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The not-so-hidden risks of 'hidden-to-maturity' accounting: on depositor runs and bank resilience

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Abstract

We build a balance sheet-based model to capture run risk, i.e., a reduced potential to raise capital from liquidity buffers under stress, driven by depositor scrutiny and further fuelled by fire sales in response to withdrawals. The setup is inspired by the Silicon Valley Bank (SVB) meltdown in March 2023 and our model may serve as a supervisory analysis tool to monitor build-up of balance sheet vulnerabilities. Specifically, we analyze which characteristics of the balance sheet are critical in order for banking system regulators to adequately assess run risk and resilience. By bringing a time series of SVB's balance sheet data to our model, we are able to demonstrate how changes in the funding and respective asset composition made SVB prone to run risk, as they were increasingly relying on held-to-maturity, aka hidden-to-maturity, accounting standards, masking revaluation losses in securities portfolios. Finally, we formulate a tractable optimisation problem to address the designation of held-to-maturity assets and quantify banks' ability to hold these assets without resorting to remarking. By calibrating this to SVB's balance sheet data, we shed light on the bank's funding risk and implied risk tolerance in the years 2020–22 leading up to its collapse.

JEL: C62, G21, G11

Keywords: bank runs, fire sales, accounting standards, funding risk

Non-technical summary

The years 2021–22 saw a significant relative rise in banks' reliance on allocation of assets into the HtM (Held-to-Maturity) portfolios, likely in order to 'hide' potential unrealized losses as rates started to rise. For this reason, the HtM framework has become described as a form of 'hidden-to-maturity' accounting. HtM was used by less capitalised banks and, worryingly, was leaving interest rate risk practically unhedged. Combined with flighty funding sources, this may create conditions for a bank run, as demonstrated in the case of the SVB meltdown in March 2023.

The first main contribution of the paper is the development of a stylized model to explain bank runs by banks' financial conditions perceived by depositors and to measure vulnerabilities of banks stemming from their balance sheet composition, in particular as it pertains to the HtM or available-for-sale (AfS) classifications of marketable securities as well as the insured versus uninsured status of deposits. We use the model to compute deposit withdrawals and assets sold by banks in equilibrium, taking into account the share of run-prone uninsured deposits in the bank's funding, a pool of liquid resources, and an impact of assets sold to raise cash on bank's profit and loss accounts. The second main contribution of this paper is a tractable optimization problem aimed at explaining the size of banks' HtM designation and assessing their resilience to a market price shock, in the sense that banks avoid being forced into selling the HtM securities. Consequently, we can shed light on banks' ability to hold HtM portfolios given banks' business model, including funding structure and risk taken.

The aim is to keep the model setup as simple as possible, while still allowing us to capture key features of the distinctive roles played by the AfS and HtM designations in relation to bank stability. First of all, this will entail two classes of liabilities, i.e., deposits: they can be either insured or uninsured. Second, we shall assume that assets of the bank can be one of three types: liquid, illiquid but marketable, or illiquid and nonmarketable. Finally, in the case of illiquid but marketable assets, these may be classified as either available-for-sale or held-to-maturity. Beyond this classification, these illiquid but marketable securities are subject to the same market price when a decision to liquidate them is made or when they need to be recognized at fair value. Specifically, we assume that the illiquid, but marketable, holdings are subject to price impacts if they need to be sold. The mark-to-market value of these assets is given by the inverse demand function. Given the observed balance sheet, the *uninsured* depositors will withdraw their funds based on whether or not the leverage ratio is in line with some maximum acceptable threshold. Consequently, we are able to derive an algorithm computing the equilibrium level of withdrawn deposits and corresponding volume of liquidated securities in six steps. The steps of the algorithm have intuitive economic interpretation; they describe the state of a bank's liquidity conditions, from most benign when a run can be covered by cash holding, through cases when the bank has to use AfS securities or even dip into the HtM, triggering reclassification and remarking, to illliquidty and solvency.

