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NO 1180 / APRIL 2010

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**NOMINAL AND  
REAL WAGE  
RIGIDITIES.**

**IN THEORY AND  
IN EUROPE**

by Markus Knell



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**WAGE DYNAMICS  
NETWORK**

# NOMINAL AND REAL WAGE RIGIDITIES. IN THEORY AND IN EUROPE<sup>1</sup>

by Markus Knell<sup>2</sup>



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<sup>2</sup> Oesterreichische Nationalbank, Economic Studies Division, Otto-Wagner-Platz 3, POB-61, A-1011 Vienna, Austria, e-mail: [Markus.Knell@oenb.at](mailto:Markus.Knell@oenb.at)

## Wage Dynamics Network

This paper contains research conducted within the Wage Dynamics Network (WDN). The WDN is a research network consisting of economists from the European Central Bank (ECB) and the national central banks (NCBs) of the EU countries. The WDN aims at studying in depth the features and sources of wage and labour cost dynamics and their implications for monetary policy. The specific objectives of the network are: i) identifying the sources and features of wage and labour cost dynamics that are most relevant for monetary policy and ii) clarifying the relationship between wages, labour costs and prices both at the firm and macro-economic level.

The WDN is chaired by Frank Smets (ECB). Giuseppe Bertola (Università di Torino) and Julián Messina (World Bank and University of Girona) act as external consultants and Ana Lamo (ECB) as Secretary.

The refereeing process of this paper has been co-ordinated by a team composed of Gabriel Fagan (ECB, chairperson), Philip Vermeulen (ECB), Giuseppe Bertola, Julián Messina, Jan Babecký (CNB), Hervé Le Bihan (Banque de France) and Thomas Mathä (Banque centrale du Luxembourg).

The paper is released in order to make the results of WDN research generally available, in preliminary form, to encourage comments and suggestions prior to final publication. The views expressed in the paper are the author's own and do not necessarily reflect those of the ESCB.

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**Address**

Kaiserstrasse 29  
60311 Frankfurt am Main, Germany

**Postal address**

Postfach 16 03 19  
60066 Frankfurt am Main, Germany

**Telephone**

+49 69 1344 0

**Internet**

<http://www.ecb.europa.eu>

**Fax**

+49 69 1344 6000

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## Abstract

In this paper I study the relation between real wage rigidity (RWR) and nominal price and wage rigidity. I show that in a standard DSGE model RWR is mainly affected by the interaction of the two nominal rigidities and not by other structural parameters. The degree of RWR is, however, considerably influenced by the modelling assumption about the structure of wage contracts (Calvo vs. Taylor) and about other institutional characteristics of wage-setting (clustering of contracts, heterogeneous contract length, indexation). I use survey evidence on price- and wage-setting for 15 European countries to calculate the degrees of RWR implied by the theoretical model. The average levels of RWR are broadly in line with empirical estimates based on macroeconomic data. In order to be able to also match the observed cross-country variation in RWR it is, however, essential to move beyond the country-specific durations of price and wages and to take more institutional details into account.

*Keywords:* Inflation Persistence, Real Wage Rigidity, Nominal Wage Rigidity, DSGE models, Staggered Contracts

*JEL-Classification:* E31, E32, E24, J51

## Non-Technical Summary

Nominal and real wage rigidities have a long tradition in the explanation of business cycle fluctuations. The recent years have shown a particular interest in the issue of real wage rigidity (RWR) since its introduction improves the explanatory power of otherwise standard models. By breaking the “divine coincidence” of standard New Keynesian models it leads, for example, to more realistic trade-offs for monetary policy (Blanchard and Galí, 2007). Similarly, it also offers a straightforward solution to the famous “Shimer puzzle” (Shimer, 2005; Hall, 2005).

As far as the reasons behind the rigidity of real wages are concerned, however, there does not exist much agreement. Explanations range from the existence of social norms to the presence of sequential real wage bargaining and often RWR is simply assumed in a short-cut formulation without specifying the exact source of the rigidity (Blanchard and Galí, 2007). None of the existing approaches deals explicitly with the possibility that RWR could simply be understood as the consequence of two nominal rigidities: a nominal price and a nominal wage rigidity. This parsimonious explanation is, however, a core element of New Keynesian DSGE models of the business cycle and it is the starting point of this paper. In particular, I study how the two nominal rigidities interact to create a RWR, how sensitive real wage rigidities react to changes in the nominal rigidities and to what extent the use of available information on price- and wage-setting is able to generate degrees of RWR that are in line with empirical evidence.

The derivations are based on the model by Erceg, Henderson and Levin (2000). It is shown that the model leads to a solution that involves an explicit and straightforward measure of RWR. Furthermore, I demonstrate that the two nominal rigidities are the main determinants of the degree of RWR. The solutions of the forward-looking New Keynesian model can also be written in a form that is very similar to a backward-looking Phillips curve specification. I show that the derived expression is closely related to the traditional “triangle” model (cf. Gordon, 1998) and that the weight of past inflation in this expression is identical to the measure of RWR.

In order to analyze how the degree of RWR implied by the EHL model corresponds to the empirical evidence, I use recent data on wage-setting practices in Europe. In particular, I take the survey evidence from the Wage Dynamics Network (WDN) of the ESCB that has collected a multitude of data on price- and wage-setting in 15 European countries (see Durant et al., 2009). A closer look at these data reveals that there exist at least four dimensions along which the assumptions about wage-setting in the basic

model with Calvo wage contracts are problematic. First, the majority of wage agreements seems to follow a predetermined pattern with given contract lengths. Second, while for most contracts ( $\approx 60\%$ ) this predetermined length is one year there exists also some heterogeneity in this context and a nonnegligible share of contracts has longer or shorter durations. Third, existing data suggest that in many countries new contracts are clustered in certain months (mostly in January). Fourth, wage indexation is a widespread practice in some countries. In order to account for these important institutional characteristics of actual wage-setting practices, I also solve the model under the assumption of Taylor wage contracts allowing also for the possibility of heterogeneous contract length, clustering and indexation. The results show that the model with standard Taylor wage contracts involves a considerable smaller degree of RWR than the model with Calvo contracts. Furthermore, the degree of RWR decreases in the share of flexible wages and the extent of asymmetric sector sizes and it increases in the prevalence of wage indexation.

In the empirical part of the paper I use the information from the WDN together with standard values for other structural parameters in order to calculate measures of RWR that are implied by the theoretical model under different assumptions about the features of wage-setting. For the standard models the implied values of quarterly RWR are between 0.6 and 0.8. These values are in line with empirical estimations and also with the assumptions that are typically made in models with short-cut formulations for RWR. The model with Calvo contracts and with standard Taylor contracts are, however, unable to match the observed cross-country variations of RWR. This follows from the fact that the average price and wage durations are rather similar across countries. In order to get a better agreement with the cross-country variation one has to take additional institutional characteristics (with respect to clustering, contract length heterogeneity and indexation) of wage-setting into account.

# 1 Introduction

The simplest explanation for the existence of real wage rigidities sees them as a consequence of two nominal rigidities: a nominal price rigidity and a nominal wage rigidity. Although this type of real wage rigidity is a crucial element of the current generation of DSGE models (cf. Christiano et al., 2005; Smets and Wouters, 2003) it is usually not in the focus of these papers and has so far not been analyzed in any detail.<sup>1</sup> In this paper I want to fill this gap. In particular, I am going to study how the two nominal rigidities interact to create real wage rigidity (RWR), how sensitive real wage rigidities react to changes in the nominal rigidities and to what extent the use of available information on price- and wage-setting is able to generate degrees of RWR that are in line with the empirical evidence.

Nominal and real wage rigidities have a long tradition in the explanation of business cycle fluctuations. While the concept of nominal wage rigidity is commonly related to the speed with which nominal wages can be changed in reaction to economic shocks, there seems to exist less unanimity about the exact meaning of real wage rigidity. The definition by Blanchard (2006) can serve as a useful reference point: “Real wage rigidities’ [capture] the speed at which real wages [adjust] to changes in warranted real wages [...]. The slower the adjustment, the higher and the longer lasting the effects of adverse shocks on unemployment” (Blanchard, 2006, p. 16). In the benchmark labor market model with complete flexibility the “warranted real wage” is equal to the marginal rate of substitution between consumption and leisure. In formal terms this flex-price labor market equilibrium can thus be written as:  $\omega_t = mrs_t$ , where  $\omega_t$  and  $mrs_t$  are the logarithms of the real wage and the marginal rate of substitution, respectively.

The recent years have shown an increased interest in the issue of RWR. This has to do with the fact that the introduction of RWR improves the explanatory power of otherwise standard models. Hall (2005) and Milgrom and Hall (2008), e.g., have shown that RWR offers a straightforward solution to the famous “Shimer puzzle” (Shimer, 2005). Blanchard and Galí (2007), on the other hand, have argued that RWR is a reasonable way to break the “divine coincidence” of standard New Keynesian models, to re-establish more plausible effects of disinflations and more realistic trade-offs for monetary policy.

As far as the reasons behind the rigidities of real wages are concerned, however, there does not exist much agreement. Blanchard and Katz (1999), in an early contribution,

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<sup>1</sup>Some discussions about this issue can be found in Woodford (2003, 231f.) and in Rabanal and Ramírez (2005).



present a model in which unemployment benefits and wages react differently to changes in productivity growth. Hall (2005), on the other hand, uses a model where RWR follows from the existence of social norms while Hall and Milgrom (2008) present an argument based on sequential (real) wage bargaining. Blanchard and Galí (2007), finally, simply assume that the real wage  $\omega_t$  is rigid for whatever reason and can be written as:  $\omega_t = \gamma\omega_{t-1} + (1 - \gamma)mrs_t$ , where  $\gamma$  is their measure of RWR. In an appendix they motivate this short-cut formulation by referring to a model with “real wage staggering”. Interestingly, however, none of these papers deals explicitly with the possibility that RWR could simply be understood as the consequence of two nominal rigidities: a nominal price and a nominal wage rigidity. This parsimonious explanation is, however, a core element of New Keynesian (and also old Keynesian) models of the business cycle and it is the starting point of this paper. In particular, I will investigate whether the parsimonious model implies a RWR that is broadly in line with the empirical evidence.

My derivations are based on the model by Erceg, Henderson and Levin [EHL] (2000). This is the benchmark model in the DSGE literature where both nominal price and nominal wage rigidities are introduced via Calvo contracts (Calvo, 1983). The EHL model leads to a solution of the form  $\omega_t = \delta^* \omega_{t-1} + f(\text{output gap, supply shocks})$ , where  $f(\cdot)$  is a linear function of the stated variables. Since the output gap itself can be expressed as a function of the marginal rate of substitution this equation is in fact close to the short-cut relation in Blanchard and Galí (2007). The parameter  $\delta^*$  measures RWR in the EHL model. I show that the two nominal rigidities are the main determinants of the degree of RWR and that  $\delta^*$  reacts rather insensitive to changes in the other structural parameters. The solutions of the forward-looking New Keynesian model can also be written in a form that is very similar to a backward-looking Phillips curve specification. I show that the derived expression is closely related to the traditional “triangle” model (cf. Gordon, 1998) and that the weight of past inflation in this expression is identical to the measure of RWR  $\delta^*$  (i.e.,  $\pi_t^p = \delta^* \pi_{t-1}^p + f(\dots)$ ).

In order to analyze how the degree of RWR implied by the EHL model corresponds to the empirical evidence, I use recent data on wage-setting practices in Europe. In particular, I take the survey evidence from the Wage Dynamics Network (WDN) of the ESCB that has collected a multitude of data on price- and wage-setting in 15 European countries (see Durant et al., 2009). A closer look at these data reveals that there exist at least four dimensions along which the assumptions about wage-setting in the basic EHL model are problematic. First, the majority of wage agreements seems to follow a predetermined pattern with given contract lengths. In other words, the ubiquitous assumption of Calvo

wage contracts in which the hazard rate of wage changes is constant is contradicted for all countries. Second, while for most contracts ( $\approx 60\%$ ) this predetermined length is one year there exists also some heterogeneity in this context and a nonnegligible share of contracts has longer or shorter durations. Third, existing data suggest that in many countries new contracts are clustered in certain months (mostly in January). Fourth, wage indexation is a widespread practice in some countries. In order to account for these important institutional characteristics of actual wage-setting practices, I also solve the EHL model under the assumption of Taylor wage contracts, i.e. contracts with a fixed and predetermined length (cf. Taylor, 1980). In light of the additional characteristics of real-world wage-setting practices, I also allow for a certain percentage of flexible wages, for the fact that the sectors might be of different size and for partial wage indexation.

The solution to this model is somewhat more involved than the one for the model with Calvo wage contracts. It can, however, again be written in a way that contains an analogous measure of RWR. Comparing the different measures of RWR leads to two conclusions. First, the model with standard Taylor wage contracts involves a considerable smaller degree of RWR than the model with Calvo contracts. Second, the degree of RWR decreases in the share of flexible wages and the extent of asymmetric sector sizes and it increases in the prevalence of wage indexation. The largest impact can be observed for the share of flexible wages. Taking all of these elements into account gives a richer and less uniform picture than the model with Calvo contracts.

In the next step, I use the information from the WDN together with standard values for other structural parameters in order to calculate the measure for RWR that is implied by the theoretical model under different assumptions about the features of wage-setting. For the basic EHL model with Calvo wage contracts the average model-based estimate of annual RWR comes out as 0.35 while it is 0.17 for the standard Taylor model (i.e., with symmetric sector sizes and without taking flexible wages and indexation into account). These year-on-year (yoy) values correspond to quarter-on-quarter (qoq) values of 0.77 and 0.64, respectively. These figures are within the range of values for  $\gamma$  that are typically assumed in the models that are based on the short-cut formulation for RWR. Blanchard and Galí (2007), e.g., use illustrative values for  $\gamma$  between 0.5 and 0.9, Duval and Vogel (2007) employ values between 0.79 and 0.93, while Faia (2008) and Blanchard and Galí (2008) calibrate  $\gamma = 0.6$  and  $\gamma = 0.5$ , respectively.

In a further step, I use macroeconomic time series data (mostly from 1990 to 2007) to provide estimates of RWR for the same group of 15 European countries. The average qoq RWR comes out as 0.7. This is close to the values of qoq RWR that are implied by the

standard EHL models. If one focuses not only on the average value of RWR but also on the variations across countries, then the agreement with the data is less convincing. While the empirical estimations result in an average standard deviation of qoq RWR across the 15 European countries of 0.26, it is only 0.02 for the values based on the EHL model with Calvo or with standard Taylor wage contracts. The reason for this nonconformity is that the average durations of prices and wages as reported in Durant et al. (2009) are very similar across the 15 European countries and this is reflected in the similar implied levels of RWR.

Taking the additional dimensions of wage-setting practices into account leads to a more differentiated picture. The average RWR decreases (but remains in the range of plausible values), whereas the cross-country variation increases and also the ranking of countries with respect to RWR changes. This follows from the fact that cross-country differences in the additional institutional characteristics are more pronounced. In some countries, e.g., automatic wage indexation is a particularly widespread phenomenon, while in other countries the share of short-term contracts is unusually high etc. Taken together, the results imply that one should take these institutional characteristics into account in order to get a realistic picture of the transmission process and of the prevalent frictions in different economies. If this is done then the results suggest that the parsimonious EHL model does in fact offer a reasonable explanation for the important phenomenon of real wage rigidity.

The paper is structured as follows. In the next section I present the standard EHL model with Calvo price and wage contracts and I derive the measure of RWR. In section 3, I study how the introduction of Taylor wage contracts and of asymmetric sector sizes, flexible wage and indexation changes the results. In section 4, I discuss the evidence of the WDN and I calculate measures of RWR that are implied by the theoretical model. For this I use the survey evidence on price- and wage-setting for 15 European countries and I compare these figures to the results of empirical estimations. Section 5 concludes.

