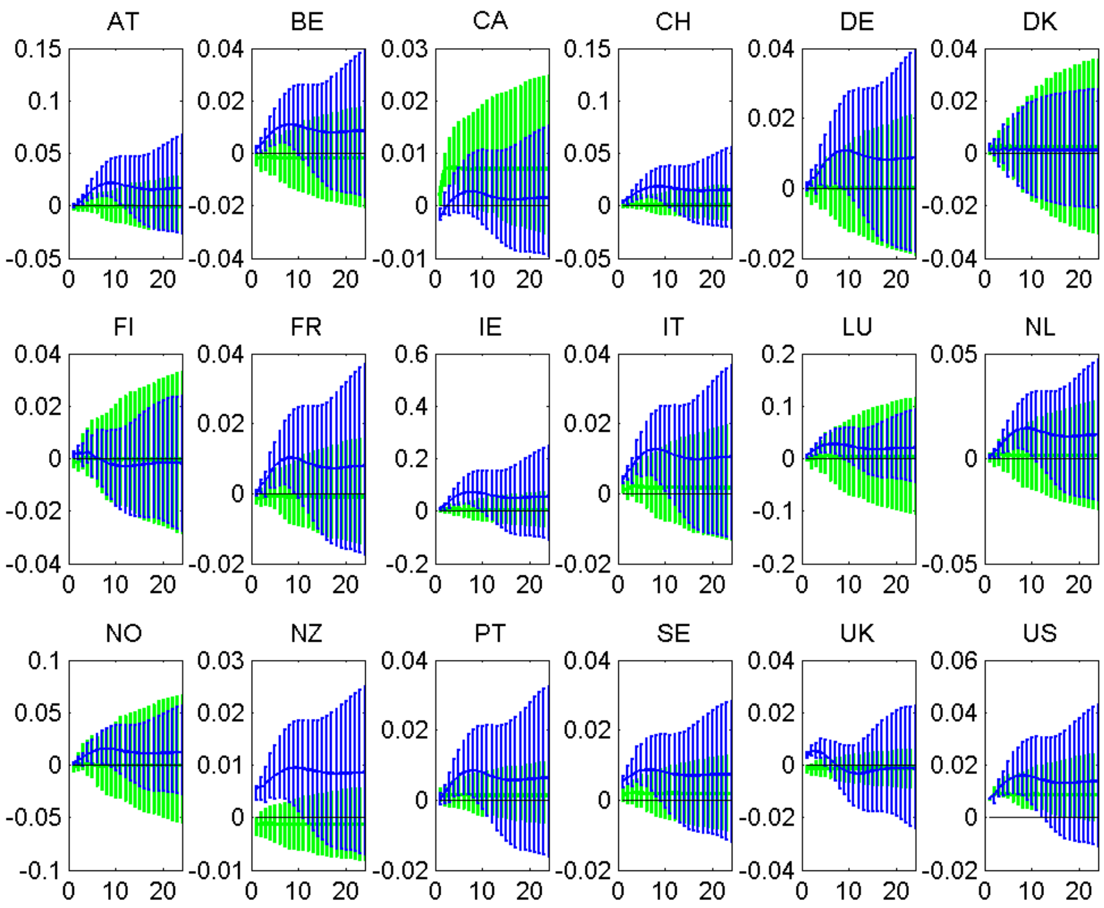
Note: The maximum of the moduli of the RS-GVAR's eigenvalues are obtained from the RS-GVAR's solution derived from the parameter space that is evaluated at point-in-time estimates of the smooth regime probabilities across markets (see also Figure 1). For the system to be stable, the maximum eigenvalue must be less than one in modulus.

Figure 4: Regime constellation for simulating shock scenarios



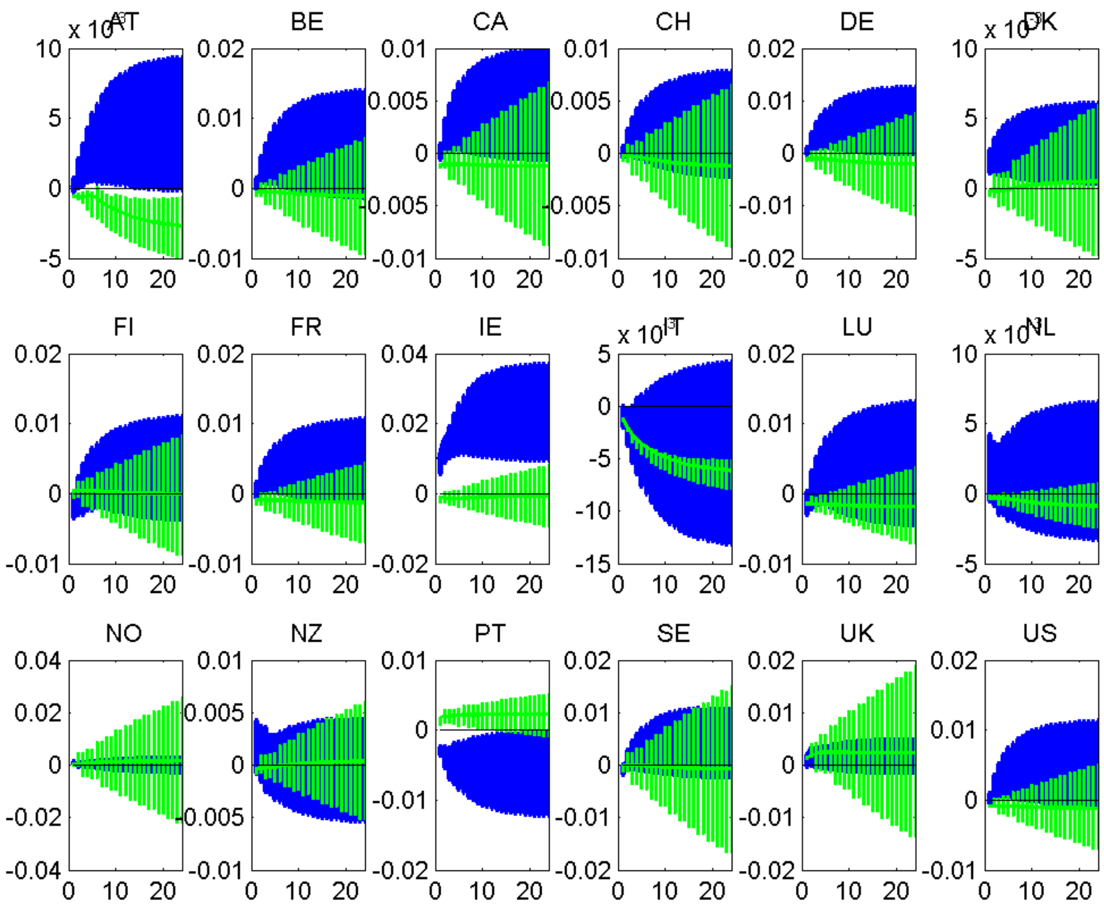
Note: Plotted are a set of smooth regime probabilities, which can also be read from Figure 1 at the two selected points in time.

Figure 5: Scenario 1 (positive shock to US GDP) - Generalized, cumulative GDP responses