Bringing the model to quarterly data on the SVB, we are able to assess build-up of vulnerabilities of the SVB ahead of its default in March 2023. We can show that the financial standing of the SVB was very sensitive to fire-sale conditions. Unrealised losses in SVB's balance sheet, related to HtM accounting, hid important weaknesses of the banks and the model can indicate periods prior to the March 2023 meltdown when the situation of the bank became dire. Beyond the SVB case, the model allows us to construct an indicator of banks' vulnerabilities — based on deposits withdrawn in equilibrium bank-run — that can help regulators to monitor dynamics of those vulnerabilities.

1 Introduction

In the FDIC's most recent quarterly profile of the U.S. banking industry FDIC (2024), it was highlighted that Q1 2024 became *"the ninth straight quarter of unusually high unrealized losses since the Federal Reserve began to raise interest rates in first quarter 2022"*. Specifically, the unrealized losses on held-to-maturity (HtM) portfolios among the FDIC insured Call Report filers amounted to more than $300 billion, an astronomical number compared to pre-2022 levels, see (FDIC, 2024, Chart 7). On its own, this need not be a cause for alarm, but it is critical to understand how the HtM accounting standards interact with any underlying balance sheet vulnerabilities and the possibility of depositor runs.

As documented in Granja (2023), the years 2021–22 saw a significant relative rise in banks' reliance on the HtM category, likely to 'hide' potential unrealized losses as rates started to rise. For this reason, the HtM framework has become described as a form of 'hidden-to-maturity' accounting. Worryingly, Granja (2023) also exposes how the HtM accounting rules were more frequently applied by less capitalised banks with significant uninsured deposits, thus prone to runs, to immunize their capital from revaluation of securities held on-balance. Further, Jiang et al. (2023*a*) argue that reclassification of assets to HtM hides actual interest rate risk that is left unhedged and not adequately recognized by capital figures reported in financial statements (see also Granja et al. (2024)). In times of distress or simply when market expectations change, these issues can be detrimental to those banks' health and—as the March 2023 banking turmoil showed—to financial stability as a whole.

In this paper, our first main contribution is the development of a stylized model to explain bank runs by banks' financial conditions perceived by depositors and to measure vulnerabilities of banks stemming from their balance sheet composition, in particular as it pertains to the HtM or available-for-sale (AfS) classifications of marketable securities as well as the insured versus uninsured status of deposits. We use the model to compute deposit withdrawals and asset sales in equilibrium, taking into account the share of run-prone uninsured deposits in the bank's funding, a pool of liquid resources, and those assets that can be mobilized with an impact on bank's profit and loss accounts, firstly because of liquidation frictions related to a price-mediated channel of contagion when banks need to sell securities to raise cash, and, secondly, due to accounting rules requiring banks to fully mark-to-market HtM portfolios when any such securities are sold. In this way, we provide a parsimonious theoretical framework that can help regulators understand and monitor some of the key drivers behind run related instabilities, in particular identifying threshold ratios of banks' balance sheets that delineate stable financial conditions from those conducive to bank runs. Moreover, our model helps to explain why changing financial ratios, such as the share of uninsured deposits or HtM securities in total assets, may result in abrupt jumps in banks' solvency or liquidity conditions.

Starting from our bank run model, the second main contribution of this paper is a tractable optimization problem aimed at explaining the size of banks' HtM designations and addressing their ability to avoid remarking HtM securities when confronted with funding risk from a market price shock. This can be of relevance to supervisory analysis and risk management. Moreover, it touches on a pertinent aspect of regulations: for securities to be classified as HtM, the U.S. GAAP rules require that a bank has both the positive *intent* and the *ability* to hold these securities until maturity. As regards the IFRS9 accounting regime adopted for instance by the European Union[1], there is no explicit mention of such ability, but a specific recognition of assets should be commensurate with the bank's business model. We note that banks deciding to hold assets to collect payments, e.g., interest payments, would follow amortised cost accounting similar to the HtM option in GAAP.