## 2 The basic model with nominal price and nominal wage rigidities à la Calvo

### 2.1 The set-up of the model

I use the standard model with sticky prices and wages by Erceg, Henderson and Levin [EHL] (2000). In order to facilitate the comparison with the existing literature I use the exact set-up and notation of the model that is presented in chapter 6 of Galí (2008) where one can also find details on the derivation of the linearized solutions of the microfounded model. The model assumes that there exists a continuum of monopolistically competitive firms that produce differentiated products where  $\varepsilon_p$  stands for the elasticity of substitution among the product varieties. There exists a Calvo constraint on price-setting and each period only a fraction  $(1 - \theta_p)$  can reset their price while a fraction  $\theta_p$  leaves the price unchanged. The average duration of a price is thus given by  $\frac{1}{1-\theta_p}$ . Nominal wage rigidity is introduced in a similar fashion. In particular, it is assumed that each household is specialized in one particular type of labor for which he is the monopolistic supplier and where each firm needs all differentiated labor types to produce its differentiated product. Households are subject to a similar Calvo constraint and in each period only a fraction  $(1 - \theta_w)$  can freely adjust the wage rate. The elasticity of substitution among the different types of labor is denoted by  $\varepsilon_w$ .

The production function for firm  $i$  is given by:  $Y_t(i) = A_t N_t(i)^{1-\alpha}$ , where  $N_t(i)$  is an index of labor inputs used in the production of good  $Y_t(i)$ . The period utility function of a representative household is given by:  $U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$ . Galí (2008) shows that the dynamic equilibrium of the model can be summarized in 5 equations:<sup>2</sup>

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t \quad (1)$$

$$\pi_t^w = \beta E_t \pi_{t+1}^w + \kappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t \quad (2)$$

$$\tilde{\omega}_t = \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta \omega_t^n \quad (3)$$

$$\tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \pi_{t+1}^p - r_t^n) + E_t \tilde{y}_{t+1} \quad (4)$$

$$i_t = \rho + \phi_p \pi_t^p + \phi_w \pi_t^w + \phi_y \tilde{y}_t + v_t, \quad (5)$$

where  $\pi_t^p = p_t - p_{t-1}$  and  $\pi_t^w = w_t - w_{t-1}$  denote price and wage inflation, respectively,  $i_t$

<sup>2</sup>These equations correspond to (15), (17), (18), (19) and (20) in chapter 6 of Galí (2008).



is the nominal interest rate,  $\omega_t \equiv w_t - p_t$  is the real wage,  $\tilde{y}_t \equiv y_t - y_t^n$  is the output gap and  $\tilde{\omega}_t \equiv \omega_t - \omega_t^n$  the real wage gap. The level of natural output  $y_t^n$  that is used in the definition of the output gap refers to the equilibrium level of output that would prevail in the absence of *both* price and wage rigidities. Similarly, the natural real wage  $\omega_t^n$  and the natural real interest rate  $r_t^n$  correspond to the real wage rate and the real interest rate in the absence of both nominal rigidities. These natural levels can be derived as:

$$y_t^n = \psi_{ya}^n a_t \quad (6)$$

$$r_t^n = \rho + \sigma E_t y_{t+1}^n \quad (7)$$

$$\omega_t^n = \log(1 - \alpha) - \mu^p + \psi_{\omega a}^n a_t = \bar{\omega}^n + \psi_{\omega a}^n a_t, \quad (8)$$

where  $\mu^p \equiv \log\left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right)$  is the log of the desired markup of firms and where  $\bar{\omega}^n \equiv \log(1 - \alpha) - \mu^p$  is defined as the real wage that would prevail in the absence of nominal rigidities *and* in the absence of technological shocks.

Equation (1) is a New Keynesian Phillips curve where inflation now also depends on the real wage gap. Equation (2) is a similar equation for wage inflation with the only difference that a positive wage gap will decrease wage inflation by moderating wage claims. (3) is an identity relating various measures of the real wage and inflation, (4) is the usual forward-looking IS curve and (5) is the monetary policy rule.

The various parameters in (1) to (8) are given as follows:

$$\lambda_p = \frac{(1 - \theta_p)(1 - \beta\theta_p)}{\theta_p} \frac{1 - \alpha}{1 - \alpha + \alpha\varepsilon_p}, \quad \kappa_p = \frac{\alpha\lambda_p}{1 - \alpha}$$

$$\lambda_w = \frac{(1 - \theta_w)(1 - \beta\theta_w)}{\theta_w(1 + \varepsilon_w\varphi)}, \quad \kappa_w = \lambda_w \left( \sigma + \frac{\varphi}{1 - \alpha} \right)$$

$$\psi_{ya}^n = \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha}, \quad \psi_{\omega a}^n = \frac{1 - \alpha\psi_{ya}^n}{1 - \alpha},$$

and  $\phi_p$ ,  $\phi_w$  and  $\phi_y$  are non-negative coefficients that denote the strength with which the central bank adjusts the nominal interest rate in response to price inflation, wage inflation and the output gap, respectively. Furthermore,  $\rho \equiv -\log \beta$  where  $\beta$  is the discount factor. The technology shocks  $a_t$  and the interest rate shock  $v_t$  are given by the AR(1) processes:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a \quad (9)$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v, \quad (10)$$

where  $\rho_a \in [0, 1]$ ,  $\rho_v \in [0, 1]$  and  $\varepsilon_t^a$  and  $\varepsilon_t^v$  are uncorrelated zero mean white noise processes. For later reference I also want to state the equation for the marginal rate of substitution:

$$mrs_t = \left( \sigma + \frac{\varphi}{1 - \alpha} \right) y_t - \frac{\varphi}{1 - \alpha} a_t \quad (11)$$

## 2.2 Measuring real wage rigidity

I am interested in the implications of the model for real wage rigidity. For this purpose it is helpful to subtract (1) from (2). Defining real wage inflation as  $\pi_t^\omega \equiv \pi_t^w - \pi_t^p$  it follows that:

$$\pi_t^\omega = \beta E_t \pi_{t+1}^\omega + (\kappa_w - \kappa_p) \tilde{y}_t - (\lambda_w + \lambda_p) \tilde{\omega}_t \quad (12)$$

Using the definitions for  $\pi_t^\omega$ ,  $\tilde{\omega}_t$  and  $\omega_t$  one can derive from (12) a second-order difference equation for the real wage  $\omega_t$ :

$$\omega_t = \frac{1}{1 + \beta + \lambda_w + \lambda_p} [\omega_{t-1} + \beta E_t \omega_{t+1} + (\kappa_w - \kappa_p) \tilde{y}_t + (\lambda_w + \lambda_p) \omega_t^n] \quad (13)$$

Equation (13) can be solved to get:<sup>3</sup>

$$\omega_t = \delta \omega_{t-1} + \delta (\kappa_w - \kappa_p) \sum_{s=0}^{\infty} (\beta \delta)^s E_t \tilde{y}_{t+s} + \delta (\lambda_w + \lambda_p) \sum_{s=0}^{\infty} (\beta \delta)^s E_t \omega_{t+s}^n \quad (14)$$

The root  $\delta$  is given by:

$$\delta = \frac{1 - \sqrt{1 - 4\beta\tilde{\lambda}^2}}{2\beta\tilde{\lambda}}, \quad (15)$$

where  $\tilde{\lambda} \equiv \frac{1}{1 + \beta + \lambda_w + \lambda_p}$ .  $\delta$  is a first approximate measure for the extent of real wage rigidity in the EHL model. It is, however, not the ultimate solution since also future variables that are present in (14) might depend on past levels of the real wage. In order to derive the general solution one has to use the complete model, in particular the assumptions about how the output gaps  $\tilde{y}_{t+s}$  are determined.

<sup>3</sup>For a difference equation of the form:  $x_t = ax_{t-1} + bE_t x_{t+1} + cz_t$  the solution is given by:  $x_t = \Upsilon x_{t-1} + \Upsilon \frac{c}{a} \sum_{s=0}^{\infty} \left( \frac{b}{a} \Upsilon \right)^s E_t z_{t+s}$ , where  $\Upsilon = \frac{1 - \sqrt{1 - 4ab}}{2b}$ .

## 2.3 Real wage rigidities under the assumption of exogenous output

Before turning to the solution that is implied by the complete EHL model (that includes the forward-looking IS-curve (4) and the monetary policy rule (5) to pin down the values of  $\tilde{y}_{t+s}$ ) I want to start with the simple assumption that the output is exogenously given and always equal to its natural level  $y_t^n$ , i.e.  $\tilde{y}_t = 0, \forall t$ .<sup>4</sup> Using (8) and (9) in (14) it follows that:

$$\omega_t = \delta\omega_{t-1} + \frac{\delta(\lambda_w + \lambda_p)}{1 - \beta\delta}\bar{\omega}^n + \frac{\delta(\lambda_w + \lambda_p)\psi_{\omega a}^n}{1 - \beta\delta\rho_a}a_t \quad (16)$$

In the case of exogenous output the root  $\delta$  captures the degree of real wage rigidity. One can use (15) to derive a straightforward and at the same time crucial result: The rigidity measure  $\delta$  goes to zero if either  $\lambda_w$  or  $\lambda_p$  go to infinity or, equivalently, if either  $\theta_p$  or  $\theta_w$  are equal to zero (see appendix A.1). As a consequence, real wages are flexible ( $\delta = 0$ ) if either prices or wages are flexible. As one would have expected, only the combination of nominal price and nominal wage rigidity creates real wage rigidity.

In the next section, I will show that this conclusion still holds for the more general case where the output gap is not assumed to be equal to zero. In fact, it will come out that the degree of real wage rigidity in the more general framework is also *quantitatively* similar to  $\delta$ .

## 2.4 Real wage rigidities in the EHL model

For the full EHL model consisting of equations (1) to (8) it is not possible anymore to derive a closed form solution for the degree of RWR. One can use, however, standard methods to solve the model numerically. In particular, in appendix A.2 I show that the solution takes the form:  $x_t = \Psi_0^x + \Psi_1^x\omega_{t-1} + \Psi_2^xa_t + \Psi_3^xv_t$ , where  $x \in \{\pi^p, \tilde{y}, \omega\}$  and  $\Psi_0^x$  to  $\Psi_3^x$  are coefficients.<sup>5</sup> The coefficient  $\Psi_1^\omega$  in the expression for  $\omega_t$  thus provides a measure for RWR in the EHL model. I prefer, however, to focus on a slightly different measure that uses the solution for  $\tilde{y}_t$  to substitute out for the interest rate shock  $v_t$ . Appendix A.2 reports the resulting expressions for  $\omega_t$  and  $\pi_t^p$ . In particular, the evolution of the real

<sup>4</sup>This model thus closely resembles a RBC model with fixed labor supply and a real wage rigidity (caused by the existence of two nominal rigidities).

<sup>5</sup>It is also shown in appendix A.2 how to write these policy functions in an equivalent way in terms of deviations of the real wage from the steady state value  $\bar{\omega}^n$ . In particular for  $x \in \{\pi^p, \tilde{y}, (\omega - \bar{\omega}^n)\}$  one gets:  $x_t = \Psi_1^x(\omega_{t-1} - \bar{\omega}^n) + \Psi_2^xa_t + \Psi_3^xv_t$ .

wage  $\omega_t$  can now be written as:

$$\omega_t = \delta^* \omega_{t-1} + \Psi_4^\omega a_t + \Psi_5^\omega \tilde{y}_t + \Psi_6^\omega, \quad (17)$$

where  $\delta^*$  and  $\Psi_4^\omega$  to  $\Psi_6^\omega$  are defined in appendix A.2. I choose the coefficient  $\delta^*$  in (17) as the measure of real wage rigidity in the EHL model since it is closely related to the existing literature and allows for straightforward comparisons among different specifications.

The degree of (annual) RWR is illustrated in Figure 1 that also includes—as a comparison—the (annual) RWR from the model with exogenous output (cf. (16)). For the illustrations I use the standard calibration of the parameters as in Galí (2009). The only difference is that Galí (2009) defines a quarter as the basic time unit while I use a semester for this purpose. This is done to later alleviate comparisons to a model with two-period Taylor wage contracts (in particular when sector sizes are asymmetric).<sup>6</sup> In using a semester as the basic time unit one has to be careful in correctly calibrating the parameters that govern the degree of nominal rigidity. In particular, an average price duration of 3 quarters corresponds to  $\theta_p = 1/3$  while an average wage duration of 4 quarters corresponds to  $\theta_w = 1/2$ . These are the baseline values for the duration of price and wage contracts used by Galí (2009) and I will refer to this in the following as the “baseline calibration”.<sup>7</sup>

Insert Figure 1 about here

Figure 1 shows that in the absence of nominal price rigidity ( $\theta_p = 0$ ) the real wage rigidity is zero. The same is true for the case of completely flexible wages ( $\theta_w = 0$ ) where  $\delta^*$  and  $\delta$  also approach zero. For the baseline calibration one gets a sizable degree of annual RWR given by  $(\delta^*)^2 = 0.31$  (which corresponds to a qoq RWR of 0.75).

$\delta^*$  reacts only very weakly to changes in the parameters  $\sigma$ ,  $\varphi$ ,  $\phi_\pi$  and  $\phi_y$ . The largest effects one can observe for changes in  $\alpha$ ,  $\varepsilon_p$  and  $\varepsilon_w$ .<sup>8</sup> For the usual range of parameter values, however, the measure of annual RWR is fairly stable and stays between 0.25 and 0.32. Overall these robustness checks show that the main determinants of the degree of

<sup>6</sup>In section 3.4 I discuss some issues related to the structure of timing more extensively.

<sup>7</sup>The rest of the parameters is calibrated as:  $\alpha = 1/3$ ,  $\beta = 0.98$  (corresponding to an annual real interest rate of roughly 4%),  $\sigma = 1$ ,  $\varphi = 5$ ,  $\varepsilon_p = 6$ ,  $\varepsilon_w = 4.52$ ,  $\phi_\pi = 1.5$ ,  $\phi_y = 0.125$ , and  $\phi_w = 0$ . The calibration in chapter 6 of Galí (2008) is similar with the exception that there  $\varepsilon_w = 6$ ,  $\varphi = 1$  and  $\phi_y = 0$ .

<sup>8</sup>For  $\sigma$  between 0.5 and 5,  $(\delta^*)^2$  stays the same, for  $\varphi$  between 0.5 and 5 it increases from 0.25 to 0.32 and for  $\phi_\pi$  between 1.1 and 10 and  $\phi_y$  between 0 and 1 it stays constant at  $(\delta^*)^2 = 0.31$ . On the other hand, annual RWR is 0.11 for  $\alpha = 0$  and it is close to 0.62 as  $\alpha$  approaches 1. For  $\varepsilon_p$  ( $\varepsilon_w$ ) close to one gets values of  $(\delta^*)^2 = 0.17$  ( $(\delta^*)^2 = 0.26$ ) which increases to  $(\delta^*)^2 = 0.37$  ( $(\delta^*)^2 = 0.33$ ) for  $\varepsilon_p = 10$  ( $\varepsilon_w = 10$ ). The extreme values for  $\alpha$ ,  $\varepsilon_p$  and  $\varepsilon_w$  are, however, not typical for the calibration of DSGE models. Furthermore,  $\delta^*$  is independent of  $\rho_a$  and very insensitive to  $\rho_v$ .