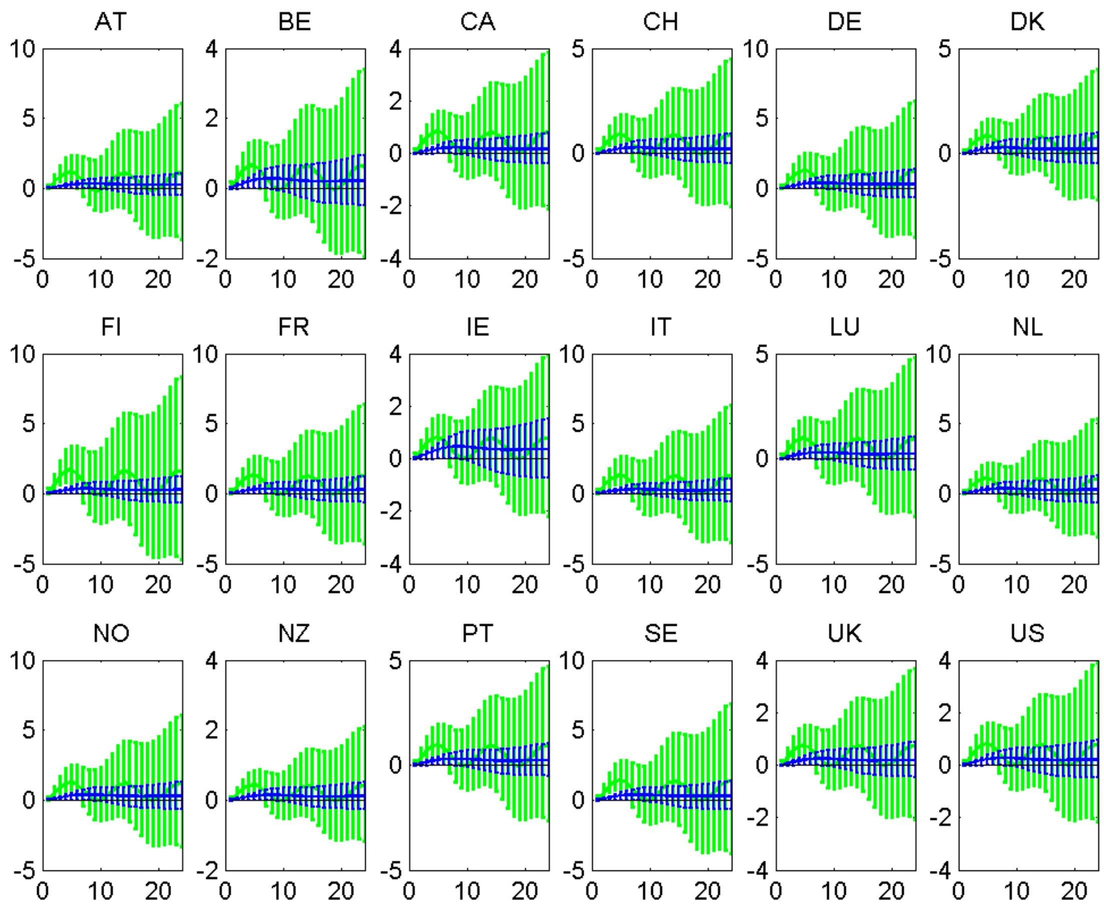
Note: The generalized impulse responses are conditional on a regime-constellation as of 2006Q1 (blue) and 2011Q4 (green). See Figure 4 for the exact regime-constellations. See text for details.

Figure 6: Scenario 1 (positive shock to US GDP) - Generalized, cumulative inflation responses



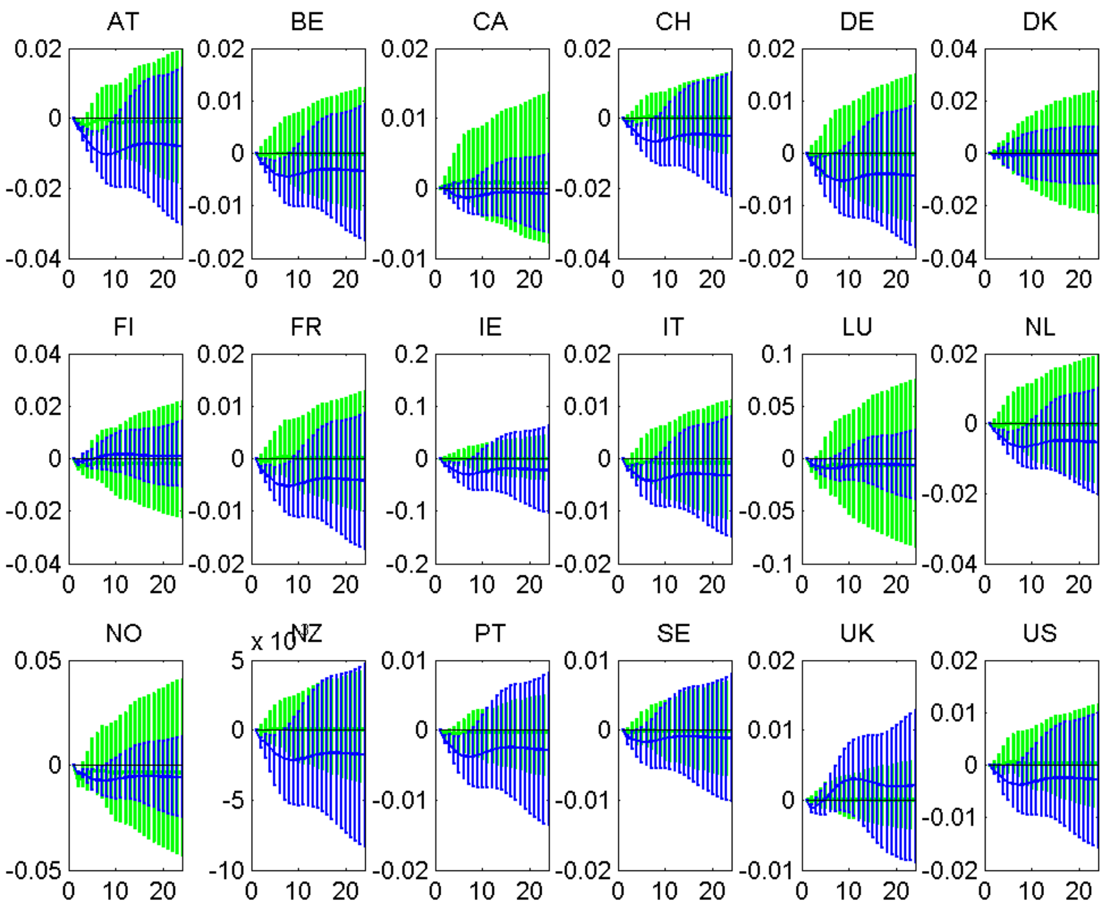
Note: The generalized impulse responses are conditional on a regime-constellation as of 2006Q1 (blue) and 2011Q4 (green). See Figure 4 for the exact regime-constellations. See text for details.

Figure 7: Scenario 1 (positive shock to US GDP) - Generalized, cumulative stock price responses



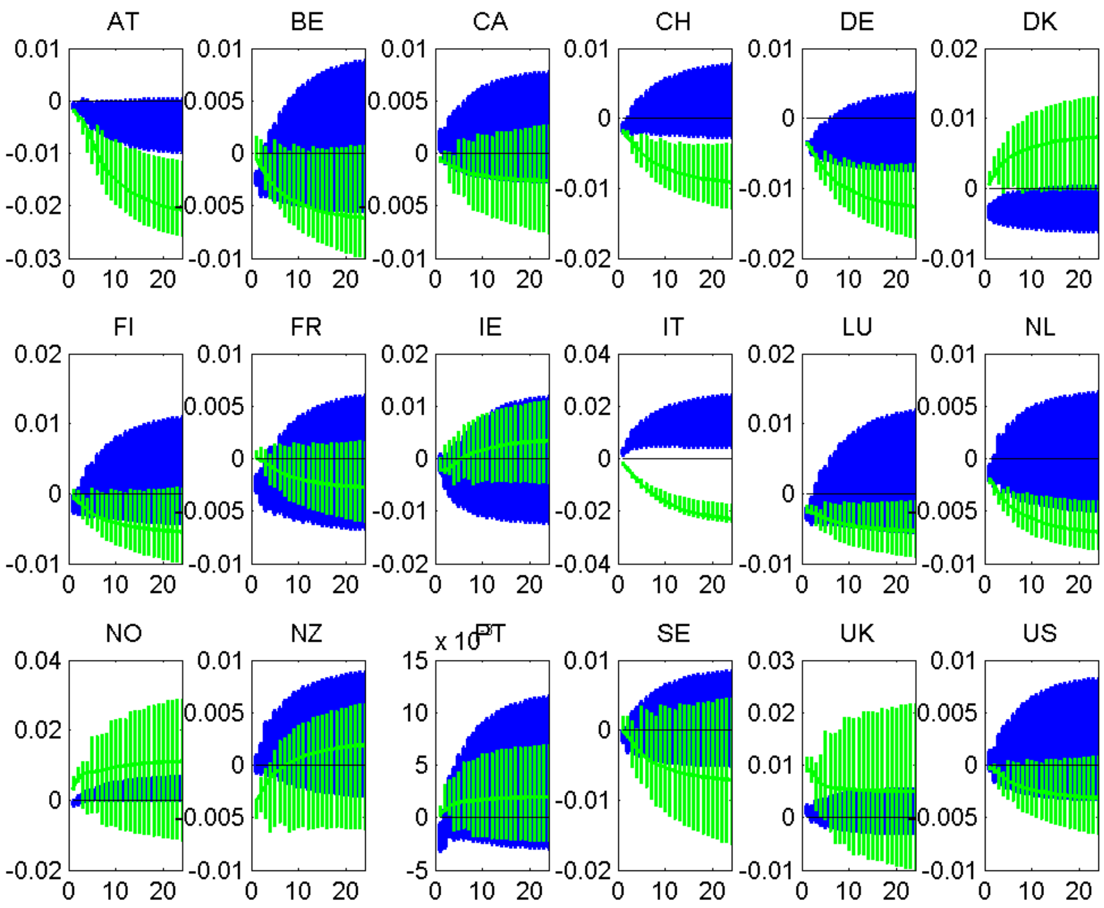
Note: The generalized impulse responses are conditional on a regime-constellation as of 2006Q1 (blue) and 2011Q4 (green). See Figure 4 for the exact regime-constellations. See text for details.