Despite these regulatory restrictions, Kim et al. (2023) provide comprehensive empirical evidence that banks' use of the HtM category often appears guided by opportunistic attempts to optimise around capital

[1] https://www.ifrs.org/issued-standards/list-of-standards/ifrs-9-financial-instruments/

requirements and accounting measures such as net income and economic value of equity, especially when there are concerns about negative valuation impact on solvency, e.g., stemming from securities holdings in a changing interest rate environment. Here we focus on a stylized version of such a setting within our bank run model, where a bank looks to maximize its HtM allocation in anticipation of a negative shock. Without accounting for the fact that this could ignite a run, the bank's incentive is to hold as much as possible in the HtM category, to reduce volatility of its earnings. When incorporating our bank run model, however, it becomes necessary for the bank to consider its ability to honor the commitment of holding HtM securities to maturity. For given levels of the bank's risk tolerance and the lenience of depositors, this can produce a measure of the maximal amount of HtM that the bank should hold. Conversely, given the observed HtM designation and assumptions on depositor lenience, it allows us to make an inference about the bank's implied risk tolerance. As discussed in Granja (2023) and Kim et al. (2023), their empirical findings indicate that neither intent nor ability are the primary concerns of banks in general. Whilst intentions may not be verifiable, the above presents a simple tool to assess the soundness of given HtM levels, when the optimisation problem is calibrated to observable balance sheet data. To comply with the intent and ability in a satisfactory way, from a resilience point of view, banks' decisions on HtM portfolios should be subject to having enough liquid assets to cover potential funding withdrawals without the need to remark and liquidate HtM portfolios in most plausible stress scenarios. Otherwise, the possible benefits from the HtM accounting rules that make income less sensitive to sudden revaluation shocks is questionable.

With the theoretical framework in place, we perform quarterly calibrations of both the bank run model and the HtM optimisation problem to a time series of Silicon Valley Bank's (SVB) balance sheet data, based only on publicly available sources. In this way, we are able to assess build-up of vulnerabilities of SVB ahead of its default in March 2023. Some of the key observations are the following. The financial standing of SVB was very sensitive to fire-sale conditions, i.e., the sensitivity of prices of securities to liquidated volumes of securities. Unrealised losses in SVB's balance sheet, related to HtM accounting, hid important weaknesses of the bank and the model can indicate periods prior to the March 2023 meltdown when the situation of the bank became dire. It is clear that fostering SVB's insured funding sources would be an important policy, or supervisory, tool to prolong the runway toward default but would not avert the default completely unless close to all deposits were insured. The implementation rests on the algorithm that we derive for the identification of clearing solutions in our bank run model (see Proposition 3.2). This algorithm provides an easy monitoring tool to assess vulnerabilities, and it also underpins our explicit characterization of solutions to the subsequent optimisation problem for HtM designations (see Proposition 5.2). Relying on the latter, we can calibrate the optimisation problem to the SVB data. This turns out to yield a stark warning sign that already from Q3 2021, and especially from Q1 2022, the HtM designations of SVB appeared incommensurate with the bank's funding risk.

In the final part of the introduction, we shall briefly discuss related work on bank runs, the instabilities that led to SVB's collapse, and the nexus of accounting standards and financial stability. The remainder of the paper is then structured as follows. In Section 2, we introduce our model and fix the assumptions that we will need for our analysis. In the subsequent Section 3, we then formulate the precise clearing problem for the resolution of a depositor run. In Section 4 we proceed to discuss case studies based on SVB, making use of the previously introduced model. Finally, Section 5 develops the optimisation problem for analyzing the allocation of assets between AfS and HtM accounting portfolios.

1.1 Related literature

Bank runs associated with risky projects and short-term, flighty funding have been studied extensively since the first comprehensive model of Diamond and Dybvig (1983) explaining banks' fragility. They tackle the duality of multiple equilibria in banks' funding conditions that may arise, implying run or no-run on banks. The mechanism described in that work is about an interplay of short-term, impatient creditors and risky return on securities held by banks. The return on the projects can only happen in the future and if banks need to liquidate securities to meet depositor withdrawals, the bank would be short of proceeds from those projects and would run out of money. Heterogeneous beliefs would create partial runs, with a fraction of depositors withdrawing cash.

The assumptions on the beliefs of depositors about banks' financial standing are key to understand how the equilibria may arise. The global games approach (Morris and Shin, 2003) was a breakthrough technique to capture the idea that depositors may be pushed to withdraw funding because of their belief that others are taking such actions. This rationalizes, or endogenizes, the funding shocks.