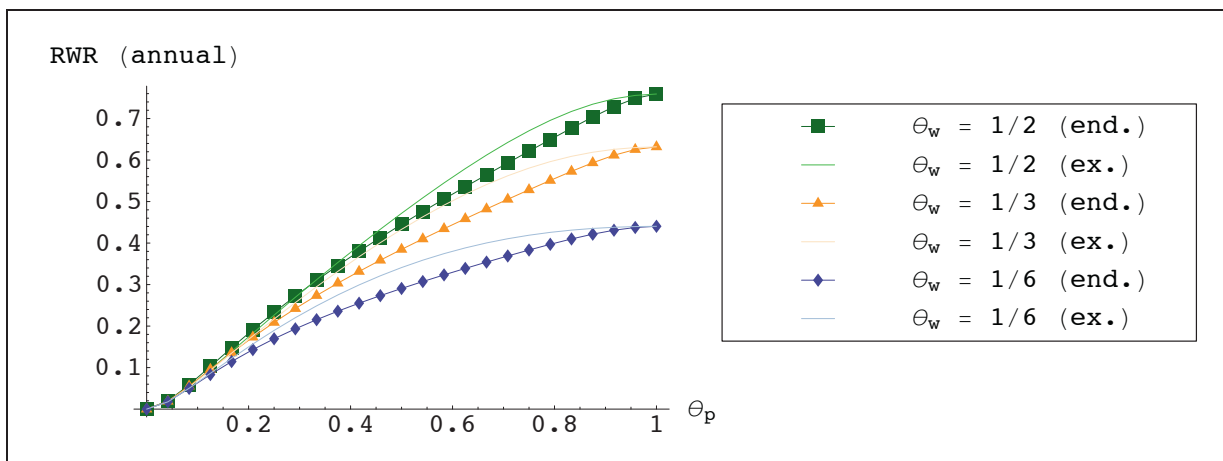


Figure 1: Comparison of the coefficients of annual real wage rigidity ( $\delta^2$  and  $(\delta^*)^2$ ) in the specifications with exogenous and endogenous  $\tilde{y}_t$ , respectively.

RWR are the two nominal rigidities. A related issue is whether the degree of RWR that is implied by the baseline calibration is reasonable and in line with the empirical evidence. I will come back to this question in section 4.

The main findings of the last two subsections can be summarized as follows:

**Result 1** *The combination of nominal price and nominal wage rigidity can give rise to a considerable degree of real wage rigidity. The assumption of completely flexible prices ( $\theta_p = 0$ ) or completely flexible wages ( $\theta_w = 0$ ) implies zero real wage rigidity ( $\delta = 0$  or  $\delta^* = 0$ ).*

**Result 2** *The degree of RWR is primarily determined by the extent of the two nominal rigidities. It is rather insensitive to changes in the other structural parameters and also to the specification of the monetary policy rule and the determination of output. In particular, the specifications with exogenous output and with endogenous output give rise to similar degrees of RWR.*

Result 1 emphasizes in a concentrated form the importance of complementarities (cf. Ascari, 2003; Huang and Liu, 2002). Nominal price rigidity without nominal wage rigidity as well as nominal wage rigidity without nominal price rigidity will result in completely flexible real wages ( $\delta^* = 0$ ). Only the interplay between the two rigidities causes real wage rigidity. By the same token, Figure 1 also nicely illustrates that one class of stickiness increases the size of overall persistence holding the degree of the other stickiness constant.

Note also that one can use (11) to transform (17) into an expression of the form  $\omega_t = \delta^* \omega_{t-1} + \gamma_1 mrs_t + \gamma_2 a_t + const.$  This is fairly close (but nevertheless not identi-

cal) to the short-cut formulation in Blanchard and Galí (2007) where they assume that  $\omega_t = \gamma\omega_{t-1} + (1 - \gamma)mrs_t$ . This highlights that the measure  $\delta^*$  (that is a function of the two nominal rigidities) is in fact closely related to the degree of RWR that is used in the models with the simple ad-hoc assumption. Despite this similarity the two models contain, however, different transmission and adjustment mechanisms and they have different dynamic properties (see Riggi, 2007).

## 2.5 Inflation persistence and a backward-looking Phillips curve

The rational expectations solution to the EHL model can be transformed into an expression that resembles a traditional, backward-looking Phillips curve. This formulation is particularly useful for empirical analyses and also for the later comparisons between the models with Calvo and with Taylor wage contracts. In appendix A.2 it is shown that one can use the solution of the model to derive an equation of the form:

$$\pi_t^p = \delta^* \pi_{t-1}^p + f(\tilde{y}_t, \tilde{y}_{t-1}, a_t, a_{t-1}), \quad (18)$$

where  $f(\cdot)$  is a linear function of the listed variables. Equation (18) is in fact fairly similar to the more traditional “triangle” model (cf. Gordon, 1998) in which the current rate of inflation is written as a function of past inflation and of current and past levels of demand factors (output gap, cyclical unemployment) and supply factors (oil price shocks, import price shocks etc.). Interestingly, the coefficient on the lagged inflation term is identical to the degree of real wage rigidity in (17). An implication of this finding is stated as the following result.

**Result 3** *The degree of intrinsic inflation persistence is the same as the degree of RWR. If there is no RWR then there will also be no intrinsic inflation persistence.*

A similar result has also been derived by Blanchard and Galí (2007, 51f.) who have shown that the presence of their (assumed) RWR leads to intrinsic inflation inertia.

## 3 The model with nominal price rigidities à la Calvo and nominal wage rigidities à la Taylor

The EHL model is based on Calvo wage contracts and Calvo price contracts. This is the standard assumption that dominates the DSGE literature. In recent years, however, this

assumption has also been criticized as being restrictive and implausible. In particular, it has been argued that a constant hazard rate for wage contracts is at odds with the empirical evidence (cf. Gottfries and Söderberg, 2008). Recent survey data by the WDN—that I will discuss more extensively in chapter 4—underline this criticism. In particular, there are at least four dimensions along which the data contradict the basic model with Calvo contracts. First, the majority of wage agreements seems to follow a predetermined pattern with given contract lengths. Second, while for most contracts this predetermined length is one year (on average 60% in the WDN survey) there exists also some heterogeneity in this context and a nonnegligible share of contracts has longer (26%) or shorter (12%) durations. Third, 54% of the firms asked in the WDN survey have indicated that they carry out wage changes in a particular month (most of them—30%—in January).<sup>9</sup> Fourth, 15% of all firms report to use automatic indexation of wages to the rate of inflation. In order to be able to take these real-world characteristics of wage-setting into account one has to move beyond the convenient but restrictive framework of Calvo wage contracts. Accordingly, in this section I am going to present a model with Taylor wage contracts that allows to incorporate all of these institutional details.

### 3.1 Wage-setting in the model with Taylor wage contracts

I use a two period Taylor model where the basic time-unit is again one semester. In order to account for the observed heterogeneity of contracts, I assume that there are three sectors: A, B and F. In the flexible sector F wages are set every period according to the flex-wage expression (see appendix A.3):  $w_t^F = \omega_t^n + p_t + (\sigma + \frac{\varphi}{1-\alpha})\tilde{y}_t$ . In sectors A and B wage contracts are fixed for two periods. Sector A negotiates the wage in periods  $t = 0, 2, 4, \dots$ , while sector B negotiates in periods  $t = 1, 3, 5, \dots$ . There is a share  $\tau$  of flexible wages and a share  $(1 - \tau)$  of two-period contracts where the relative size of sector A (B) among all staggered wages is given by  $s_A$  ( $s_B = 1 - s_A$ ). Furthermore, a share  $\gamma_w$  of all fixed contracts is indexed to the rate of (current) inflation. Finally, it is assumed that firms' price-setting decisions are still characterized by a Calvo structure and that all firms use labor from all sectors in proportion to their relative sizes  $\tau$ ,  $(1 - \tau)s_A$  and  $(1 - \tau)s_B$ , respectively.

When compared to the model of section 2 one has to change two equations (see ap-

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<sup>9</sup>Cf. also Olivei and Tenreyro (2007) and Knell and Stiglbauer (2009). The WDN data show also a concentration of price changes in certain months. The degree of clustering is, however, less pronounced than in the area of wage-setting (only 35% of the firms indicate to follow such a strategy) and I will concentrate in the following on the clustering of wage changes.

pendix A.3). The following wage-setting equation takes the place of (2):

$$w_t^i = \frac{1}{1 + \varepsilon_w \varphi} \sum_{k=0}^1 \frac{\beta^k}{1 + \beta} E_{t+k} \left\{ (1 + \varepsilon_w \varphi) w_{t+k} - \tilde{w}_{t+k} + \tilde{y}_{t+k} \left( \sigma + \frac{\varphi}{1 - \alpha} \right) \right\}, \quad (19)$$

where  $i = A$  for  $t = 0, 2, 4, \dots$  and  $i = B$  for  $t = 1, 3, 5, \dots$ . For the periods where sector  $i \in \{A, B\}$  does not adjust wages it holds that  $w_t^i = w_{t-1}^i + \gamma_w \pi_t$ . Instead of (3) one has to use the following definition of aggregate wages:

$$w_t = (1 - \tau) \left( s^i w_t^i + s^{-i} (w_{t-1}^{-i} + \gamma_w \pi_t) \right) + \tau \left( \omega_t^n + p_t + \left( \sigma + \frac{\varphi}{1 - \alpha} \right) \tilde{y}_t \right) \quad (20)$$

The complete model is now given by the five equations (1), (4), (5), (19) and (20).

### 3.2 Real wage rigidity in the model with Taylor wage contracts

One can again use standard methods to solve the model (see appendix A.3). A direct comparison between the solutions and the degrees of RWR for the formulations with Calvo and with Taylor contracts is, however, not straightforward. First, even in the case with symmetric sector sizes (i.e.  $s_A = s_B = 1/2$ ) the Taylor model does not lead to a formulation where the average real wage  $\bar{w}_t$  depends just on  $\bar{w}_{t-1}$ ,  $a_t$  and  $\tilde{y}_t$  (as in (17)).<sup>10</sup> In particular, for the case with symmetric Taylor wage contracts  $\bar{w}_t$  depends on  $\bar{w}_{t-1}$ ,  $a_t$ ,  $a_{t-1}$ ,  $\tilde{y}_t$ ,  $\tilde{y}_{t-1}$ ,  $\pi_t^p$  and  $\pi_{t-1}^p$ .

Second, for the case of asymmetric sector sizes the period-on-period RWR differs between the two subperiods and depends on the sector that sets the new wage. In order to deal with these difficulties and to allow for comparisons I use a year-on-year formulation. In appendix A.3 it is shown that the evolution of the average real wage can be written as:

$$\bar{w}_t^i = \tilde{\delta} \bar{w}_{t-2}^i + f^i(\tilde{y}_t^i, \tilde{y}_{t-1}^{-i}, \tilde{y}_{t-2}^i, a_t, a_{t-1}, a_{t-2}, \pi_t^{p,i}, \pi_{t-2}^{p,i}) \quad (21)$$

The coefficient  $\tilde{\delta}$  measures the yoy rigidity of the average real wage. It is the same in both sectors of the economy, independent of which sector sets the new wage. The reaction of  $\bar{w}_t^i$  to supply shocks, output gaps and inflation rates is, however, different in the two

<sup>10</sup>I have to write  $\bar{w}_t$  for the average real wage in order to distinguish it from the real wage of the two individual sectors. In particular:  $\omega_t^A \equiv w_t^A - p_t$ ,  $\omega_t^B \equiv w_t^B - p_t$ ,  $\bar{\omega}_t^A \equiv w_t - p_t$  in periods when sector  $A$  is changing the wage while  $\bar{\omega}_{t+1}^B \equiv w_{t+1} - p_{t+1}$  in periods when sector  $B$  is changing. For symmetric sector sizes the dynamics of  $\bar{\omega}_t^A$  and  $\bar{\omega}_{t+1}^B$  are described by the same equation and one can thus drop the sectoral index.

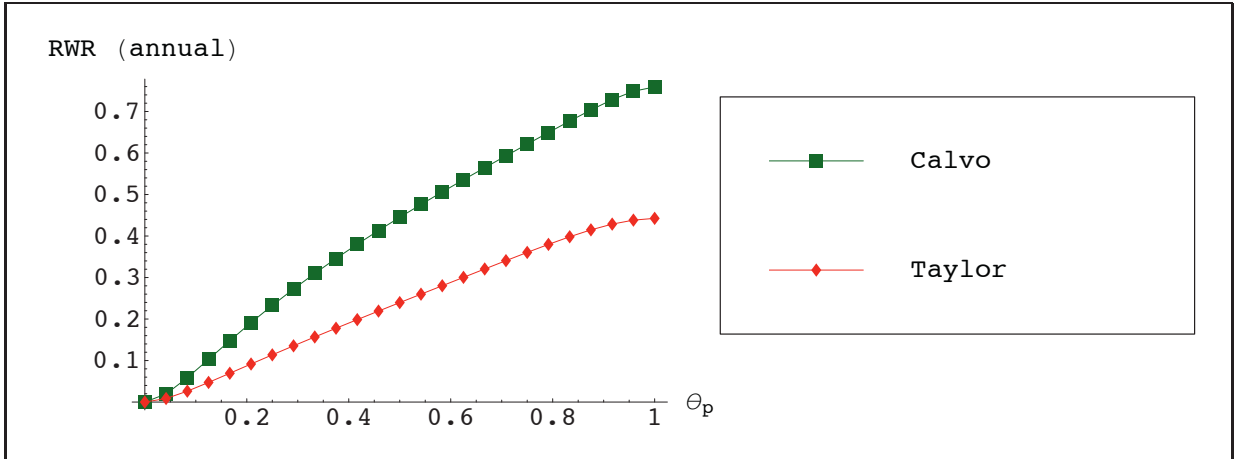


Figure 2: Comparison of annual RWR in the model with Calvo  $((\delta^*)^2)$  and with standard Taylor  $(\tilde{\delta})$  wage contracts ( $s_A = 1/2, \tau = 0, \gamma_w = 0$ ). The measures of annual RWR are based on a model where the basis time period is one semester and the average duration of wage contracts is one year. For the case of Calvo wage contracts this means that  $\theta_w = 1/2$ .

subperiods (as indicated by the indexation of the function  $f^i(\cdot)$ ). This annual measure of RWR  $\tilde{\delta}$  can be compared with the annual measure of RWR in the Calvo model (given by  $(\delta^*)^2$ ).

The correspondence between  $\tilde{\delta}$  and  $(\delta^*)^2$  is further emphasized if one again derives a backward-looking Phillips curve for the model with Taylor wage contracts. In appendix A.3 I show how it can be written as:

$$\pi_t^{p,i} = \tilde{\delta} \pi_{t-2}^{p,i} + f^i(\tilde{y}_t^i, \tilde{y}_{t-1}^{-i}, \tilde{y}_{t-2}^i, a_t, a_{t-1}, a_{t-2}) \quad (22)$$

Using (18) one observes that for the model with Calvo contracts the expression for yoy inflation persistence has exactly the same form as (22):  $\pi_t^p = (\delta^*)^2 \pi_{t-2}^p + f(\tilde{y}_t, \tilde{y}_{t-1}, \tilde{y}_{t-2}, a_t, a_{t-1}, a_{t-2})$ .

In Figure 2 I contrast  $(\delta^*)^2$  with  $\tilde{\delta}$  for the standard model with Taylor contracts but without flexible wages, indexation or asymmetric sector sizes.