Figure 8: Scenario 2 (negative shock to DE inflation) - Generalized, cumulative GDP responses



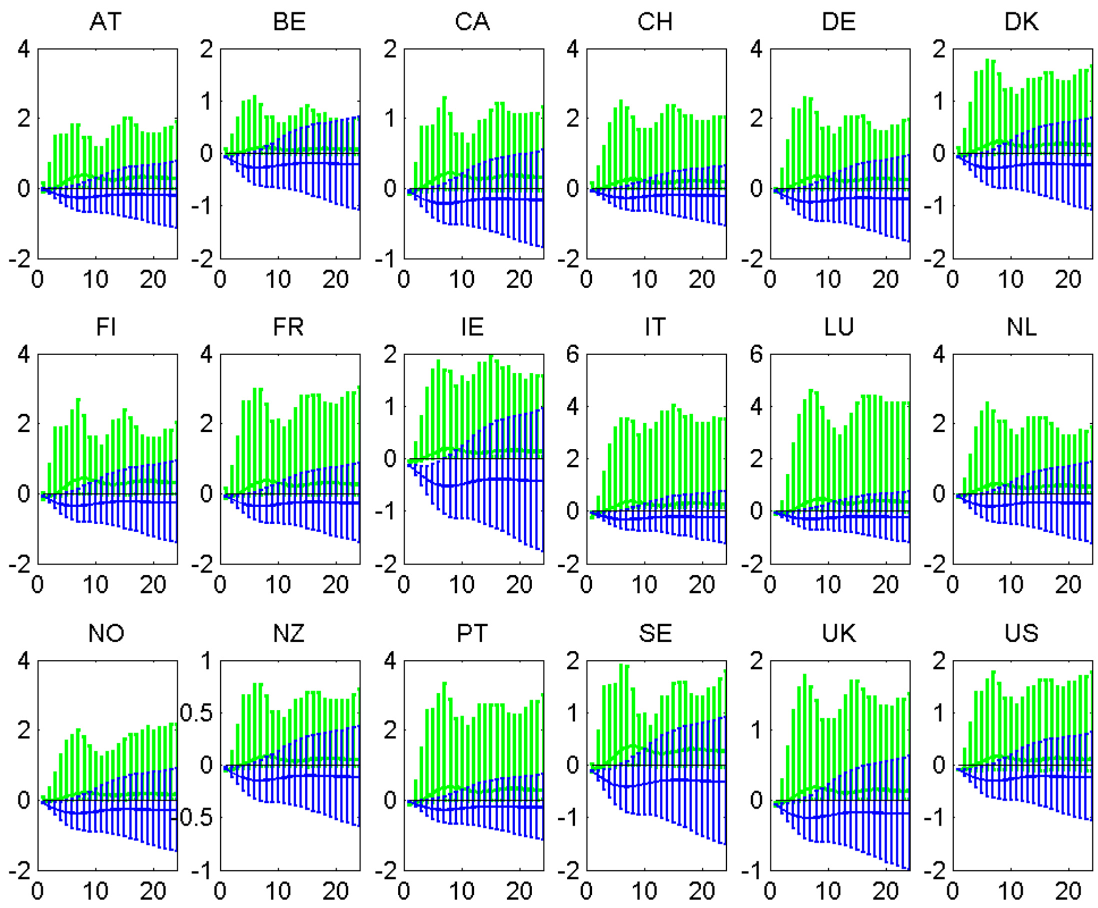
Note: The generalized impulse responses are conditional on a regime-constellation as of 2006Q1 (blue) and 2011Q4 (green). See Figure 4 for the exact regime-constellations. See text for details.

Figure 9: Scenario 2 (negative shock to DE inflation) - Generalized, cumulative inflation responses



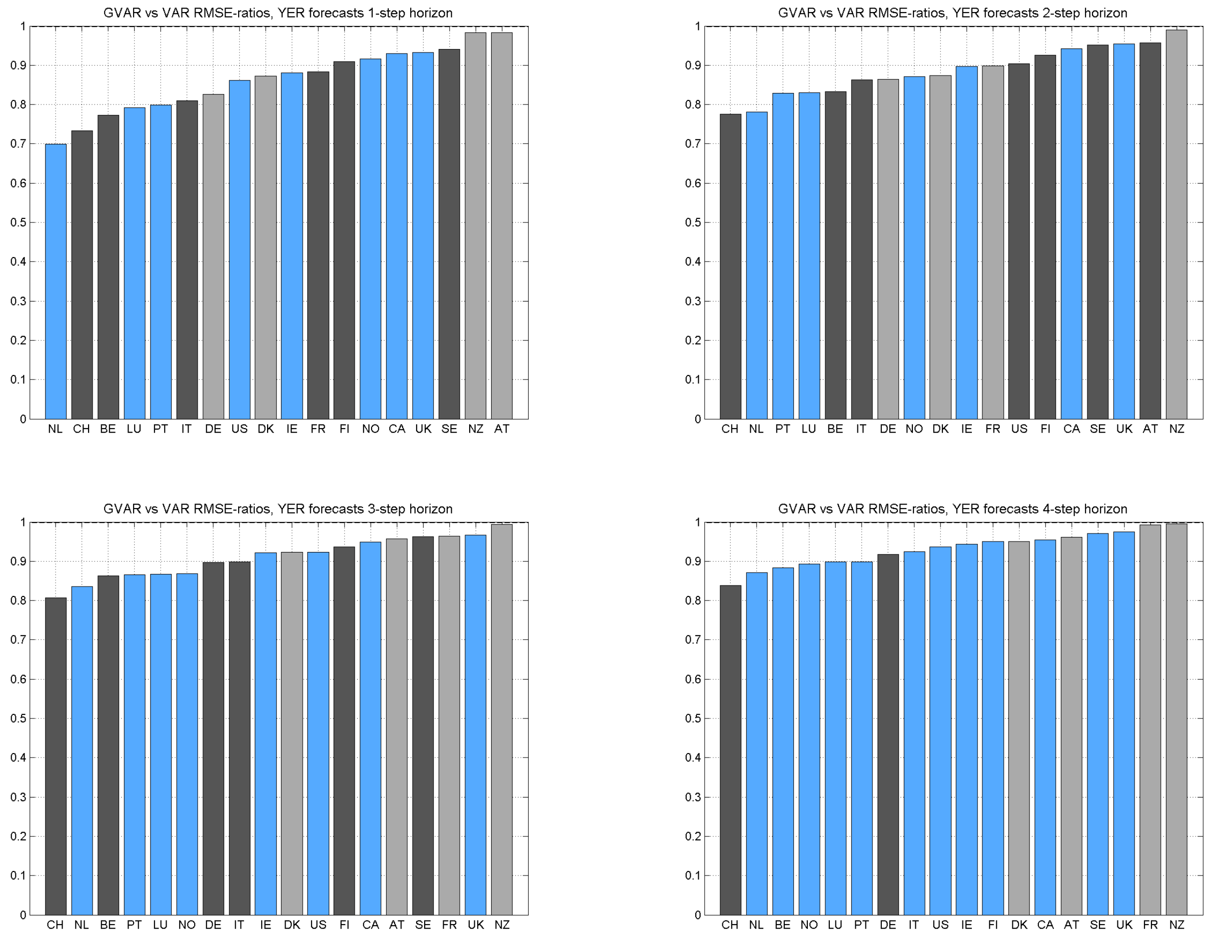
Note: The generalized impulse responses are conditional on a regime-constellation as of 2006Q1 (blue) and 2011Q4 (green). See Figure 4 for the exact regime-constellations. See text for details.