Despite the topic being extensively researched with stringent liquidity regulations and policy intervention frameworks in place, liquidity risk forcefully materializes time and again. In March 2023, Silicon Valley Bank became a textbook example of a bank run, revitalising the discussion on bank on risk (Vo and Le, 2023). When SVB imploded, authorities looked deeper into the unrealised losses that were the root cause of its meltdown (see, e.g., relevant FDIC and ECB reports).

As shown by Drechsler et al. (2023), runs may occur in the rising interest rate environment since hedging may not be able to fully and concurrently eliminate interest and liquidity risk given the negative convexity of bank deposit franchise value and a typical long duration of bank assets. They focus on a question about hedging of either liquidity or interest rate risk to prevent runs, which is different to our primary goal of characterizing liquidity and solvency vulnerabilities in the balance sheet. Dependence of the fragility of banks on changes of rates in different rate environments (low vs high) was studied by Ahnert et al. (2023) demonstrating the risk of runs increases more when rates rise from low levels. SVB was exposed to this risk of rising interest rates—especially from low levels observed during the COVID-19 pandemic—but, additionally those losses were hidden given the accounting treatment of held-to-maturity assets. One lesson learned from March 2023 is that the accounting rules masking the adverse changes in fair value of banks assets and the very unstable funding sources (concentrated and easy to call back) may create conditions for the outflow of deposits to happen. Further, accompanying fire sales may exacerbate banks' solvency and the overall market conditions. As shown by Liu (2023), small shocks to the balance sheet of banks may be amplified to systemic events.

Within this work, we contribute to the growing literature on the impact of accounting standards on financial stability. Reporting frameworks, but also incentives, play a role in market participants' assessment of financial conditions (Bischof et al., 2021). Especially relevant are the recent works Granja (2023); Kim et al. (2023) on HtM accounting and how it is employed by banks in times of stress. Our model allows us to study policy options to avert the bank run risk, especially, looking at deposit insurance. This is different to Altermatt et al. (2022) who study the role of redemption penalties. Another important aspect is that our model gives supervisors a simple tool to watch for dangerous trends developing in balance sheets of supervised banks. This could make possible the early detection of those institutions that require deeper scrutiny.

Finally, we wish to highlight the recent work of Granja et al. (2024), which appeared while we were working on this project, as we find it to be the closest in scope to our aims and general approach. The authors build upon Jiang et al. (2023*b*), who constructed a model of bank runs driven by the level of interest rates (e.g., following monetary policy) and franchise value of deposits in relationship with the present value of bank assets, by considering capital regulation recognizing income stabilizing effects of HtM and a cost of asset reclassification from HtM to AfS. However, there are some significant differences compared to our setup and assumptions. In our model, there is no exogenous cost to reclassification of HtM portfolios, instead a cost only appears endogenously, as determined by assets sold in order for the bank to raise cash to satisfy deposit withdrawals in equilibrium. On the other hand, Granja et al.

(2024) assume a constant unit cost of the HtM reclassification without which banks would only opt for the HtM category. Our setup allows us to study the impact of fire sales and price-impact sensitivity of liquidated securities on market prices of assets. To this end, we assume that banks would sell outright securities from their portfolios instead of pledging those as collateral at the central bank. In this way we can measure how banks can withstand funding shocks on a standalone basis. We explicitly consider a funding structure that captures the run-prone uninsured deposits within which the model determines a fraction of withdrawn funding in equilibrium. Furthermore, the focus of Granja et al. (2024) is on solvency aspects of bank runs, with recapitalization as an instrument to mitigate the risk, whereas, in our approach, funding and liquidity risk are at the centre of the model.