Insert Figures 2 and 3 and about here

One gets the following result:

**Result 4** *For the same average durations of price and wage contracts, the assumption of Taylor wage contracts implies a considerably lower degree of real wage rigidity than the assumption of Calvo wage contracts.*

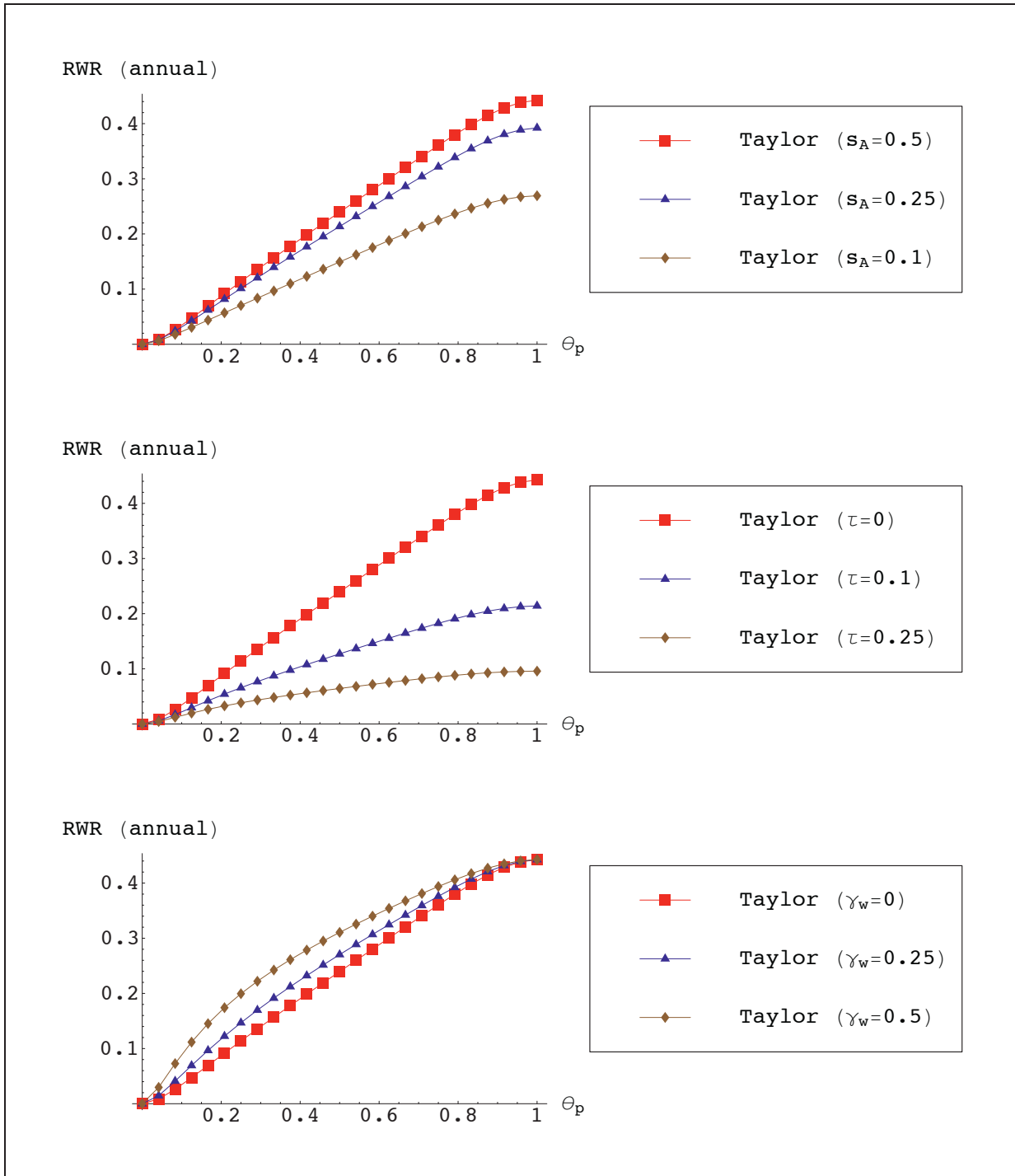


Figure 3: Comparison of annual RWR in three variants of the model with Taylor wage contracts. In the upper panel the sector size  $s_A$  is varied, in the middle panel a share  $\tau$  of all wages is assumed to be flexible, while in the lower panel a fraction  $\gamma_w$  of all staggered wages is indexed to current inflation.

For the baseline calibration (with  $\theta_p = 1/3$ ), the annual RWR implied by the model with Calvo contracts is given by  $(\delta^*)^2 = 0.31$  while for Taylor contracts it is  $\tilde{\delta} = 0.16$ . The reason for the considerably lower rigidity in the model with Taylor contracts is the fact that under the latter assumption there is an exactly given duration for every contract. In the Calvo framework, on the other hand, some contracts might last for a very long time span. This (unrealistic) feature considerably increases the extent of intrinsic persistence. This fact is known from the literature (cf. Dixon and Kara, 2006) although it is mostly ignored when calibrating the models.

In Figure 3 I show what happens if the additional institutional characteristics of wage-setting are taken into account. The results can be summarized in the following way:

**Result 5** *In a model with Taylor wage contracts real wage rigidity is lower if*

- (i) the relative size of the sectors with fixed wages is more asymmetric,*
- (ii) the share of the sector with flexible wages  $\tau$  is higher,*
- (iii) the percentage of indexation is lower.*

RWR is related to the institutional characteristics in the expected direction. As far as the relative magnitudes are concerned, Figure 3 reveals that the share of flexible wages has the largest impact. For a reasonable value of  $\tau = 0.1$  annual RWR is almost halved to  $\tilde{\delta} = 0.09$ . The effect is less pronounced but still sizable for the other two parameters. For  $\gamma_w = 0.25$  RWR increases to  $\tilde{\delta} = 0.19$  while for  $s_A = 0.25$  it decreases to  $\tilde{\delta} = 0.14$ .

Results 4 and 5 together suggest that the assumption of Calvo wage contracts is not innocuous. In particular, it might be highly misleading to simply translate the available information about the average duration of wage contracts into a parameter  $\theta_w$  that is then used in a model with Calvo wage contracts. Institutional details about the wage-setting practices matter and they can have a considerable impact on the implied degree of persistence and thus the dynamics of adjustment.

### 3.3 Discussion of the comparison between Calvo and Taylor models

In an important article on the comparison between models with Calvo and with Taylor contracts, Dixon and Kara (2006) have argued that one should use a comparison between the average age or the average lifetime of the two kinds of contracts. Both of these

criteria amount to set  $\theta_w = \frac{N-1}{N+1}$ , where  $N$  is the length of the Taylor wage contract. This is different from my calibration where (following the majority of the literature) I have used  $\theta_w = \frac{N-1}{N}$ . For the two-period framework the Dixon-Kara approach thus implies to set  $\theta_w = \frac{1}{3}$  instead of  $\theta_w = \frac{1}{2}$ . Figures 1 and 2 suggest, however, that this does not make a qualitative and only a small quantitative difference. The implied annual RWR for the Calvo model drops from 0.31 to 0.27. If one also adapts the parameter for price stickiness to  $\theta_p = \frac{1}{5}$  then RWR comes out as 0.17 (for the Calvo model) and 0.09 (for the Taylor model)—a similar difference as before. In the following, I will stick to the original approach for two reasons. First, it is the prevalent approach in the related literature. Second, and more importantly, the later empirical comparisons are based on firm surveys in which the relevant questions did not refer to the average age or lifetime of contracts but rather to the *frequency* of wage and price changes (Druant et al., 2009). In this case, as shown in Dixon and Kara (2006), an accurate comparison does in fact involve to set  $\theta_i = \frac{N-1}{N}$ .

### 3.4 Discussion of the time structure

Before turning to the empirical data I want to briefly deal with two issues that are related to the time structure of the model. First, how large is the bias introduced by working with a semester as the basic time-unit in both the Calvo and the Taylor model? Second, is it possible to stick to the standard Taylor structure of two-period-staggering while still allowing for an average wage duration that is longer or shorter than one year? The latter question is particularly important when trying to match the model with the empirical data.

In appendix B I deal with both questions and I show there that these are in fact nonnegligible issues. As far as the first issue is concerned, appendix B.1 illustrates, e.g., that the baseline calibration of a model with Calvo wage contracts and with semesters as the basic time units underestimates the RWR by 36% when compared to a model with quarters as the basic time unit. The underestimation is even larger when it is compared to a model with months (53%) or days (62%) as the basic time unit. For cross-country comparisons, however, this issue is less important since the effects on the relative ranking and the cross-country variation are moderate.

As far as the second timing issue is concerned I present in appendix B.2 a straightforward method to allow in the Taylor framework for average wage durations that are different from one year. The main idea behind the procedure is to redefine the length of



the basic time unit as one half of the average wage duration. The time discount factor and the parameter capturing the degree of price stickiness have then to be adapted such as to conform to this new timing. The results suggest that the approximate method will lead to plausible results as long as average price duration is not too much shorter than average wage duration.

## 4 Survey evidence on nominal rigidities and what they imply for real wage rigidity

In this section I am going to analyze whether the EHL model implies plausible sizes and cross-country patterns of RWR when calibrated to real-world data on price- and wage-setting. Furthermore, I will study how sensitive the results are to different assumption concerning the detailed characteristics of wage-setting institutions.

To this end, I use the results from firm surveys that have been conducted in a number of European countries in the context of the ESCB's Wage Dynamics Network (WDN). Aggregate data and a discussion of the results can be found in Druant et al. (2009). In Table 1 I summarize some country-specific details that are relevant for the calibration of the EHL model.

Insert Table 1 about here

Columns 1 and 2 contain the measures for the average duration of prices and wages (Table 5 in Druant et al., 2009). They indicate that the degree of price stickiness is rather similar across European countries. It ranges from 8.4 months (Lithuania) to 10.7 months (Hungary) and for the Euro-area countries the span is even smaller (from 9 to 10 months). The duration of wages is on average higher than the one for prices and also cross-country differences are more pronounced. This is the expected result given the differences in wage-setting institutions and practices. Wage duration is shortest for Slovenia, Lithuania and Spain (around 12 months) and ranges up to 15 months for countries like the Netherlands, the Czech Republic and Poland. Italy seems to be a special case since it has an average duration of wages of almost two years.<sup>11</sup>

Columns 3 to 5 report summary statistics about other important characteristics of wage-setting. First, I have used the raw data on the percentages of new wage agreements

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<sup>11</sup>This creates problems for some of the calculations based on the assumption of Taylor wage contracts. Therefore I omit Italy from the following cross-country comparisons.

Table 1: Survey evidence on nominal rigidities and other features of wage-setting

	(1)	(2)	(3)	(4)	(5)
	Duration (in months)		Sector Size	Flexibility	Indexation
	Prices	Wages	$s_A$	$\tau$	$\gamma_w$
Austria (AUT)	9.1	12.5	0.35	0.07	0.1
Belgium (BEL)	9.9	12.6	0.45	0.23	0.98
Spain (ESP)	9.7	11.9	0.2	0.12	0.55
France (FRA)	10.1	12	0.46	0.2	0.06
Greece (GRC)	10.2	11.9	0.38	0.34	0.2
Ireland (IRL)	8.5	12.8	0.42	0.15	0.05
Italy (ITA)	9.5	20.3	0.44	0.04	0.02
Netherlands (NLD)	9.1	13.9	0.28	0.11	0
Portugal (PRT)	9.5	12.9	0.19	0.06	0.09
Czech Republic (CZE)	9.7	14.6	0.34	0.12	0.08
Estonia (EST)	10	12.7	0.41	0.21	0.04
Hungary (HUN)	10.7	13.8	0.27	0.03	0.11
Lithuania (LTU)	8.4	11.4	0.47	0.45	0.11
Poland (POL)	9.5	15.4	0.45	0.14	0.07
Slovenia (SVN)	9.6	11.8	0.45	0.28	0.23
Total (ALL)	9.6	14.9	0.37	0.12	0.15

*Note:* The numbers contained in this table are primarily based on results from the WDN survey as presented in Druant et al. (2009). The information in columns 1 and 2 stems from their Table 5, the numbers in columns 4 and 5 from their Tables 4 and 7, respectively. For the calculation of  $s_A$  I have used the raw data and the procedure described in footnote 12.

that are concluded in each month to calculate an indicator for the clustering of wage contracts  $s_A$ . The data show that despite the typically rather large share of wages that are renewed in January, the calculated amount of asymmetry is on average rather small (average:  $s_A = 0.37$ ).<sup>12</sup> Second, for the numbers in column 4 I take the information in Table 4 of Druant et al. (2009) about the percentage of wages that are set more frequently than once a year and I use them as my approximate measure for the share  $\tau$  of flexible wages. The average value of 12% looks plausible, in particular if one takes the existence of new jobs and job-to-job changes into account for which wages are typically set in a rather flexible manner.<sup>13</sup> Finally, I measure the extent of wage indexation  $\gamma_w$ —as reported in column 5—by the percentage of firms that have reported to use automatic indexation of wages to either past or expected inflation (Table 7 in Druant et al., 2009). This is true for an average of 15% of all firms, where in Belgium and Spain the percentages are exceptionally high.

I use the numbers in Table 1 to calculate country-specific degrees of RWR. Figure 4 compares the estimations that are based on the model with Calvo wage contracts with the one based on standard Taylor contracts ( $s_a = 1/2, \tau = 0, \gamma_w = 0$ ).

Insert Figure 4 about here

In line with the results of section 3 the annual RWR based on the model with Taylor contracts is considerably lower (average: 0.17) than the one based on the model with Calvo contracts (average: 0.35). The ranking of countries is not drastically affected by the assumptions about the nature of wage contracts although there are some notable changes in position (involving, e.g., the Netherlands and Poland). Given that the average duration of wages (15 months) and prices (10 months) is close to the assumption that is used in the baseline calibration by Galí (2009) (12 months and 9 months, respectively) it is not surprising that the average values for RWR based on European data are close

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<sup>12</sup>In particular, I take the data to derive the proportion of new contracts in the first relative to the second semester, depending on different assumptions about the beginning of a year: i.e.  $\frac{\% \text{ new contracts Jan-Jun}}{\% \text{ new contracts Jul-Dec}}$ ,  $\frac{\% \text{ new contracts Feb-Jul}}{\% \text{ new contracts Aug-Jan}}$ , ...,  $\frac{\% \text{ new contracts Jun-Nov}}{\% \text{ new contracts Dec-May}}$ . From these ratios one can derive specific sector sizes  $s_A^1$  to  $s_A^6$ . The country-specific measure for clustering reported in column 3 of Table 1 is the average of these values. Using just the first element  $s_A^1$  (i.e. the first half-year is assumed to last from January to June) leads to similar values (average: 0.34; the correlation to the values in Table 1 is 0.93). The biggest differences are observed for Spain ( $s_A^1 = 0.14$ ) and Portugal ( $s_A^1 = 0.1$ ).

<sup>13</sup>The exact numbers can of course only be regarded as a rough approximation for wage flexibility. Complications might, e.g., arise if a fraction of wages that are said to be changed within a year are adjusted according to some predetermined schedule (as is, e.g., the case in Greece). Nevertheless, the data in Table 1 can serve as a first indication for cross-country variation in wage flexibility.

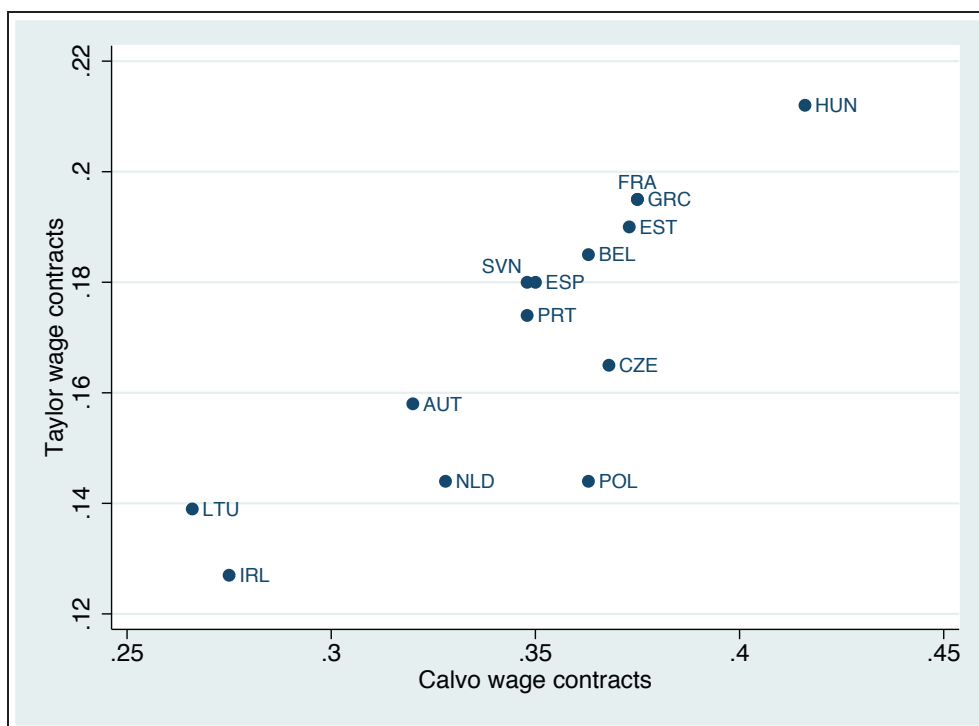


Figure 4: Comparison of the annual RWR implied by the theoretical model with either Calvo or standard Taylor wage contracts (where  $s_a = 1/2$ ,  $\tau = 0$ ,  $\gamma_w = 0$ ).

to the results based on the baseline calibration derived in sections 2 and 3. For either assumption, however, one observes remarkably small differences in the implied RWR for the large bulk of countries. For Calvo contracts most values are in the interval between 0.26 and 0.37 and for Taylor contract they range from 0.13 to 0.21. Only Ireland and Lithuania stand out at the lower end and Hungary at the higher end.