Figure 10: Scenario 2 (negative shock to DE inflation) - Generalized, cumulative stock price responses



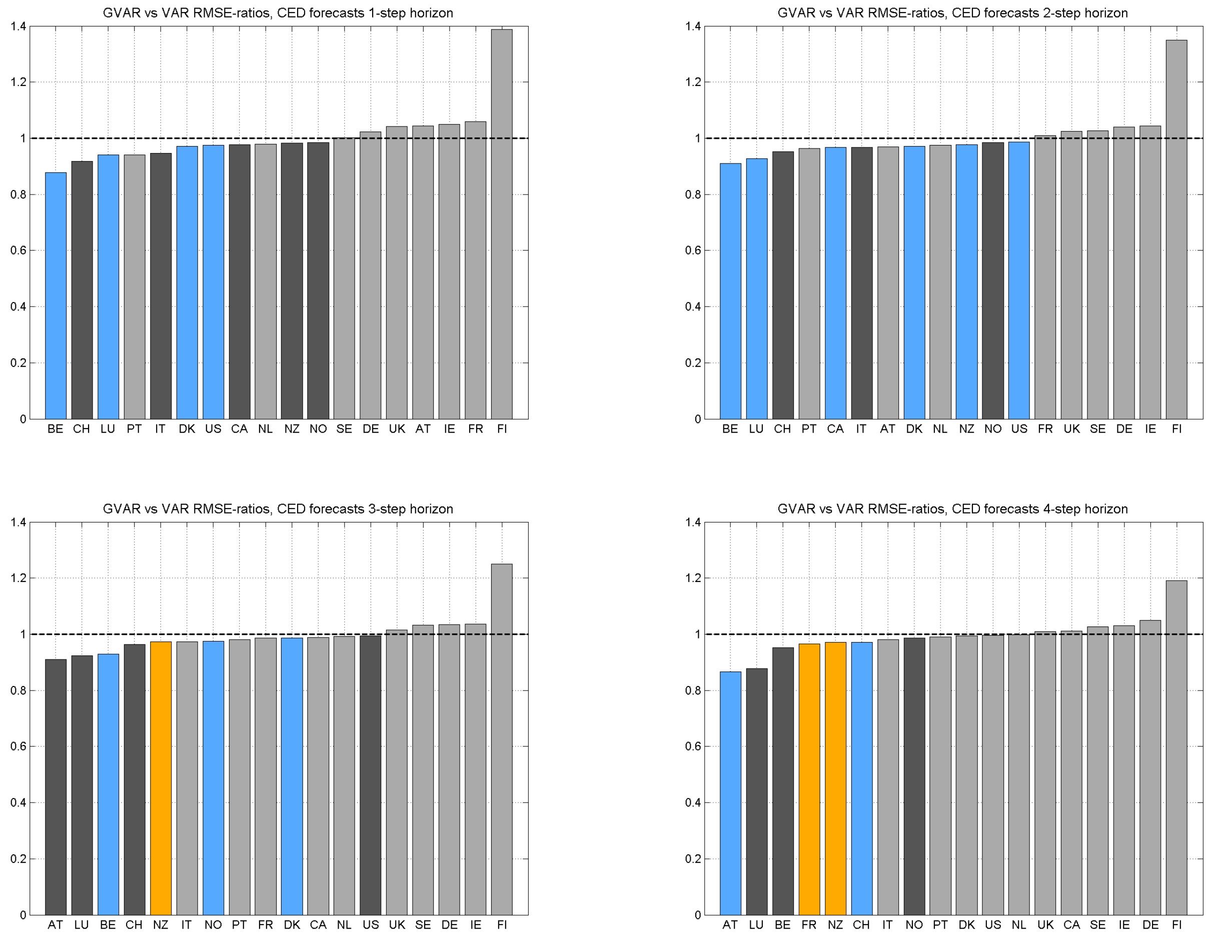
Note: The generalized impulse responses are conditional on a regime-constellation as of 2006Q1 (blue) and 2011Q4 (green). See Figure 4 for the exact regime-constellations. See text for details.

Figure 11: GVAR vs VAR, Real GDP, Out-of-sample forecast performance 2008Q1-2011Q4



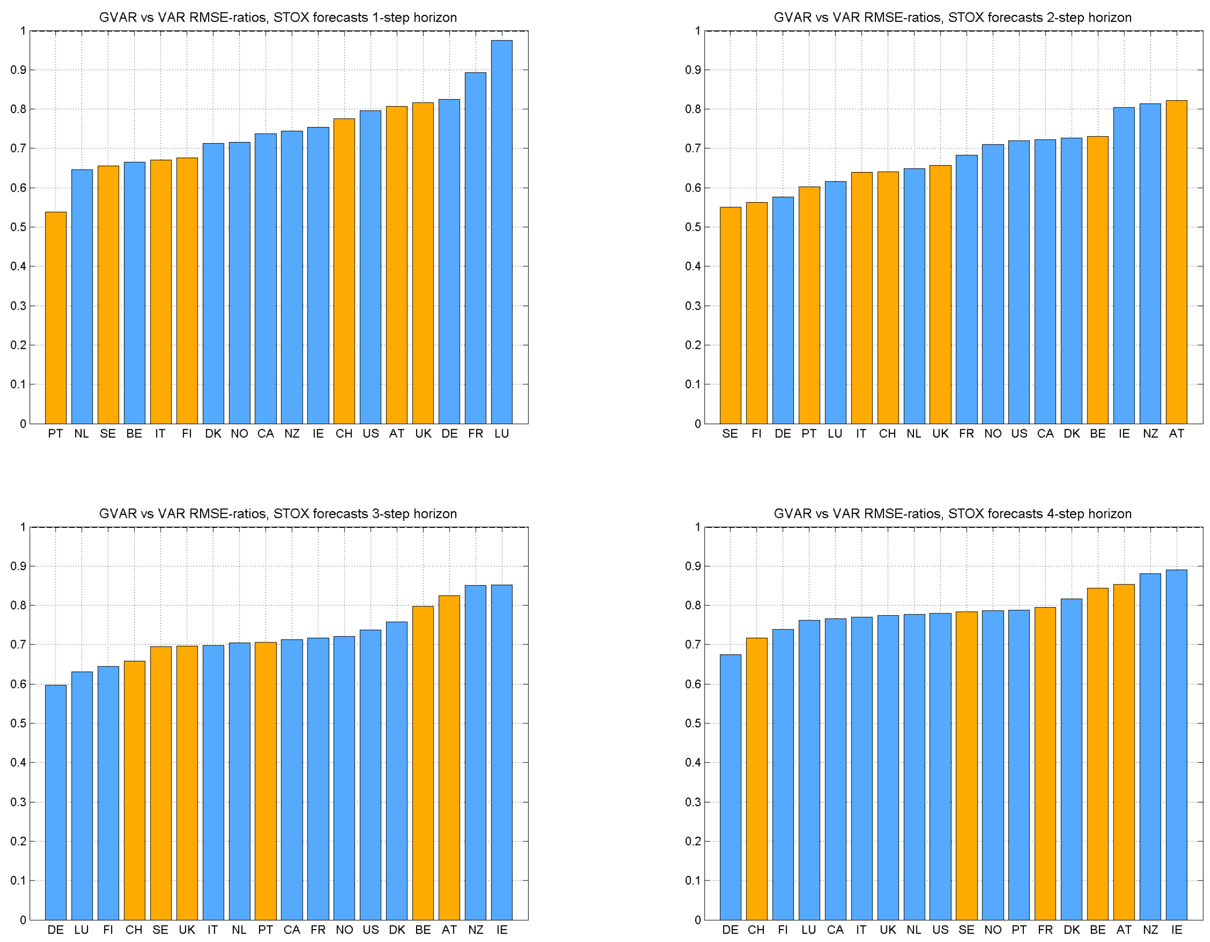
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 12: GVAR vs VAR, Price inflation, Out-of-sample forecast performance 2008Q1-2011Q4