2 Balance sheet construction and model setup

We begin by specifying the balance sheet that underpins our model. The aim is to keep this as simple as possible, while still allowing us to capture key features of the distinctive roles played by the AfS and HtM designations in relation to bank stability. First of all, this will entail two classes of liabilities, i.e., deposits: they can be either insured $L_I \geq 0$ or uninsured $L_U \geq 0$ with the total liabilities given by $L := L_I + L_U$. Uninsured deposits are assumed to be flighty and subject to run risk. However, the model can be parameterise to reflect a different classification of liabilities, for instance stable and unstable funding considered in FINREP reporting. Next, we shall assume that assets of the bank can be one of three types: liquid, illiquid but marketable, or illiquid and nonmarketable. Finally, in the case of illiquid but marketable assets, these may be classified as either available-for-sale or held-to-maturity. Beyond this classification, these illiquid but marketable securities are subject to the same market price. In summary, the stylized balance sheet will consist of the following four asset classes:

(i) liquid (cash) assets $x \geq 0$;

(ii) available-for-sale (AfS) illiquid assets $s \geq 0$ with an initial mark-to-market value of sp for some unit price $p > 0$;

(iii) held-to-maturity (HtM) illiquid assets $h \geq 0$ valued in full (despite being subject to the same market price as the AfS assets); and

(iv) nonmarketable illiquid assets $\ell \geq 0$.

With the above notation, the total assets are given by $A = x + sp + h + \ell$. The bank's equity is then the difference between this value and the total liabilities (L). These quantities determine the initial balance sheet before any consideration of a run.

Remark 2.1. For now, we take the classification of AfS versus HtM as fixed and given. In Section 5, we shall address this allocation through an optimisation problem, as briefly discussed in the introduction.

Following Banerjee and Feinstein (2021), we assume that the illiquid, but marketable, holdings are subject to price impacts if they need to be sold. The mark-to-market value of these assets is given by the inverse demand function $f : [0, s + h] \to [0, p]$ for the initial price $p > 0$. As these liquidations are realized, the bank realizes the volume weighted average price $\bar{f}(\gamma) := \frac{1}{\gamma} \int_0^\gamma f(t)dt$, for $\gamma \in (0, s + h]$, with $\bar{f}(0) := p$. Note that $\bar{f}$ is continuous at $\gamma = 0$ if f is continuous there. In this way, any unsold AfS assets are valued at the price determined by f, any sold AfS assets are valued at the price determined by $\bar{f}$, and any HtM assets are (initially) valued at a fixed price of 1. Similarly, we will assume throughout that the liquid (x) and nonmarketable (ℓ) assets have fixed value throughout this study.

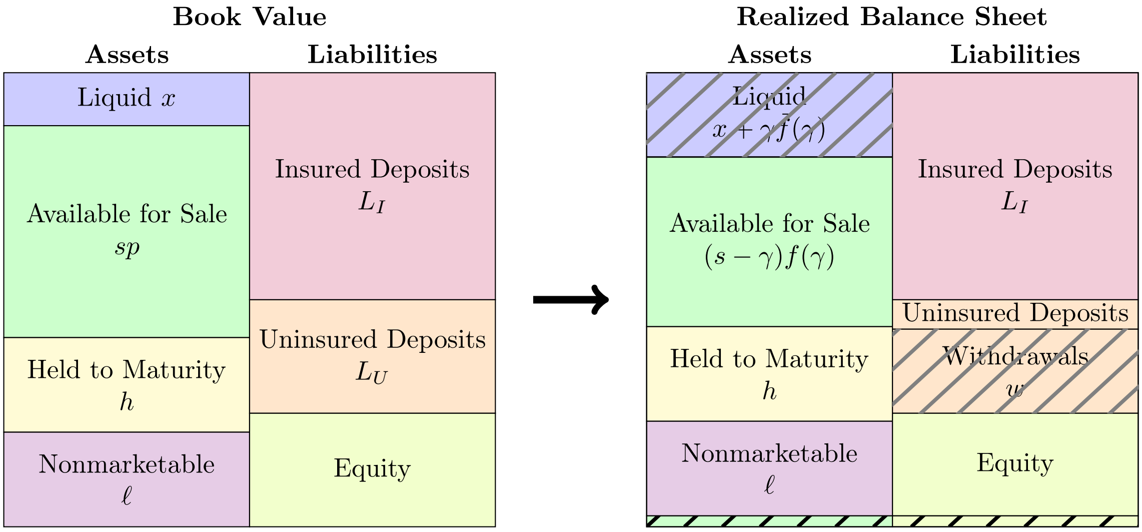

Figure 1: Stylized book and balance sheet for the bank subject to withdrawal risks where held to maturity assets need not be sold.