Insert Figure 5 about here

In Figure 5 I use the institutional details reported in columns 3 to 5 of Table 1. Including the information about asymmetric sector sizes does not have a huge impact. The average RWR (0.16) is close to the value in the standard Taylor model (0.17) and also the standard deviation remains almost unchanged. This follows from the fact that the extent of asymmetry is in general rather small and also similar for most countries. The only exceptions are Portugal ( $s_A = 0.19$ ) and Spain ( $s_A = 0.2$ ) for which one can observe lower degrees of RWR and also a change in the country ranking.

The impact of the prevalence of flexible wages is much more pronounced. Average RWR decreases to 0.08, whereas cross-country variation increases. One can also observe

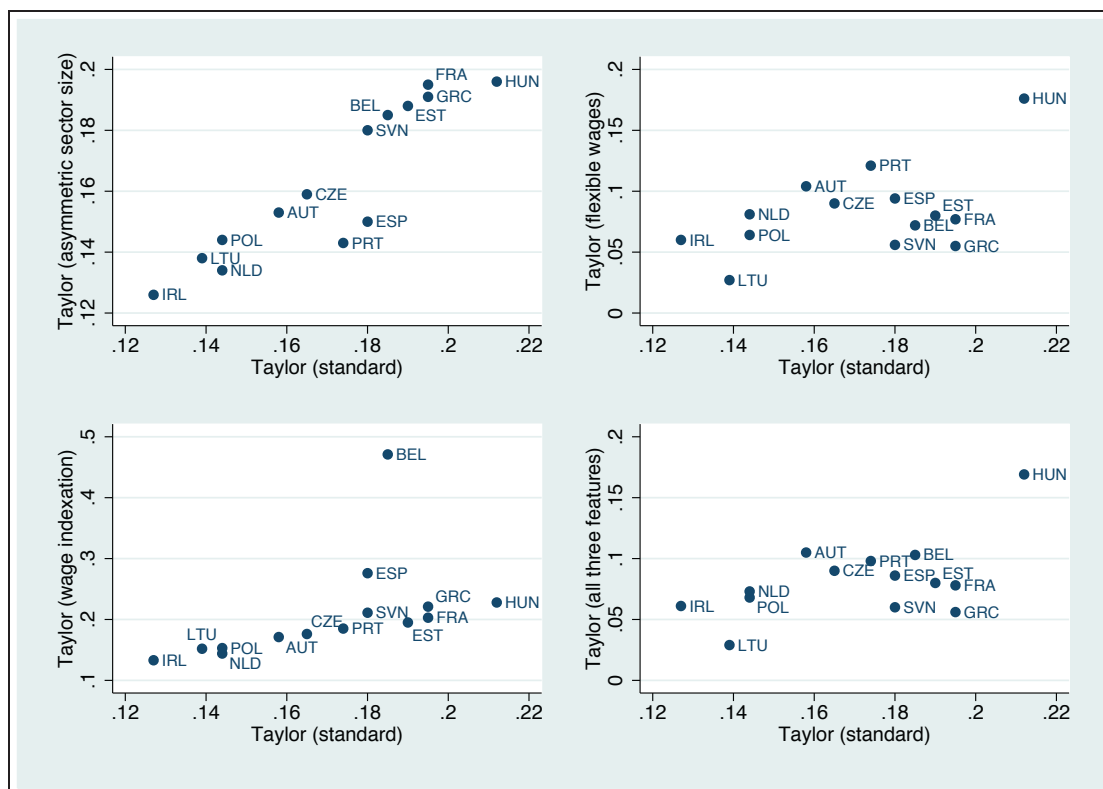


Figure 5: Comparison of the RWR implied by different variants of the model with Taylor contracts. The pictures show the cases where (a)  $\tau = 0$ ,  $\gamma_w = 0$  and  $s_A$  is country-specific, (b)  $s_A = 1/2$ ,  $\gamma_w = 0$  and  $\tau$  is country-specific, (c)  $s_A = 1/2$ ,  $\tau = 0$  and  $\gamma_w$  is country-specific and (d) where all  $s_A$ ,  $\tau$  and  $\gamma_w$  are country-specific. The correlations (rank-correlation) of the four cases with the standard model are given by: 0.91 (0.94), 0.5 (0.24), 0.51 (0.85) and 0.56 (0.34).

some remarkable changes in the relative ranking of countries, e.g. for Greece, Lithuania and Slovenia that are characterized by the largest values for  $\tau$ . The correlation (0.5) and rank-correlation (0.24) with the standard Taylor model are rather low.

Taking wage indexation into account leads to considerable changes for two countries where this is an widely-used practice. While for the other 13 countries the average RWR increases only slightly (to 0.18) the increases is much larger for Belgium (from 0.19 to 0.47) and Spain (from 0.18 to 0.28).

Taking all three additional wage-setting characteristics into account one can observe that they have a noticeable impact on the measure of RWR. The average is lower (0.08) than in the standard case and one can again also observe some changes in the relative position of countries. In fact, the rank correlation with the standard Taylor model is only 0.34.

To conclude this section it would be interesting to compare the model-based measures for RWR with existing empirical evidence. Unfortunately, however, the latter is rather scarce. Blanchard and Galí (2007), e.g., do not provide an estimate or a “reasonable” value for the (assumed) magnitude of RWR. In discussing the results they employ illustrative values between 0.5 and 0.9 (on a qoq basis), whereas Duval and Vogel (2007), in a related set-up, choose values between 0.79 and 0.93. Others use a fixed calibrated value of  $\gamma = 0.5$  (Blanchard and Galí, 2008) or  $\gamma = 0.6$  (Faia, 2008). Using the European survey data on price and wage-setting one can calculate the implied values for qoq RWR based on the EHL model. They come out as 0.77 (Calvo wage contracts), 0.64 (standard Taylor wage contracts) and 0.54 (Taylor wage contracts with institutional heterogeneity). These theory-plus-survey-based values thus fall within the range of values assumed in the framework based on the short-cut formulation.

In order to expand the scarce evidence on RWR in Europe, I have also performed a regression analysis using various macroeconomic time series for the same set of 15 countries that are covered in the WDN survey. Details of the analysis can be found in appendix C. The empirical specification is based on the theoretical model as given by (17) or (21) and the time period is from 1990–2007 (for most EU15 countries) and 1995–2007 (for most new member states). The estimated average level of qoq RWR comes out as 0.7 with a standard deviation of 0.27.<sup>14</sup> The estimates are thus in the neighborhood of the values

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<sup>14</sup>The only other directly comparable cross-country estimates of RWR can be found in Abbritti and Weber (2008). They use data from 13 OECD countries and for the time period 1970–1999. They report an average qoq RWR of 0.7 (SD: 0.12). These values are similar to my own estimates despite the fact that the two analyses differ along important dimensions: they refer to different countries (only four are contained in both), they use different time spans and they employ different empirical specifications

based on the theoretical models—slightly lower than the ones based on the model with Calvo contracts (0.77) and larger than the ones based on the model with Taylor contracts (0.64 and 0.54, respectively). It must be stressed, however, that the use of shorter basic time units would increase the implied values of RWR for both assumptions about wage contracts (see appendix B.1). Using quarters, e.g., increases average qoq RWR to 0.82 (for Calvo contracts) and to 0.72 (for standard Taylor wage contracts). Taken together, one can thus state that the EHL model is in fact capable of generating degrees of RWR that are of a similar magnitude as the assumed values in short-cut formulations and also as empirical estimates.

Turning to the cross-country differences one can observe that the variations in RWR implied by the standard theoretical models are much smaller than their empirical counterparts. While my qoq estimates show a standard deviation of 0.27 it is only 0.02 for the calculations based on the model with Calvo contracts and also only 0.02 for the one based on standard Taylor contracts. The corresponding coefficients of variations (CV) are given by 0.39, 0.03 and 0.04, respectively. The inclusion of additional institutional details increases the spread of RWR between countries. In particular, for the case where all three characteristics are taken into account (cf. Figure 5 (d)) the cross-country variation as measured by the SD is 0.05 despite the lower average value of RWR, implying a CV of 0.1. This is still below the observed value of 0.39 but larger than in the case of the standard models.

As a last point one might ask how the country rankings with respect to the calculated and to the estimated degrees of RWR compare. The outcome is not completely conclusive but it also supports the argument that institutional variety is important. Using all 15 countries, Spearman's rank correlation coefficient is -0.09 for the model with Calvo contracts but it increases to 0.24 if one uses the model with Taylor contracts and with institutional heterogeneity.

Summing up, the results of this section show that the parsimonious EHL model is in fact capable to explain reasonable amounts of average RWR. The additional inclusion of three crucial characteristics of European wage-setting improves the ability of the theoretical model to also explain the observed degree of cross-country variation in RWR. In this context it is important to note that I have only picked out three features that could be incorporated into the standard model in a straightforward way and for which cross-country evidence is readily available. It seems obvious that the inclusion of further

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(Abbritti and Weber (2006), e.g., estimate their equation in levels while I use growth rates).

institutional details would cause an even higher degree of cross-country variation. In order to get an accurate description of the dynamic properties and transmission channels in different economies it is imperative to reflect cross-country differences in the institutional set-up in an accurate way. It does not seem enough to focus just on the observed (or estimated) *average* duration of prices and wages and to translate these figures into symmetric hazard rates  $\theta_p$  and/or  $\theta_w$ . Reasonable cross-country comparisons require a closer look at institutional peculiarities and the detailed organization of industrial relations.

## 5 Conclusion

In this paper I have used a standard DSGE model to show that the synchronous presence of a nominal price and a nominal wage rigidity leads to real wage rigidity. I found that the institutional details of wage determination can have a considerable impact on the extent of RWR. If wages are assumed to be set for a fixed length of time (Taylor wage contracts) then the resulting real wages are much less rigid than in the case of Calvo wage contracts with an identical average duration. Furthermore, RWR is lower if wage-setting is clustered in particular months, if the share of flexible wages is higher and if there is less wage indexation.

I have used recent survey evidence on price- and wage-setting practices for 15 European countries in order to study whether the predictions of the parsimonious theoretical model are in line with empirical estimates. The calibrated EHL model based on the survey data for the average duration of prices and wages implies a quarterly RWR between 0.6 and 0.8. These values are in line with empirical estimations and also with the assumptions that are typically made in models with short-cut formulations for RWR. The model with Calvo contracts and with standard Taylor contracts are, however, unable to match the observed cross-country variations of RWR. This follows from the fact that the average price and wage durations are rather similar across countries. In order to get a better agreement with the cross-country variation one has to take additional institutional characteristics of wage-setting into account. The survey data document, e.g., that European countries show larger differences with respect to clustering, contract length heterogeneity and indexation. The inclusion of these dimensions increases the cross-country variation (as measured by the standard deviation or the coefficient of variation), although it is still somewhat below the empirically observed extent of variation.

This shortfall is likely to be due to further institutional differences from which this



paper abstracts. First, I have captured the heterogeneity of contract length by the percentage of flexible wages. In reality, however, one can also observe country-specific shares of long-term wage contracts ( $\geq 2$  years) and this could lead to more cross-country variation in the implied degrees of RWR. Second, wage-setting is also influenced by the structure of industrial relations. There are important differences in European countries with respect to coordination and centralization of wage bargaining. Knell and Stiglbauer (2009), e.g., have shown that the institution of wage leadership (that can be observed in Germany, Austria and some Scandinavian countries) implies a lower degree of inflation persistence and will—pari passu—also be associated with a lower degree of RWR. Finally, cross-country differences in employment protection and the unemployment benefit system and also in the degree of openness and in the structure of financial intermediation are likely to have an effect on the transmission mechanism and on real wage rigidity. The incorporation of these and related institutional details into standard DSGE models is a promising area for future research.

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# Appendices

## A Derivations and proofs

### A.1 The model with exogenous output (section 2.3)

From the definitions of  $\lambda_p$  and  $\lambda_w$  it follows that:  $\lim_{\theta_p \rightarrow 0} \lambda_p = \infty$  and  $\lim_{\theta_w \rightarrow 0} \lambda_w = \infty$ . So using the definition  $\tilde{\lambda} \equiv \frac{1}{1+\beta+\lambda_w+\lambda_p}$  it follows that  $\lim_{\theta_p \rightarrow 0} \tilde{\lambda} = 0$  and  $\lim_{\theta_w \rightarrow 0} \tilde{\lambda} = 0$ . From the definition of  $\delta$  (equation (15)) one thus gets that:  $\lim_{\tilde{\lambda} \rightarrow 0} \delta = 0/0$ . Using l'Hospital's rule one can conclude that  $\lim_{\tilde{\lambda} \rightarrow 0} \delta = \lim_{\tilde{\lambda} \rightarrow 0} 2\tilde{\lambda} \left(1 - 4\beta\tilde{\lambda}^2\right)^{-\frac{1}{2}} = 0$ .

In a similar vein one can calculate that  $\lim_{\tilde{\lambda} \rightarrow 0} \frac{\delta(\lambda_w+\lambda_p)}{1-\beta\delta}\bar{\omega}^n = \bar{\omega}^n$  and  $\lim_{\tilde{\lambda} \rightarrow 0} \frac{\delta(\lambda_w+\lambda_p)\psi_{\omega a}^n}{1-\beta\delta\rho_a} = \psi_{\omega a}^n$ . Therefore for  $\theta_p \rightarrow 0$  or  $\theta_w \rightarrow 0$  one can write that  $\omega_t = \bar{\omega}^n + \psi_{\omega a}^n a_t$  or (using the definitions of  $mrs_t$ ,  $\psi_{\omega a}^n$  and  $\psi_{y_a}^n$ ):  $\omega_t - \bar{\omega}^n = mrs_t$ . This is in fact the expected result for the situation with completely flexible prices.

For complete persistence ( $\theta_p = 1$ ,  $\theta_w = 1$ ), on the other hand, one gets that  $\lambda_p = 0$ ,  $\lambda_w = 0$  and  $\tilde{\lambda} = \frac{1}{1+\beta}$  and thus  $\delta = 1$ . In this case it thus holds that  $\omega_t = \omega_{t-1}$ .