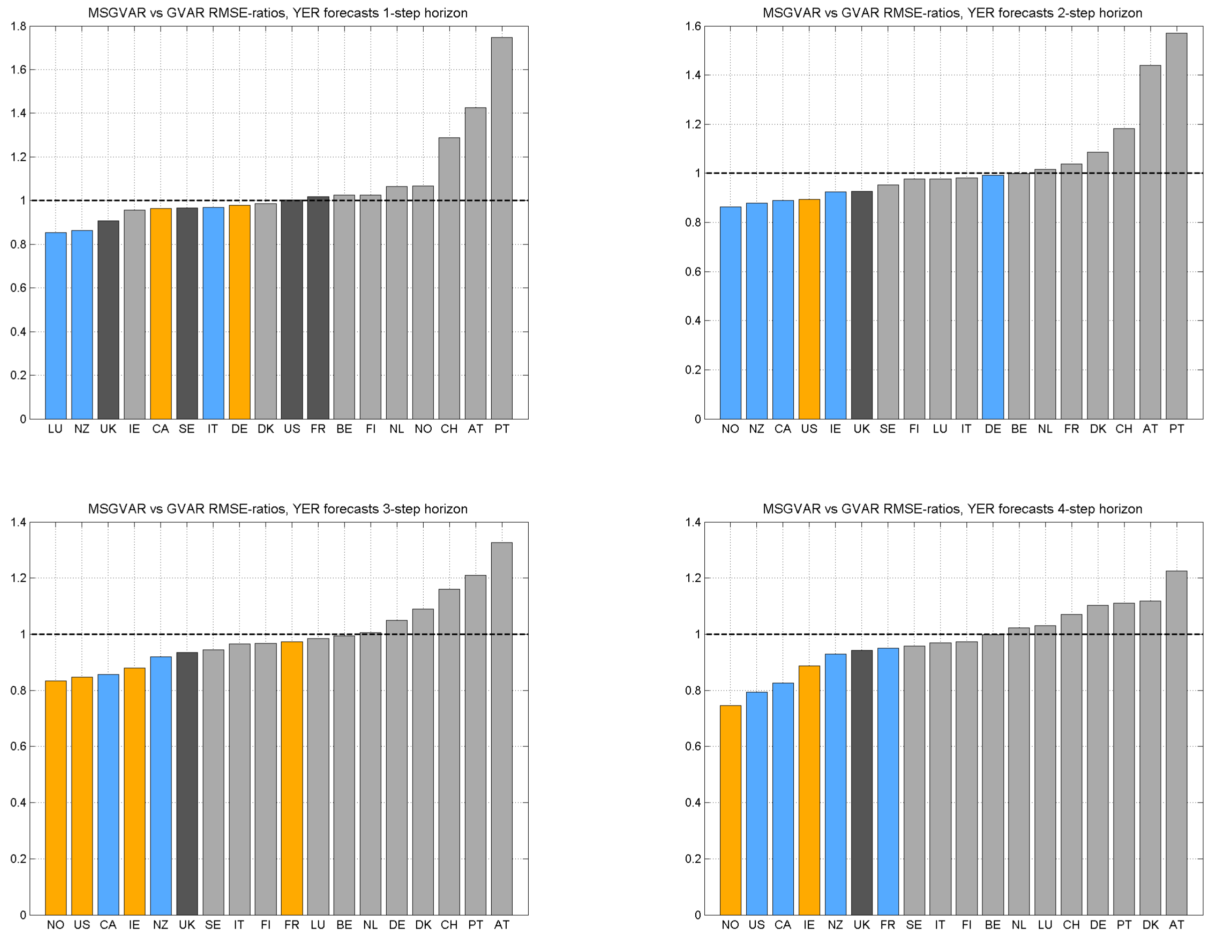
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 13: GVAR vs VAR, Stock prices, Out-of-sample forecast performance 2008Q1-2011Q4



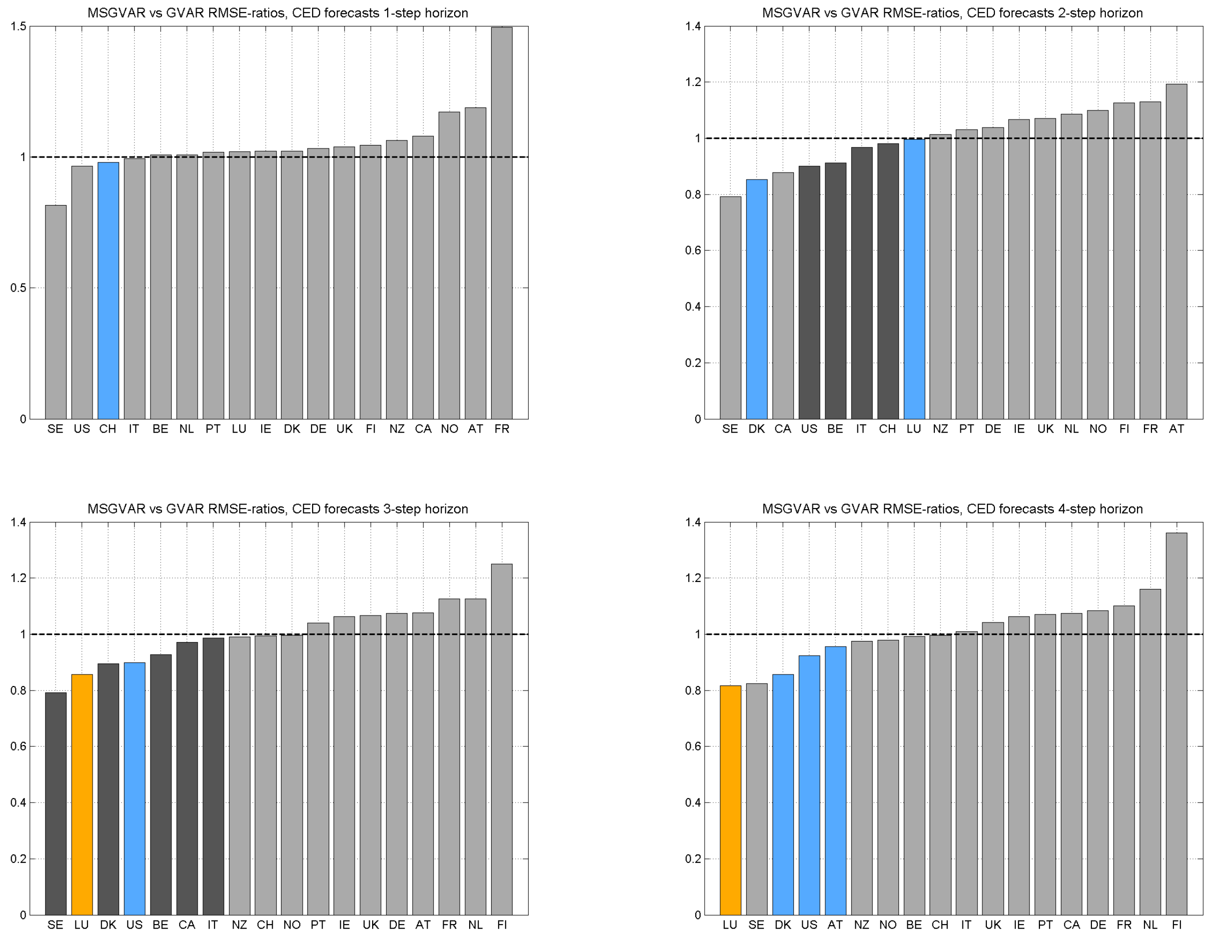
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 14: RS-GVAR vs GVAR, Real GDP, Out-of-sample forecast performance 2008Q1-2011Q4