### A.2 Solution to the model with Calvo wage contracts (section 2)

#### A.2.1 Basic Solution

One can use the methods of undetermined coefficients to derive a solution of the following form:

$$\pi_t^p = \Psi_0^p + \Psi_1^p \omega_{t-1} + \Psi_2^p a_t + \Psi_3^p v_t \quad (23)$$

$$\tilde{y}_t = \Psi_0^y + \Psi_1^y \omega_{t-1} + \Psi_2^y a_t + \Psi_3^y v_t \quad (24)$$

$$\omega_t = \Psi_0^\omega + \Psi_1^\omega \omega_{t-1} + \Psi_2^\omega a_t + \Psi_3^\omega v_t \quad (25)$$

The coefficient  $\Psi_1^\omega$  thus gives the degree of real wage rigidity in a specification where one corrects for the realization of the two shocks  $a_t$  and  $v_t$ . It holds that  $\Psi_0^p = -\Psi_1^p \bar{\omega}^n$ ,  $\Psi_0^y = -\Psi_1^y \bar{\omega}^n$  and  $\Psi_0^\omega = (1 - \Psi_1^\omega) \bar{\omega}^n$  and thus the system (23) to (25) could as well be written in terms of deviation of  $\omega_t$  from its flexible-prices-no-shocks value  $\bar{\omega}^n$ :  $\pi_t^p = \Psi_1^p (\omega_{t-1} - \bar{\omega}^n) + \Psi_2^p a_t + \Psi_3^p v_t$ ,  $\tilde{y}_t = \Psi_1^y (\omega_{t-1} - \bar{\omega}^n) + \Psi_2^y a_t + \Psi_3^y v_t$  and  $(\omega_t - \bar{\omega}^n) = \Psi_1^\omega (\omega_{t-1} - \bar{\omega}^n) + \Psi_2^\omega a_t + \Psi_3^\omega v_t$ .

$\Psi_3^\omega v_t$ .<sup>15</sup>

### A.2.2 Solution in terms of $\omega_{t-1}$ , $a_t$ and $\tilde{y}_t$

Using (24)  $v_t$  can be expressed in terms of  $\tilde{y}_t$ :  $v_t = \frac{1}{\Psi_3^y} (\tilde{y}_t - \Psi_0^y - \Psi_1^y \omega_{t-1} - \Psi_2^y a_t)$ . Inserting this in (23) and (25) equilibrium inflation and the equilibrium real wage can be written just in terms of the past real wage ( $\omega_{t-1}$ ), the output gap ( $\tilde{y}_t$ ) and a supply shock ( $a_t$ ) which corresponds loosely to a specification one can frequently find in the empirical literature:

$$\pi_t^p = \left( \Psi_0^p - \Psi_0^y \frac{\Psi_3^p}{\Psi_3^y} \right) + \left( \Psi_1^p - \Psi_1^y \frac{\Psi_3^p}{\Psi_3^y} \right) \omega_{t-1} + \left( \Psi_2^p - \Psi_2^y \frac{\Psi_3^p}{\Psi_3^y} \right) a_t + \frac{\Psi_3^p}{\Psi_3^y} \tilde{y}_t \quad (26)$$

$$\omega_t = \left( \Psi_0^\omega - \Psi_0^y \frac{\Psi_3^\omega}{\Psi_3^y} \right) + \left( \Psi_1^\omega - \Psi_1^y \frac{\Psi_3^\omega}{\Psi_3^y} \right) \omega_{t-1} + \left( \Psi_2^\omega - \Psi_2^y \frac{\Psi_3^\omega}{\Psi_3^y} \right) a_t + \frac{\Psi_3^\omega}{\Psi_3^y} \tilde{y}_t \quad (27)$$

The degree of RWR used in the text is defined as  $\delta^* \equiv \left( \Psi_1^\omega - \Psi_1^y \frac{\Psi_3^\omega}{\Psi_3^y} \right)$ . The other coefficients used in equation (17) are:  $\Psi_4^\omega \equiv \left( \Psi_2^\omega - \Psi_2^y \frac{\Psi_3^\omega}{\Psi_3^y} \right)$ ,  $\Psi_5^\omega \equiv \frac{\Psi_3^\omega}{\Psi_3^y}$  and  $\Psi_6^\omega \equiv \left( \Psi_0^\omega - \Psi_0^y \frac{\Psi_3^\omega}{\Psi_3^y} \right)$ .

### A.2.3 A Phillips curve ( $\pi_t^p$ depending on $\pi_{t-1}^p$ )

One can also derive an equation that is fairly close to the traditional Phillips curve formulation. First, lag (23) by one period and then use  $(\omega_{t-1} - \bar{\omega}^n) = \Psi_1^\omega (\omega_{t-2} - \bar{\omega}^n) + \Psi_2^\omega a_{t-1} + \Psi_3^\omega v_{t-1}$  to substitute for  $(\omega_{t-2} - \bar{\omega}^n)$ . One gets:

$$\pi_{t-1}^p = \frac{\Psi_1^p}{\Psi_1^\omega} (\omega_{t-1} - \bar{\omega}^n) + \left( \Psi_2^p - \Psi_2^\omega \frac{\Psi_1^p}{\Psi_1^\omega} \right) a_{t-1} + \left( \Psi_3^p - \Psi_3^\omega \frac{\Psi_1^p}{\Psi_1^\omega} \right) v_{t-1}$$

This can be used to find an expression for  $(\omega_{t-1} - \bar{\omega}^n)$  which can then plugged into (23) to get:

$$\pi_t^p = \Psi_1^\omega \pi_{t-1}^p + \Psi_2^p a_t + \Psi_3^p v_t - (\Psi_2^p \Psi_1^\omega - \Psi_2^\omega \Psi_1^p) a_{t-1} - (\Psi_3^p \Psi_1^\omega - \Psi_3^\omega \Psi_1^p) v_{t-1} \quad (28)$$

<sup>15</sup>It can be easily seen that only if the restrictions on  $\Psi_0^p$ ,  $\Psi_0^y$  and  $\Psi_0^\omega$  are fulfilled it holds that in a steady state  $\pi_t^p = 0$ ,  $\tilde{y}_t = 0$  and  $\omega_t = \bar{\omega}^n$ .

Again one can use (24) (and a lagged version of (24)) to express  $v_t$  and  $v_{t-1}$  in terms of the other variables. It follows:

$$\begin{aligned} \pi_t^p &= \delta^* \pi_{t-1}^p + \left( \Psi_2^p - \Psi_2^y \frac{\Psi_3^p}{\Psi_3^y} \right) a_t + \frac{\Psi_3^p}{\Psi_3^y} \tilde{y}_t + \frac{1}{\Psi_3^y} (\Psi_1^p \Psi_3^\omega - \Psi_1^\omega \Psi_3^p) \tilde{y}_{t-1} + \\ &\quad \frac{1}{\Psi_3^y} [\Psi_1^\omega (\Psi_3^p \Psi_2^y - \Psi_2^p \Psi_3^y) + \Psi_2^\omega (\Psi_1^p \Psi_3^y - \Psi_3^p \Psi_1^y) + \Psi_3^\omega (\Psi_2^p \Psi_1^y - \Psi_1^p \Psi_2^y)] a_{t-1} \end{aligned} \quad (29)$$

This corresponds to equation (18) in the text.

### A.3 Solution to the model with Taylor wage contracts (section 3)

#### A.3.1 The wage-setting equation

I use a variant of the Taylor model where there are two sectors A and B with contracts of a fixed (two-period) length and one sector F where wages are set in a flexible way. I start the description with the sectors with staggered wages.

There is no explicit treatment of the model with Taylor contracts in Galí (2008).<sup>16</sup> It is, however, straightforward to derive a wage-setting equation following analogous steps as in Galí (2008, chap. 6.1.2.1). Instead of equation (10) in chapter 6 the two-period Taylor model implies the following optimal wage-setting equation (for sector  $i \in \{A, B\}$  that is allowed to choose a new wage in period  $t$ ):

$$w_t^i = \frac{1}{1 + \varepsilon_w \varphi} \sum_{k=0}^1 \frac{\beta^k}{1 + \beta} E_{t+k} \{ \mu_w + mrs_{t+k} + \varepsilon_w \varphi w_{t+k} + p_{t+k} \}$$

One can also follow Galí (2008) and define  $\hat{\mu}_t^w \equiv \mu_t^w - \mu^w$  as the deviation of the economy's (log) average wage markup  $\mu_t^w \equiv (w_t - p_t) - mrs_t$  from its steady state level  $\mu^w \equiv \log \mu^w = \log \left( \frac{\varepsilon_w}{\varepsilon_w - 1} \right)$ . The marginal rate of substitution is given by (11) and thus one can also write:

$$w_t^i = \frac{1}{1 + \varepsilon_w \varphi} \sum_{k=0}^1 \frac{\beta^k}{1 + \beta} E_{t+k} \{ (1 + \varepsilon_w \varphi) w_{t+k} - \hat{\mu}_{t+k}^w \}$$

Using other definitions and transformations one can also derive that  $\hat{\mu}_t^w = \tilde{\omega}_t - \tilde{y}_t \left( \sigma + \frac{\varphi}{1-\alpha} \right)$ .

<sup>16</sup>The only exception is the end-of-chapter exercise 3.5 that deals with optimal price-setting in the Taylor model.

Thus another equivalent expression is:

$$w_t^i = \frac{1}{1 + \varepsilon_w \varphi} \sum_{k=0}^1 \frac{\beta^k}{1 + \beta} E_{t+k} \left\{ (1 + \varepsilon_w \varphi) w_{t+k} - \tilde{\omega}_{t+k} + \tilde{y}_{t+k} \left( \sigma + \frac{\varphi}{1 - \alpha} \right) \right\}$$

This formulation is used in the text as equation (19). For the periods where sector  $i \in \{A, B\}$  does not adjust wages I assume that a fraction of firms  $\gamma_w$  indexes the previously determined wage to the rate of (current) inflation, i.e.  $w_t^i = w_{t-1}^i + \gamma_w \pi_t$ .

The flexible wage set in sector F follows from the flex-wage condition (cf. Galí, 2008, p. 133) that  $\omega_t^{flex} = (\sigma + \frac{\varphi}{1-\alpha}) \tilde{y}_t$ . From this one can derive the equation for  $w_t^F$  as:  $w_t^F = \omega_t^n + p_t + (\sigma + \frac{\varphi}{1-\alpha}) \tilde{y}_t$ .

The average wage in period  $t$  is the weighted average of the three sectoral wages, i.e.:

$$w_t = (1 - \tau) (s^i w_t^i + s^{-i} (w_{t-1}^{-i} + \gamma_w \pi_t)) + \tau \left( \omega_t^n + p_t + (\sigma + \frac{\varphi}{1 - \alpha}) \tilde{y}_t \right)$$

This is equation (20) in the text. The rest of the model is the same as in the standard case.

### A.3.2 Basic Solution

One can again use the method of undetermined coefficients to derive a solution of the form:

$$w_t^i = \Gamma_0^{wi} + \Gamma_1^{wi} w_{t-1}^j + \Gamma_2^{wi} p_{t-1}^j + \Gamma_3^{wi} a_t + \Gamma_4^{wi} v_t \quad (30)$$

$$p_t^i = \Gamma_0^{pi} + \Gamma_1^{pi} w_{t-1}^j + \Gamma_2^{pi} p_{t-1}^j + \Gamma_3^{pi} a_t + \Gamma_4^{pi} v_t \quad (31)$$

$$\tilde{y}_t^i = \Gamma_0^{yi} + \Gamma_1^{yi} w_{t-1}^j + \Gamma_2^{yi} p_{t-1}^j + \Gamma_3^{yi} a_t + \Gamma_4^{yi} v_t \quad (32)$$

For  $t = 0, 2, 4, \dots$  one has that  $i = A$  and  $j = B$  while for  $t = 1, 3, 5, \dots$  it holds that  $i = B$  and  $j = A$ . For the symmetric case with  $s^A = s^B = \frac{1}{2}$  the various coefficients are the same across sectors, i.e.  $\Gamma_0^{wA} = \Gamma_0^{wB}, \dots, \Gamma_4^{yA} = \Gamma_4^{yB}$ . This, however, is no longer true for the asymmetric case with  $s^A \neq s^B$  where it is thus important to distinguish between the determination of the key variables in sectors  $A$  and  $B$ . The numerical calculations



have shown a useful result:<sup>17</sup>

$$\Gamma_1^{wi} + \Gamma_2^{wi} = 1, \Gamma_1^{pi} + \Gamma_2^{pi} = 1, \Gamma_1^{yi} + \Gamma_2^{yi} = 0 \quad (33)$$

### A.3.3 The period-on-period and year-on-year RWR

For the sake of comparison it is better to express all relations in terms of  $\tilde{y}_t$  instead of  $v_t$ . One can use (32) to write  $v_t = \frac{1}{\Gamma_4^{yi}} (\tilde{y}_t - \Gamma_0^{yi} - \Gamma_1^{yi} w_{t-1}^j - \Gamma_2^{yi} p_{t-1}^j - \Gamma_3^{yi} a_t)$ . From this one can derive an expression for the real wage  $\omega_t^i \equiv w_t^i - p_t^i$  in sector  $i$ :

$$\begin{aligned} \omega_t^i = & \left( \Gamma_0^{wi} - \Gamma_0^{pi} - \frac{(\Gamma_4^{wi} - \Gamma_4^{pi}) \Gamma_0^{yi}}{\Gamma_4^{yi}} \right) + \left( \Gamma_1^{wi} - \Gamma_1^{pi} - \frac{(\Gamma_4^{wi} - \Gamma_4^{pi}) \Gamma_1^{yi}}{\Gamma_4^{yi}} \right) \omega_{t-1}^j + \\ & \left( \Gamma_3^{wi} - \Gamma_3^{pi} - \frac{(\Gamma_4^{wi} - \Gamma_4^{pi}) \Gamma_3^{yi}}{\Gamma_4^{yi}} \right) a_t + \left( \frac{(\Gamma_4^{wi} - \Gamma_4^{pi})}{\Gamma_4^{yi}} \right) \tilde{y}_t^i \end{aligned} \quad (34)$$

Since the coefficients might be different across the two sectors also the reaction of  $\omega_t^A$  to  $\omega_{t-1}^B$  might be different from the reaction of  $\omega_{t+1}^B$  to  $\omega_t^A$  etc. For this reason (and in order to make the cases with different weights  $s_A$  and  $s_B$  comparable) it is useful to write the real wage  $\omega_t^i$  in sector  $i$  as a function of its own last optimally set real wage  $\omega_{t-2}^i$ . This comes out as:

$$\omega_t^i = \Gamma_0^{\omega i} + \Gamma_1^{\omega i} \omega_{t-2}^i + \Gamma_2^{\omega i} a_t + \Gamma_3^{\omega i} a_{t-1} + \Gamma_4^{\omega i} \tilde{y}_t + \Gamma_5^{\omega i} \tilde{y}_{t-1}, \quad (35)$$

where:

$$\begin{aligned} \Gamma_0^{\omega i} = & \frac{[\Gamma_1^{yi} (\Gamma_4^{wi} - \Gamma_4^{pi}) - \Gamma_4^{yi} (\Gamma_1^{wi} - \Gamma_1^{pi})] (\Gamma_4^{wj} - \Gamma_4^{pj}) \Gamma_0^{yj}}{\Gamma_4^{yi} \Gamma_4^{yj}} + \\ & \frac{[\Gamma_4^{yi} (\Gamma_1^{wi} - \Gamma_1^{pi}) - \Gamma_1^{yi} (\Gamma_4^{wi} - \Gamma_4^{pi})] (\Gamma_0^{wj} - \Gamma_0^{pj}) \Gamma_4^{yj}}{\Gamma_4^{yi} \Gamma_4^{yj}} + \\ & \frac{[\Gamma_4^{yi} (\Gamma_0^{wi} - \Gamma_0^{pi}) - \Gamma_0^{yi} (\Gamma_4^{wi} - \Gamma_4^{pi})] \Gamma_4^{yj}}{\Gamma_4^{yi} \Gamma_4^{yj}} \\ \Gamma_1^{\omega i} = & \frac{[\Gamma_1^{yi} (\Gamma_4^{wi} - \Gamma_4^{pi}) - \Gamma_4^{yi} (\Gamma_1^{wi} - \Gamma_1^{pi})] [\Gamma_1^{yj} (\Gamma_4^{wj} - \Gamma_4^{pj}) - \Gamma_4^{yj} (\Gamma_1^{wj} - \Gamma_1^{pj})]}{\Gamma_4^{yi} \Gamma_4^{yj}} \end{aligned}$$

<sup>17</sup>Intuitively, these relationships follow from the fact that the “order” of normal variables on each side of equations (30) to (32) has to be the same. Unfortunately, I have not been able to show analytically that these relationships have to hold. Nevertheless, they have been confirmed for all numerical cases that have been scrutinized.

$$\Gamma_2^{\omega i} = \Gamma_3^{\omega i} - \Gamma_3^{p i} - \frac{\Gamma_3^{y i} (\Gamma_4^{\omega i} - \Gamma_4^{p i})}{\Gamma_4^{y i}}$$

$$\Gamma_3^{\omega i} = \frac{[\Gamma_1^{y i} (\Gamma_4^{\omega i} - \Gamma_4^{p i}) - \Gamma_4^{y i} (\Gamma_1^{\omega i} - \Gamma_1^{p i})] [\Gamma_3^{y j} (\Gamma_4^{\omega j} - \Gamma_4^{p j}) - \Gamma_4^{y j} (\Gamma_3^{\omega j} - \Gamma_3^{p j})]}{\Gamma_4^{y i} \Gamma_4^{y j}}$$

$$\Gamma_4^{\omega i} = \frac{\Gamma_4^{\omega i} - \Gamma_4^{p i}}{\Gamma_4^{y i}}$$

$$\Gamma_5^{\omega i} = \frac{-[\Gamma_1^{y i} (\Gamma_4^{\omega i} - \Gamma_4^{p i}) - \Gamma_4^{y i} (\Gamma_1^{\omega i} - \Gamma_1^{p i})] (\Gamma_4^{\omega j} - \Gamma_4^{p j})}{\Gamma_4^{y i} \Gamma_4^{y j}}$$

Note that the coefficient  $\Gamma_1^{\omega i}$  that determines the extent of (year-on-year) rigidity of the sectoral real wage is the same in both sectors, i.e.  $\Gamma_1^{\omega A} = \Gamma_1^{\omega B} = \Gamma_1^{\omega}$  and:

$$\Gamma_1^{\omega} \equiv \frac{[\Gamma_1^{y A} (\Gamma_4^{\omega A} - \Gamma_4^{p A}) - \Gamma_4^{y A} (\Gamma_1^{\omega A} - \Gamma_1^{p A})] [\Gamma_1^{y B} (\Gamma_4^{\omega B} - \Gamma_4^{p B}) - \Gamma_4^{y B} (\Gamma_1^{\omega B} - \Gamma_1^{p B})]}{\Gamma_4^{y A} \Gamma_4^{y B}} \quad (36)$$

For comparisons across models and across countries etc. one is, however, not so much interested in the rigidity of the sectoral real wage but in the rigidity of the average real wages given by  $\bar{\omega}_t^A \equiv w_t - p_t^A$  and  $\bar{\omega}_{t+1}^B \equiv w_{t+1} - p_{t+1}^B$ . Using (20) one can derive:

$$\bar{\omega}_t^A = (1 - \tau) [s_A \omega_t^A + s_B \omega_{t-1}^B - s_B \pi_t^A (1 - \gamma_w)] + \tau \left[ \bar{\omega}^n + \psi_{\omega a} a_t + \left( \sigma + \frac{\varphi}{1 - \alpha} \right) \tilde{y}_t \right] \quad (37)$$

$$\bar{\omega}_{t+1}^B = (1 - \tau) [s_A \omega_t^A + s_B \omega_{t+1}^B - s_A \pi_{t+1}^B (1 - \gamma_w)] + \tau \left[ \bar{\omega}^n + \psi_{\omega a} a_{t+1} + \left( \sigma + \frac{\varphi}{1 - \alpha} \right) \tilde{y}_{t+1} \right] \quad (38)$$

I use here  $\bar{\omega}_t^A$  and  $\bar{\omega}_{t+1}^B$  to distinguish clearly between the average real wage in periods when sector  $A$  sets the new wage and when sector  $B$  does so. One can take the expressions for  $\omega_t^A$  and  $\omega_{t-1}^B$  (from (35)) and insert them into (37). Noting that  $(1 - \tau)(s_A \omega_{t-2}^A + s_B \omega_{t-3}^B) = \bar{\omega}_{t-2}^A + (1 - \tau)s_B \pi_{t-2}^A - \tau [\bar{\omega}^n + \psi_{\omega a} a_{t+1} + (\sigma + \frac{\varphi}{1 - \alpha}) \tilde{y}_{t+1}]$  one can derive an expressions for  $\bar{\omega}_t^A$  as a function of  $\bar{\omega}_{t-2}^A$ ,  $a_t$ ,  $a_{t-1}$ ,  $a_{t-2}$ ,  $\tilde{y}_t^A$ ,  $\tilde{y}_{t-1}^B$ ,  $\tilde{y}_{t-2}^A$ ,  $\pi_t^A$  and  $\pi_{t-2}^A$ .<sup>18</sup> Following similar steps one can also write  $\bar{\omega}_{t+1}^B$  as a function of  $\bar{\omega}_{t-1}^B$ ,  $a_{t+1}$ ,  $a_t$ ,  $a_{t-1}$ ,  $\tilde{y}_{t+1}^B$ ,  $\tilde{y}_t^A$ ,  $\tilde{y}_{t-1}^B$ ,  $\pi_{t+1}^B$  and  $\pi_{t-1}^B$ . These derivations lead to the result that the coefficient on  $\bar{\omega}_{t-2}^A$  in the first expression and on  $\bar{\omega}_{t-1}^B$  in the second expression are identical and given by  $\Gamma_1^{\omega}$ . This means that the yoy rigidity of the average real wage is the same in both periods, independent of which

<sup>18</sup>The complete equations are rather long and they have been calculated in a *Mathematica* file which is available upon request.

sector sets the new wages. Note, however, that the reaction of the average real wage to supply shocks and monetary policy shocks is different in the two subperiods. And note furthermore that the average real wage also depends on current and past inflation rates.<sup>19</sup>

### A.3.4 A Phillips curve

One can again also derive an expression that is similar to a backward-looking Phillips curve. Using  $\pi_t^A = p_t^A - p_{t-1}^B$ ,  $\pi_{t-1}^B = p_{t-1}^B - p_{t-2}^A$  and equations (30), (31) and (32) for  $w_t^A$ ,  $w_{t-1}^B$ ,  $p_t^A$ ,  $p_{t-1}^B$ ,  $\tilde{y}_t^A$  and  $\tilde{y}_{t-1}^B$  to solve (using (33)) for  $\pi_t^A$  as a function of  $\pi_{t-1}^B$ ,  $a_t$ ,  $a_{t-1}$ ,  $\tilde{y}_t^A$  and  $\tilde{y}_{t-1}^B$ . Similarly,  $\pi_{t-1}^B$  can be written as a function of  $\pi_{t-2}^A$ ,  $a_{t-1}$ ,  $a_{t-2}$ ,  $\tilde{y}_{t-1}^B$  and  $\tilde{y}_{t-2}^A$ . Taking these two together one can thus write  $\pi_t^A$  as a function of last year's inflation  $\pi_{t-2}^A$  and present and past levels of  $a_t$  and  $\tilde{y}_t$ .<sup>20</sup>

Following these steps the term on lagged inflation comes out as  $\Gamma_1^\omega$ .<sup>21</sup> This is the same result as in the case of the standard model where the coefficient of RWR  $\delta^*$  in (27) is the same as in the backward looking Phillips curve (see (29)). So  $\Gamma_1^\omega$  in the model with (symmetric or asymmetric) Taylor wage contracts corresponds to  $(\delta^*)^2$  in the model with Calvo wage contracts. This is the magnitude I focus in the text when I compare different models and specifications (i.e.,  $\tilde{\delta} \equiv \Gamma_1^\omega$ ).

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<sup>19</sup>It is interesting in this context to point out a mirror-inverted result. In the Taylor model  $\bar{\omega}_t$  depends on  $\bar{\omega}_{t-1}$ ,  $a_t$ ,  $a_{t-1}$ ,  $\tilde{y}_t$ ,  $\tilde{y}_{t-1}$ ,  $\pi_t^P$  and  $\pi_{t-1}^P$  while the newly set real wage  $\omega_t^i$  can be written just as a function of  $\omega_{t-1}^i$ ,  $a_t$  and  $\tilde{y}_t$ . In the Calvo formulation, on the other hand, it is exactly the opposite. The average real wage is just a function of  $\bar{\omega}_{t-1}$ ,  $a_t$  and  $\tilde{y}_t$  while  $\omega_t^i$  depends on  $\omega_{t-1}^i$ ,  $a_t$ ,  $a_{t-1}$ ,  $\tilde{y}_t$ ,  $\tilde{y}_{t-1}$ ,  $\pi_t^P$  and  $\pi_{t-1}^P$ .

<sup>20</sup>Details can again be found in a *Mathematica* file available upon request

<sup>21</sup>Note that this is also the coefficient of  $\pi_{t-1}^B$  that one gets if  $\pi_{t+1}^B$  is expressed as a function of  $\pi_{t-1}^B$  etc. So  $\Gamma_1^\omega$  is the relevant persistence term in both periods and sectors.

## B Notes on the time structure of the models

### B.1 The impact of the choice of the basic time unit on RWR

The standard models with Calvo and with Taylor wage contracts is based on a structure where the basic time unit corresponds to one semester. In the two-period Taylor model this implies that a wage contract lasts for one year (=two semesters). This assumption has been primarily made for convenience and in order to be able to deal with the case of asymmetric sector sizes in a coherent and comprehensible way. The assumption differs, however, from the related literature where the basic time unit is normally defined as one quarter (which is in line with the frequency of the available macroeconomic data). When calibrating the model I had to be careful to choose the correct parameter values. E.g., in the baseline case I have used a discount rate of  $\beta = 0.98$  and a price adjustment probability of  $(1 - \theta_p) = 1/3$  which implies an average price duration of  $\frac{1}{1-\frac{1}{3}} = 1.5$  semesters (or 270 days) which is a common value in the related literature (cf. Galí, 2008, 2009).

Despite the identical average duration of price and wage contracts it is nevertheless clear that the choice of the basic time unit has an effect on the dynamic properties of the model. In particular, a system where the “Calvo fairy” appears on a daily basis will imply higher persistence (for the same average contract duration) than a system where changes are only allowed on a quarterly or semiannually frequency. In order to study the extent of this effect I have solved the basic models under the assumption of shorter basic time units.

For the model with Calvo wage contracts this has been straightforward since it only involves some reparameterizations. In particular, if *frequ* denotes the length of the basic time unit (measured in days), the structural parameter that corresponds to an average contract duration of  $x$  days is given by:  $\theta_p = 1 - \frac{frequ}{x}$ , where  $x \geq frequ$ . The time discount rate is given by  $\beta = 0.96^{\frac{frequ}{360}}$ . Following the same steps as sketched in appendix A.2 one gets an estimation for  $\delta_{frequ}^*$ . This can be transformed into an annual measure of RWR by calculating  $\delta_{frequ}^{*,annual} = (\delta_{frequ}^*)^{\frac{360}{frequ}}$ . The results of this exercise are illustrated in Figure B.1 where I have used the baseline calibration and held the average length of wage contracts constant at 360 days.

Insert Figure B.1 about here

One observes that the choice of the basic time units has a nonnegligible effect on the estimated degree of RWR. The shorter the basic time unit, the higher the RWR. For an

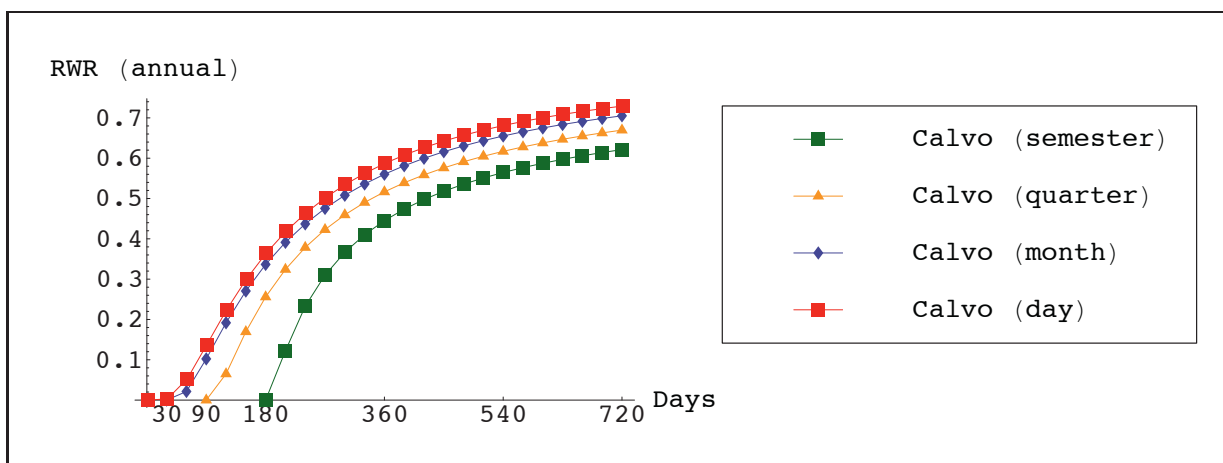


Figure B.1: Comparison of the model with Calvo wage contracts with different durations of the basic time period. The figures are based on the parameters of the baseline calibration. The average duration of wage contracts is set equal to 360 days for all specifications while the average duration of price contracts is shown on the x-axis.

average price duration of 270 days, e.g., the annual RWR is given by 0.31 (semester), 0.42 (quarter), 0.48 (month) and 0.50 (day). The intuition behind this result is clear. For the case of quarterly frequencies of price changes an average duration of 90 days is the most flexible situation one can imagine (and  $\theta_p = 0$  in this case). If one takes into account, however, that prices can in general be changed more frequently than an average duration of 90 days looks already rather sticky. Assuming a day as the correct basic time unit, the “true RWR” is higher than indicated by the values based on longer time units: by 62% (semester), 19% (quarter) and 6% (month). The larger the average price duration, the smaller the bias gets. For a price duration of 360 days, e.g., the corresponding percentages are reduced to: 32% (semester), 14% (quarter) and 5% (month).