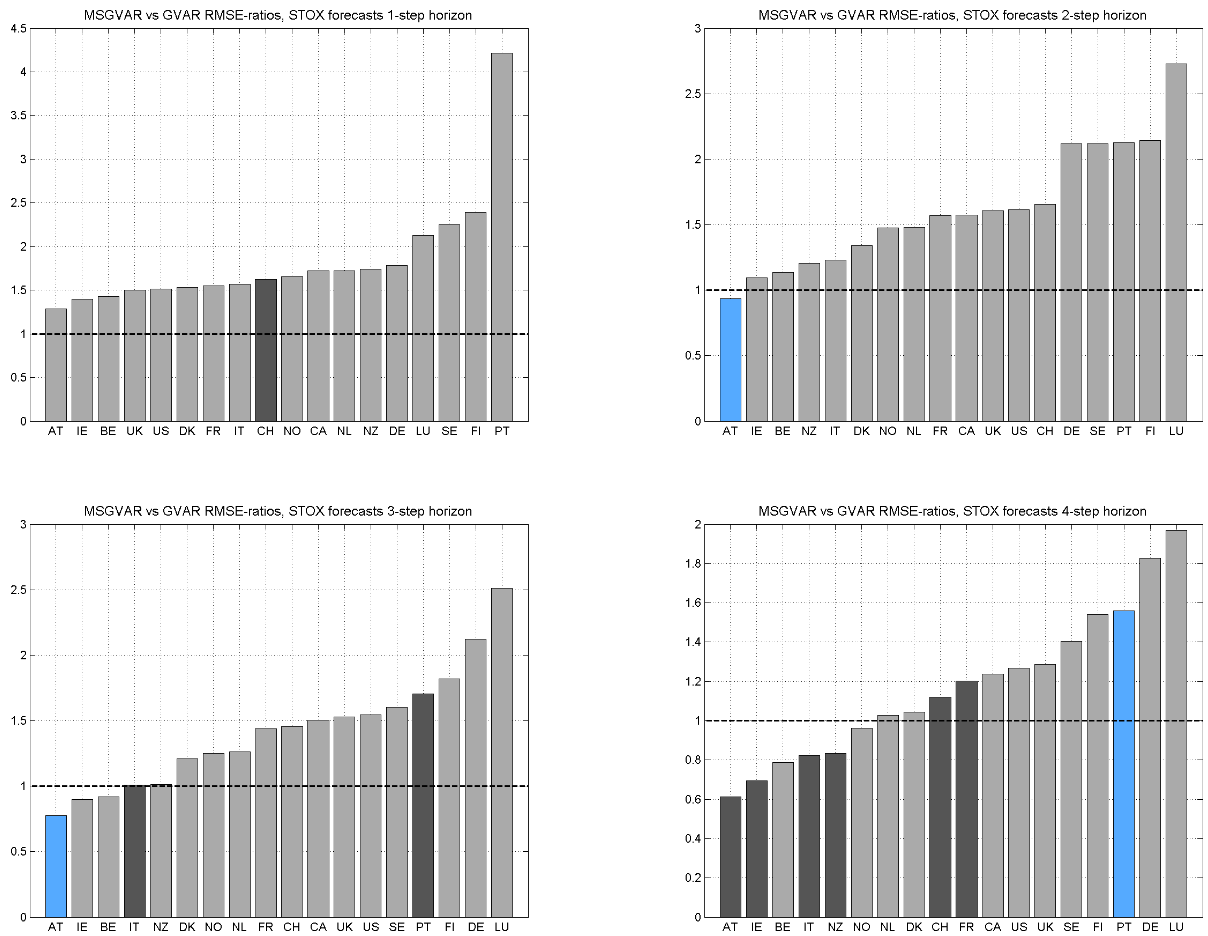
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 15: RS-GVAR vs GVAR, Price inflation, Out-of-sample forecast performance 2008Q1-2011Q4



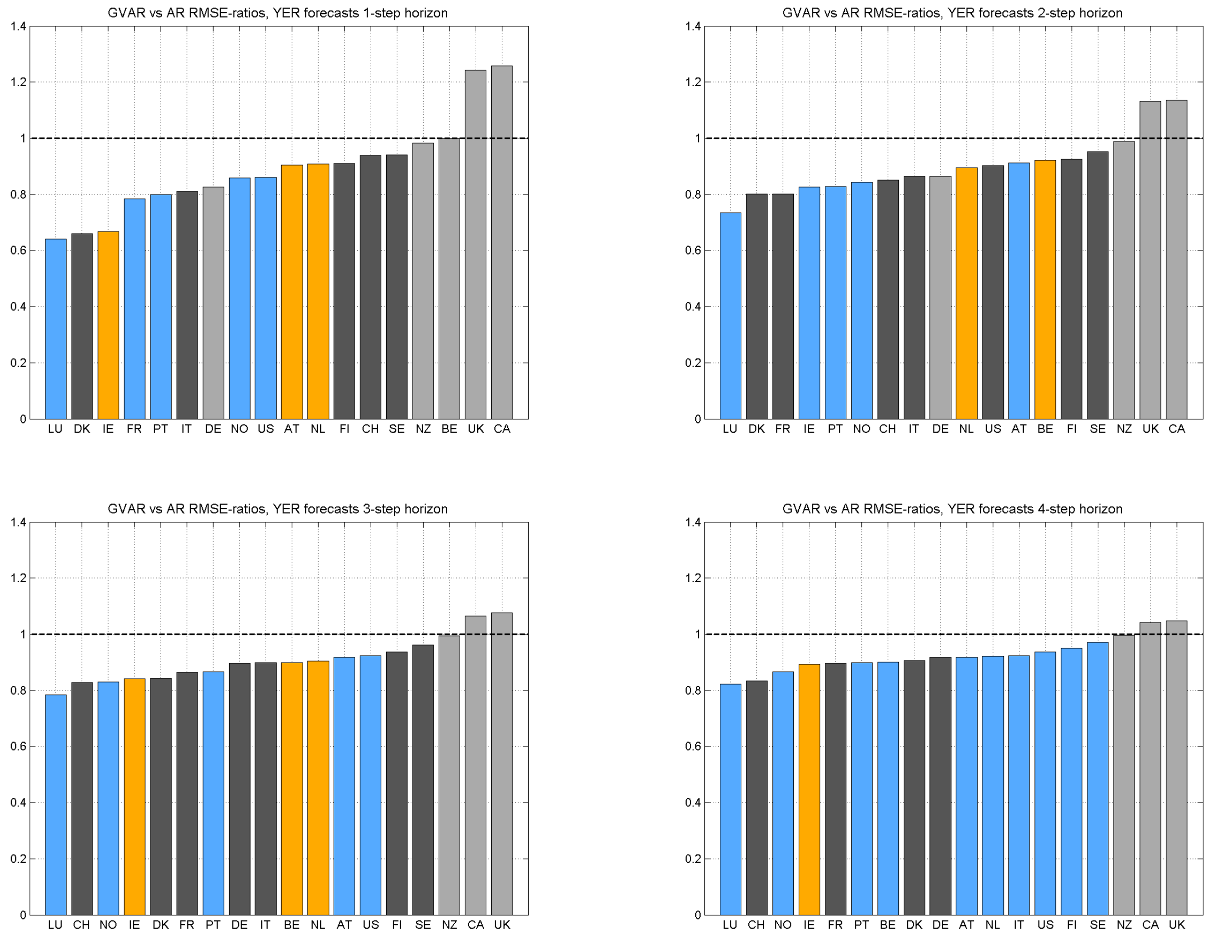
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 16: RS-GVAR vs GVAR, Stock prices, Out-of-sample forecast performance 2008Q1-2011Q4



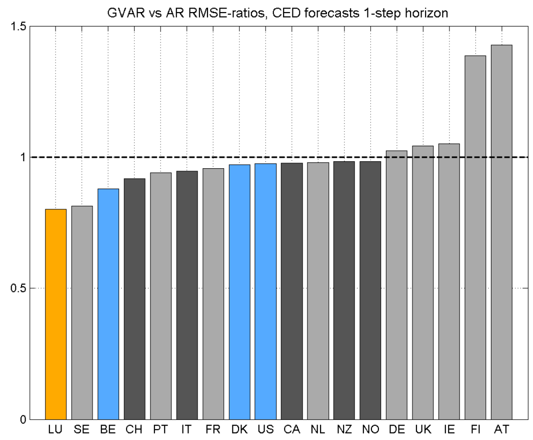
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

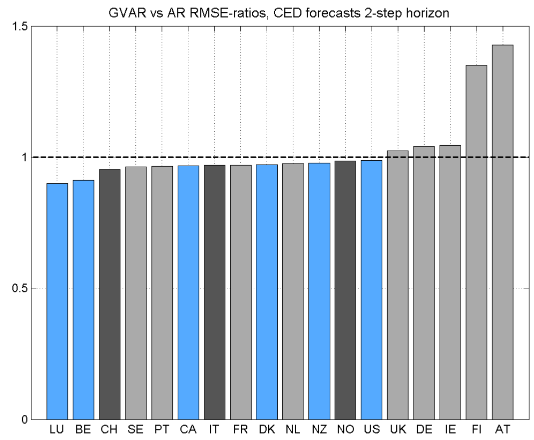
Figure 17: GVAR vs AR, Real GDP, Out-of-sample forecast performance 2008Q1-2011Q4