The same exercise can also be performed for the (symmetric) Taylor model, even though in this case the calculations are less straightforward. For the Calvo model the degree of annual RWR  $\delta_{frequ}^{*,annual}$  can be directly derived from the solution of the period model (i.e. from  $\delta_{frequ}^*$ ). This is not possible (or at least intractable) for the model with Taylor wage contracts. In fact, already for the two-period structure it has been rather difficult to derive an equation of the form (22) (see also appendix A.3). For the cases with shorter basic time units such an explicit derivation of  $\tilde{\delta}_{frequ}$  is no longer feasible. Therefore I have chosen an alternative strategy to come up with comparable measures of RWR for different timing assumptions. In particular, the derivations of the two-period model have suggested that the extent of RWR can be accurately inferred from an empirical

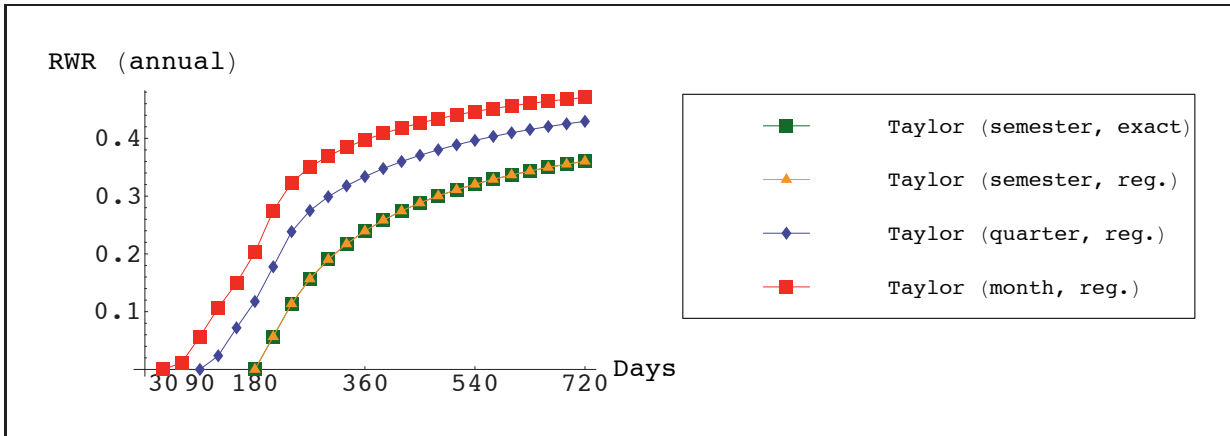


Figure B.2: Comparison of the model with Taylor wage contracts with different durations of the basic time period. The figures are again based on the parameters of the baseline calibration, the average duration of wage contracts is set equal to 360 days for all specifications and the average duration of price contracts is shown on the x-axis. The green line is based on the exact measure  $\tilde{\delta}$  as stated in (36) while the different-colored lines are based on regressions of current inflation  $\pi_t$  on the annual lag of inflation and on other explanatory variables as described in the text.

estimation where the rate of inflation  $\pi_t$  is regressed on the year-on-year lagged inflation  $\pi_{t-2}$  and measures for the output gap and the supply shocks for all intermediate periods (i.e. from  $a_t$  to  $a_{t-2}$  and from  $\tilde{y}_t$  to  $\tilde{y}_{t-2}$ ). As shown in (34) the coefficient on  $\pi_{t-2}$  is equal to  $\tilde{\delta}$  and the estimated regression coefficient on  $\pi_{t-2}$  should thus give an accurate estimation of RWR. I have simulated 50.000 data points (assuming  $\rho_a = 0 = \rho_v = 0$  and  $\sigma_a = \sigma_v = 1$ ) and ran a regression like that. The result is plotted as the orange line in Figure B.2, together with the exact (i.e. analytically derived) measure (green line) given by  $\tilde{\delta}$  (as given in (36)). The two lines are indistinguishable.

Insert Figure B.2 about here

Taking the regression results for the two-period model as a suggestive starting point I have also solved the Taylor model with 4 and with 12 subperiods.<sup>22</sup> I have then again simulated a large number of datapoints and I have run regressions that allow me to infer the degree of RWR. In particular, these regressions are of the form:

$$\pi_t = \tilde{\delta}_{quarter} \pi_{t-4} + f(a_t, \dots, a_{t-4}, \tilde{y}_t, \dots, \tilde{y}_{t-4}) \quad (39)$$

<sup>22</sup>The case of daily basic time units (360 subperiods) was too cumbersome to analyze, as was the case with asymmetric sector sizes.

and

$$\pi_t = \tilde{\delta}_{month} \pi_{t-12} + f(a_t, \dots, a_{t-12}, \tilde{y}_t, \dots, \tilde{y}_{t-12}) \quad (40)$$

The results are plotted in Figure B.2. They are qualitatively similar to the case of Calvo wage contracts, although now the underestimation of RWR due to a longer basic time unit is somewhat larger. For quarters as the basic time unit one gets  $\tilde{\delta} = 0.28$  instead of  $\tilde{\delta} = 0.16$  (while for the model with Calvo contracts the difference is  $(\delta^*)^2 = 0.42$  vs.  $(\delta^*)^2 = 0.31$ ). The bias again decreases for larger price durations. In drawing Figures B.1 and B.2 I have kept the length of the wage contract constant (at 360 days). For this assumption the figures illustrate that for both the case with Calvo and with Taylor contracts the choice of the basic time unit does not affect the ranking of countries. This might, however, change once one compares countries that differ both in their duration of prices and of wages. In the next section I say more on this issue.

## B.2 Accounting for different durations of wage contracts in the two-period Taylor model

The standard two-period Taylor model fixes the average duration of the staggered wage contracts at two semesters=one year. In order to be able to use the simple two-period framework also for cross-country comparisons and to allow for longer or shorter average wage durations it is necessary to make some adaptations. I use a straightforward method to make these adjustments that is based on the idea to take the average duration of wage contracts  $dur$  from the data and define  $frequ = \frac{dur}{2}$  as the length of the basic time unit. Due to this change in units one has to re-specify the discount rate and the parameter that captures price stickiness. In particular,  $\beta = 0.96 \frac{frequ}{360}$  and  $\theta_p = 1 - \frac{frequ}{x}$ , where  $x$  stands for the average duration of price contracts (in days) and  $x \geq frequ$ . Using these values one can then follow the same steps as in chapter A.3 to calculate a value  $\check{\delta}$  as a measure of period-to-period RWR. The annual RWR can then be derived from  $\tilde{\delta}_{frequ}^{annual} = \left(\check{\delta}\right)^{\frac{360}{frequ}}$ .<sup>23</sup>

The results for some alternative assumptions about the average duration of wage contracts are shown in Figure B.3.

Insert Figure B.3 about here

Intuition (and the experience from working with Calvo wage contracts) suggests that a longer average duration of wages should be associated—*ceteris paribus*—with a higher

<sup>23</sup>Note that for the symmetric standard model one gets that  $\tilde{\delta} = \check{\delta}^2$  (cf. (36)).

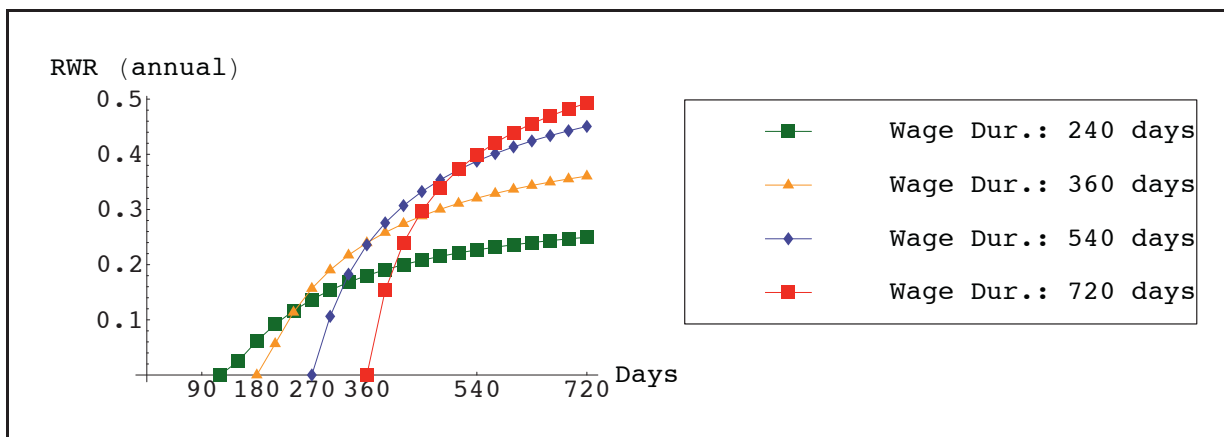


Figure B.3: Comparison of the two-period Taylor model with different durations of the average wage contract (in days). The figures are based on the parameters of the baseline calibration. The average duration of price contracts is shown on the x-axis.

RWR. As one can observe from Figure B.3 this requirement is in fact borne out by the method described above as long as the average price duration is not too much shorter than the average wage duration. Assuming, e.g., that the two nominal variables are characterized by the same average duration one gets RWRs equal to 0.12, 0.24, 0.39 and 0.49 when the duration is 240, 360, 540 and 720 days, respectively. On the other hand, one sees that if this condition about the relative duration is not fulfilled then one might get erroneous results. If, e.g., wage agreements are written for two years while prices last on average for less than one year then the approximate method would imply a RWR of zero, which is obviously wrong. The data used for the cross-country comparisons, however, do not show such vast discrepancies in relative duration for most countries. On the other hand, Figure B.3 also suggests that the procedure is less than perfect and that country rankings might change if the difference between the two durations is too large. One should thus best regard it as an approximate method that is helpful to make countries with different wage durations comparable.<sup>24</sup>

## C Empirical estimations of real wage rigidity

The starting point of the empirical specification are equations (17) and (21) that are derived from theoretical models. The solution to the model with Calvo wage contracts

<sup>24</sup>Needless to say that a model based on monthly basic time units would allow for more accurate results. Such a finer timing structure is, however, computationally rather involved, especially in as far as asymmetric sector sizes are concerned.



implies that the current real wage  $\omega_t$  is a function of  $\omega_{t-1}$ ,  $\tilde{y}_t$  and  $a_t$ , while the model based on (symmetric) Taylor wage contracts implies that it depends on  $\omega_{t-1}$ ,  $\tilde{y}_t$ ,  $\tilde{y}_{t-1}$ ,  $a_t$ ,  $a_{t-1}$ ,  $\pi_t^p$ ,  $\pi_{t-1}^p$ . The latter model thus encompasses the first one and it is chosen as the benchmark specification.

I use quarterly data and in order to deal with issues of stationarity and seasonality I estimate the model in yoy differences (i.e. in yoy growth rates). In particular, the estimation equation for each country is:

$$\Delta_4\omega_t = \beta_0 + \beta_1\Delta_4\omega_{t-1} + \beta_2\Delta_4\tilde{y}_t + \beta_3\Delta_4\tilde{y}_{t-1} + \beta_4\Delta_4a_t + \beta_5\Delta_4a_{t-1} + \beta_6\Delta_4\pi_t^p + \beta_7\Delta_4\pi_{t-1}^p \quad (41)$$

I measure the real wage  $\omega_t$  by nominal compensation per employee deflated by the GDP deflator; labor productivity  $a_t$  by real GDP divided by the level of employment; the output gap  $\tilde{y}_t$  by the deviation of real GDP from a Hodrick-Prescott-trend ( $\lambda = 1600$ ); inflation  $\pi_t^p = p_t - p_{t-1}$  as the period-on-period change in the GDP deflator. The main data source is the OECD Economic Outlook database with some exceptions. The data on wages for Belgium, Spain and the Netherlands are from national accounts and for Portugal, Poland and Hungary from the OECD Main Economic Indicators (MEI) database. In addition, GDP for Hungary is from the MEI and employment for Estonia from the International Financial Statistics (IFS) database provided by the IMF. The IFS is also the source of all data for Lithuania and Slovenia.

For the EU15 I typically have the complete set of time series from 1980 onwards with the exception of Greece (only from 2000) and Portugal (only from 1995). For the new member states I have complete datasets from 1995 onwards with the exception of Lithuania and Slovenia where the data only start in 1999. In order to get comparable results the focus is on a benchmark time period (when available) from 1990–2007.

Insert Table C.1 and Figure C.1 about here

In table C.1 I list the estimates for qoq RWR from countrywise estimations based on (41). The average RWR is given by 0.7 with a SD of 0.27. Focusing just on EU15 countries gives a similar average of 0.74 (SD: 0.27). The use of unemployment rates as a measure of business cycle conditions also leads to similar results. In figure C.1 I contrast these empirical with the theory-plus-survey-based values for the model with Calvo contracts and the one with extended Taylor contracts. The correspondence (as measured by the coefficients of correlation and rank correlation) between these two classes of measures of RWR increases if one uses the Taylor model with institutional heterogeneity.

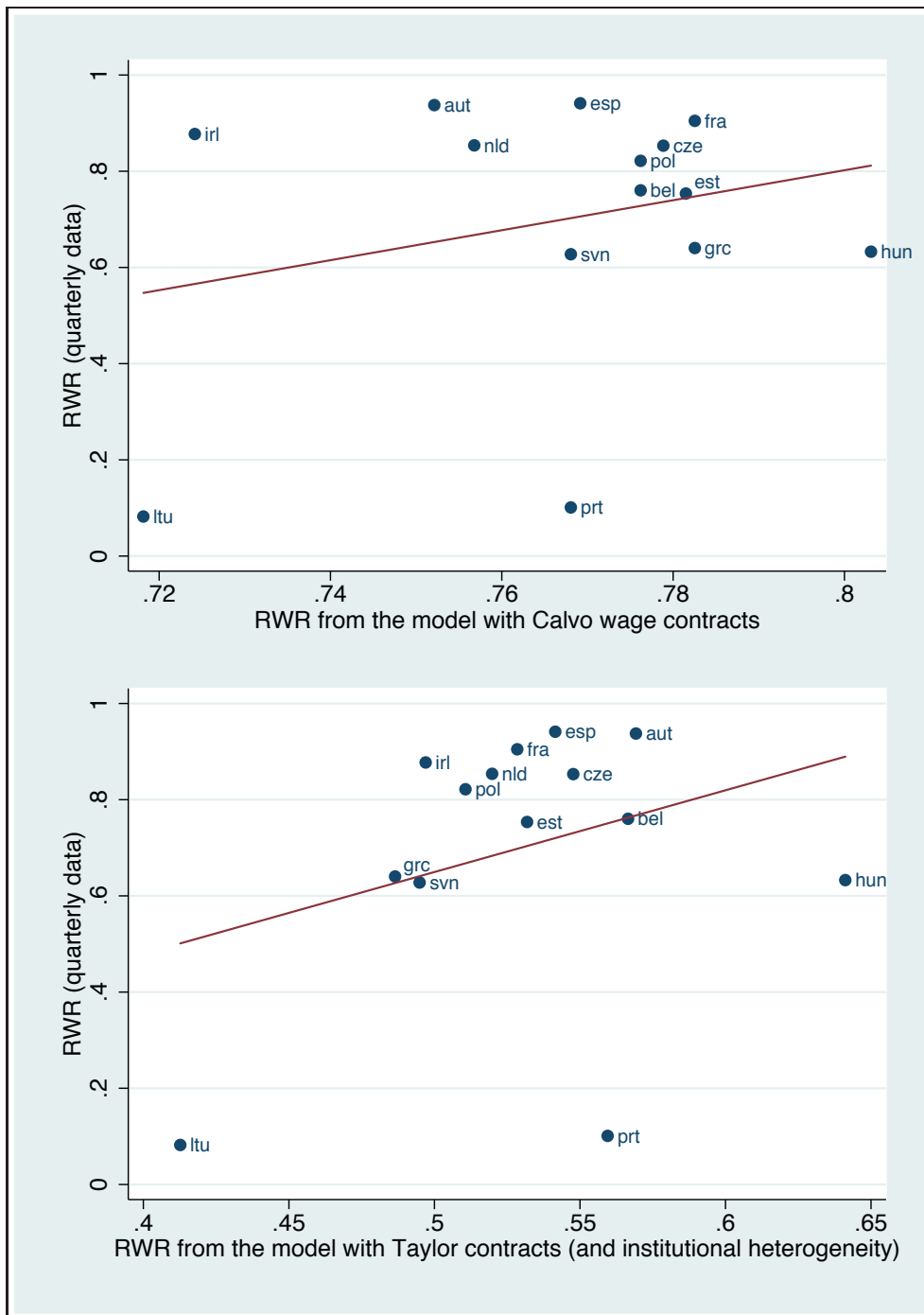


Figure C.1: Comparison of the quarterly RWR based on macroeconomic data and RWR implied by the theoretical model with Calvo and with Taylor wage contracts (including heterogeneous contracts, clustering and indexation). The correlation (rank-correlation) of the theory-based and the empirical values is given by: 0.25 (-0.09) [Calvo] and 0.32 (0.24) [extended Taylor].

Table C.1: Empirical estimates of real wage rigidity

Country	RWR ( $\beta_1$ )
Austria (AUT)	0.937
Belgium (BEL)	0.76
Spain (ESP)	0.941
France (FRA)	0.905
Greece (GRC)	0.64
Ireland (IRL)	0.877
Italy (ITA)	0.681
Netherlands (NLD)	0.854
Portugal (PRT)	0.101
Czech Republic (CZE)	0.853
Estonia (EST)	0.754
Hungary (HUN)	0.633
Lithuania (LTU)	0.082
Poland (POL)	0.822
Slovenia (SVN)	0.628
Average	0.698
Standard Deviation	0.269