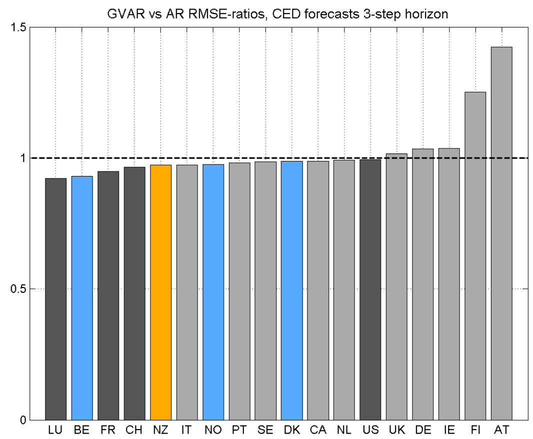


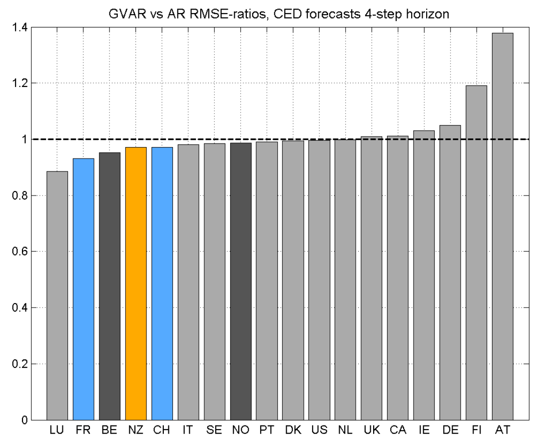
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 18: GVAR vs AR, Price inflation, Out-of-sample forecast performance 2008Q1-2011Q4



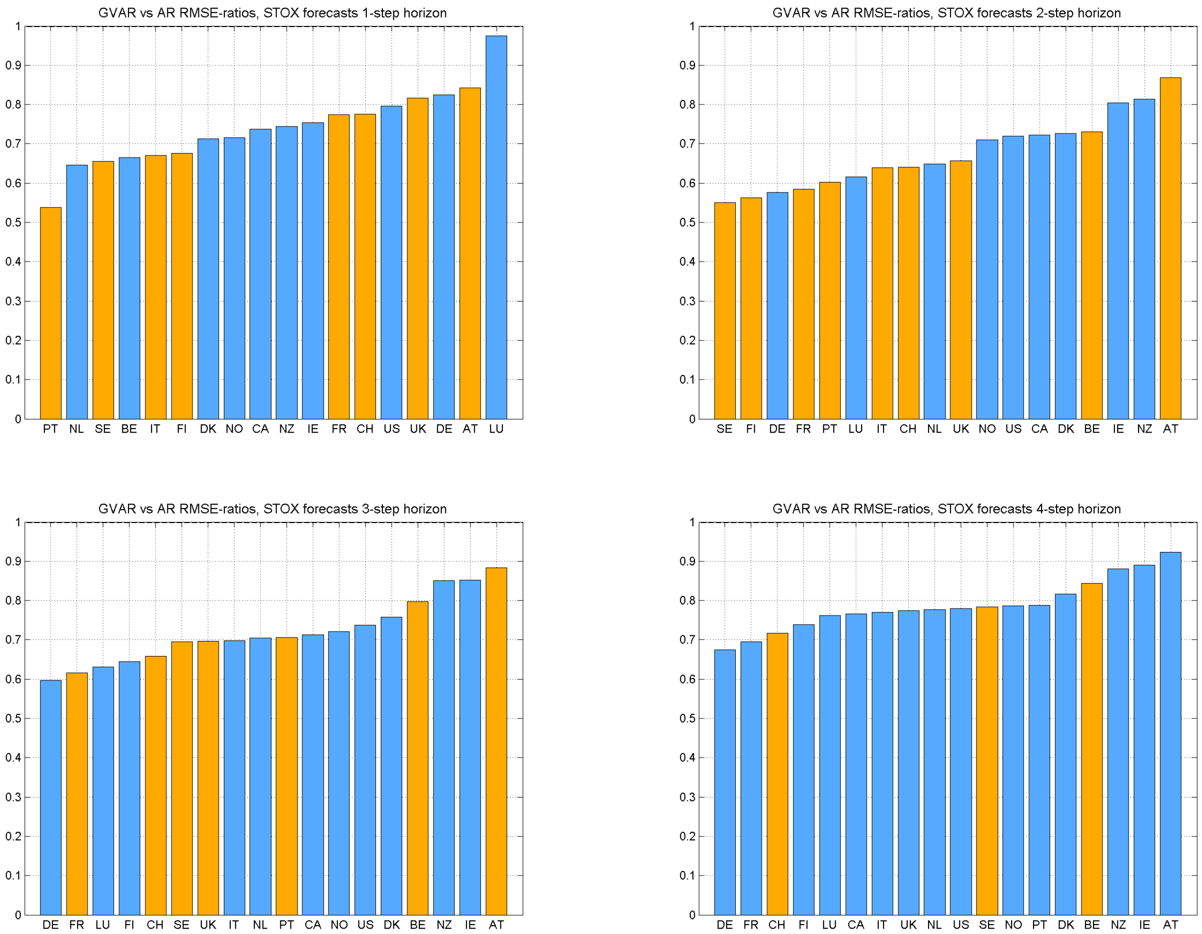






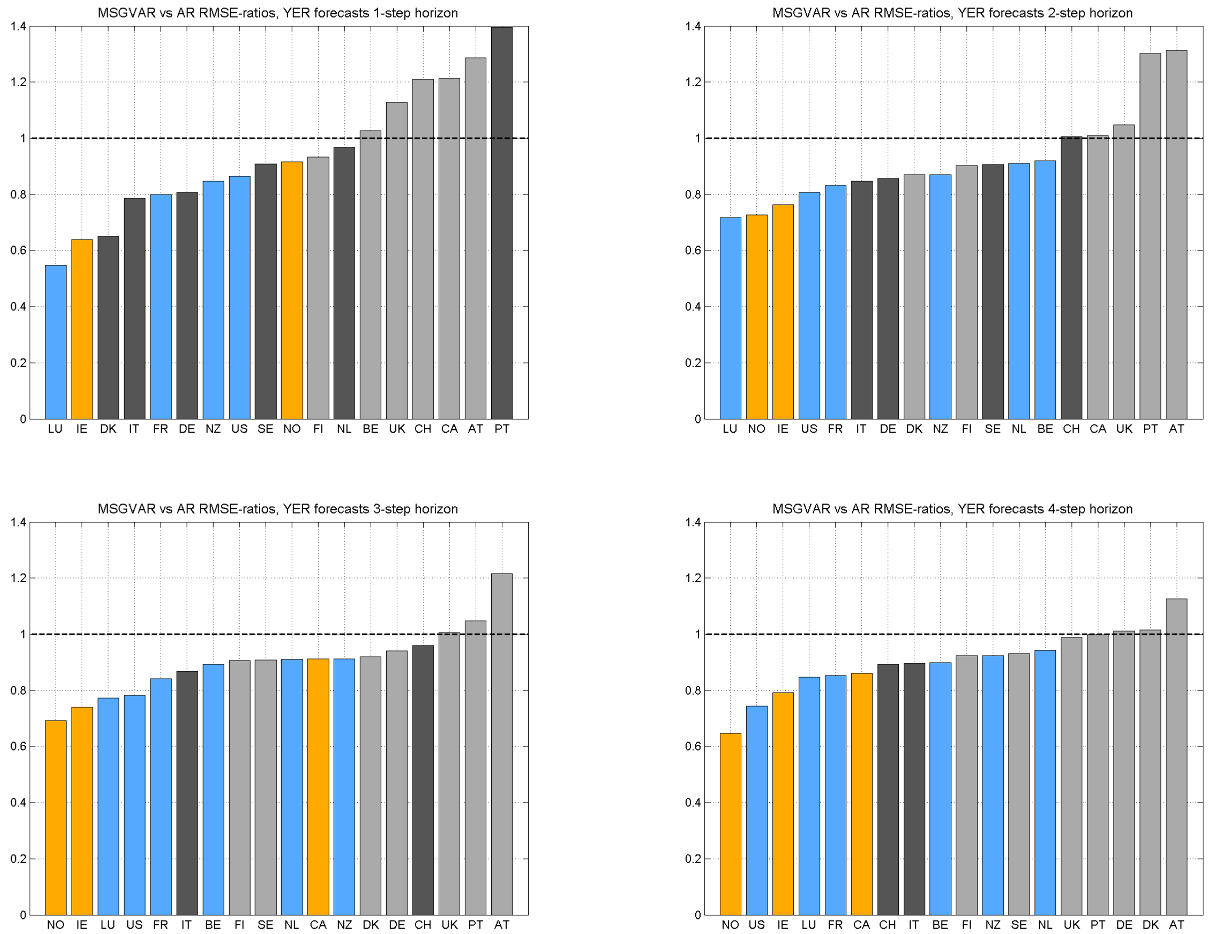
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 19: GVAR vs AR, Stock prices, Out-of-sample forecast performance 2008Q1-2011Q4



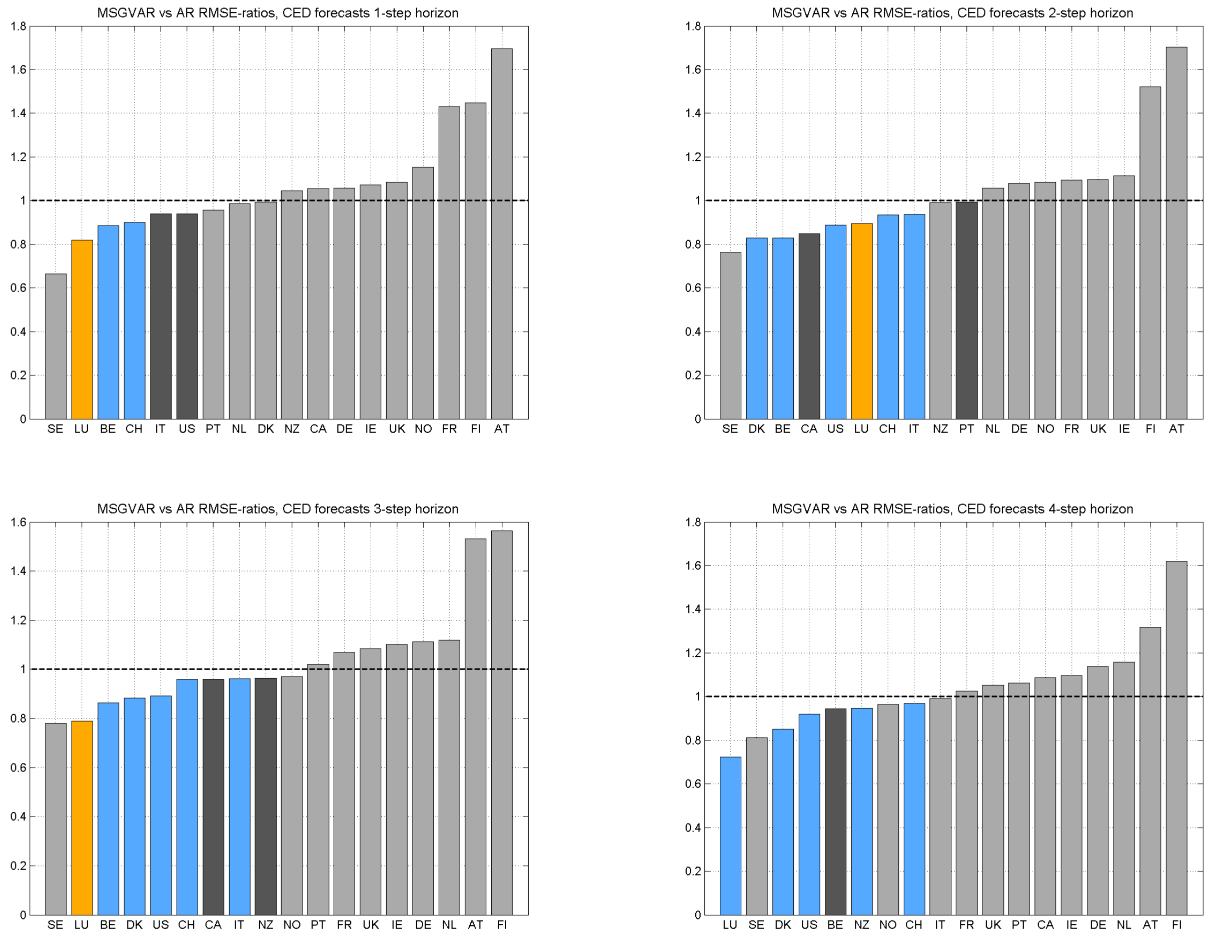
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 20: RS-GVAR vs AR, Real GDP, Out-of-sample forecast performance 2008Q1-2011Q4



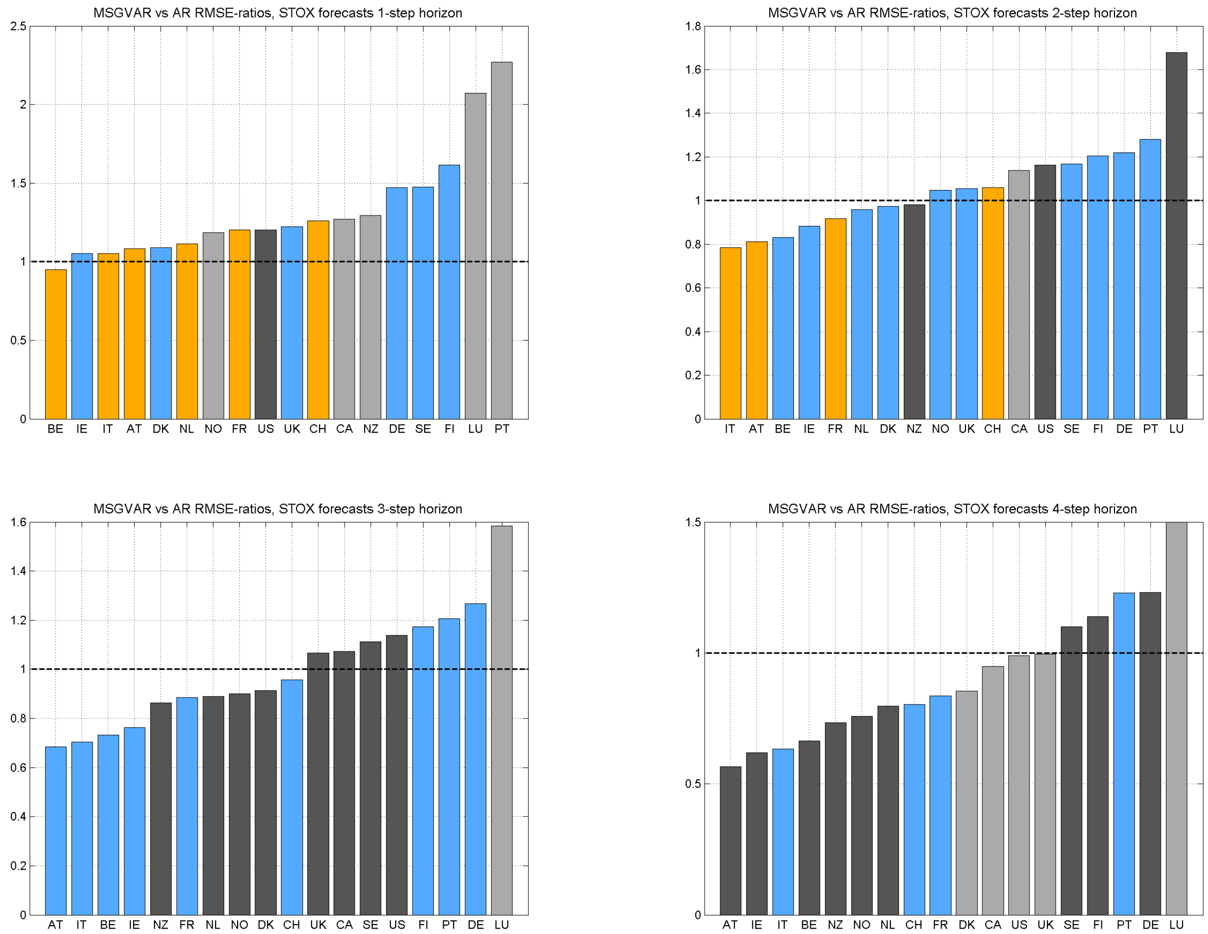
Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 21: RS-GVAR vs AR, Price inflation, Out-of-sample forecast performance 2008Q1-2011Q4



Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.

Figure 22: RS-GVAR vs AR, Stock prices, Out-of-sample forecast performance 2008Q1-2011Q4



Note: Ratios smaller one indicate that the model (first mentioned) outperformed the benchmark (second mentioned) by one minus the ratio times 100 in percent. The colors of the bars indicate whether the relative gain in performance was significant according to the Clark-West (2007) test-statistic. Orange, blue and dark grey indicate significance at the 1%, 5%, and 10% probability levels.