The composition of the initial balance sheet and an example of a realized balance sheet after withdrawals are illustrated in Figure 1. Given the observed balance sheet, the *uninsured* depositors will withdraw their funds based on whether or not the leverage ratio is in line with some maximum acceptable threshold. The total withdrawals that result from this are denoted by $w \in [0, L_U]$. Specifically, the uninsured investors have a maximum leverage ratio $\lambda_{\max} > 1$ that they are willing to accept before withdrawals are initiated. The actual leverage ratio λ is defined as the ratio of assets over equity, accounting for withdrawals and the corresponding losses that must be recognized on the balance sheet. Notably, leverage is one of the key financial indicators determining banks' stability and has been used as an important variable in seminal bank run models (Gertler and Kiyotaki, 2015).[2] Furthermore, the leverage ratio is one of two key solvency indicators that is regulated by capital standards, most importantly by Basel III regulation introduced after the Global Financial Crisis. In particular, banks should keep it above the regulatory minimum and typically retain a voluntary buffer, so as to minimize the risk that the leverage ratio falls below requirements.[3] The investors could also look at some other indicators, also related to funding and liquidity position of the banks, however, those are more difficult to track as they are reported with a considerable lag and the investors may want to react to more representative signals coming from the solvency angle. For these reasons, we focus on the leverage ratio as the sole signal tracked by depositors. Naturally, the *insured* depositors leave their funds at the bank even in a stress scenario—in particular, a bank run—due to the guarantee of recovery in case of a bank failure.

Remark 2.2. While bank runs are typically equated with liquidity issues, we take the view of Michael S. Barr, Vice Chair of for Supervision at the Federal Reserve, that "*while the proximate cause of SVB's failure was a liquidity run, the underlying issue was concern about its solvency.*"[4] In this way, the solvency concerns of uninsured depositors can manifest as liquidity problems for the bank.

Although our model is static, we will eventually consider data for multiple points in time, so it is worth mentioning that the acceptable maximum leverage ratio $\lambda_{\max}$ could in principle be changing in time. The acceptable maximum leverage ratio could be a function of changing macro-financial environment or changing risk tolerance of the depositors. For example, risk aversion might have changed in 2020 due to wealth effects (or expected wealth effects) implied by COVID-19 crisis (see drivers of risk aversion

[2]It is one of several key financial stress indicators (Duca and Peltonen, 2013).
[3]See https://www.bis.org/basel_framework/standard/LEV.htm
[4]See https://www.federalreserve.gov/publications/files/svb-review-20230428.pdf

studied by Guiso et al. (2018)), while in 2021-2022 it might have been more stable given no new shocks with magnitudes comparable to the pandemic. One could then think of sudden drops in $\lambda_{\max}$ as the reason for a run, spurred by changing depositor sentiments, but we shall not pursue such an angle here. Rather, we will view $\lambda_{\max}$ as a given, albeit unobservable, characteristic of depositors that would remain fixed as the balance sheet evolves over some time period of interest. Thereby, changes in run risk are explained by observable changes to the bank's balance sheet composition. In particular, when we perform quarterly simulations based on a time series of SVB's balance sheet data in Section 4, we assume that $\lambda_{\max}$ stays constant throughout. That way, we can study vulnerabilities of SVB exclusively stemming from its evolving balance sheet structure. Naturally, this will involve inferring plausible levels for $\lambda_{\max}$ and we will consider a range of such values for some of the simulations.

Assumption 2.3. *The inverse demand function* $f : [0, s+h] \to (0, p]$ *is non-increasing with initial price* $f(0) = p$*, where* $p \in (0, 1]$.

Assumption 2.4. *We assume* $L_U > 0$ *as no withdrawals would occur otherwise.*

For modelling purposes, we stress that the quantities L_I, L_U, and L remain fixed, as they capture the liabilities of the initial balance sheet, before a run. The uninsured liabilities after withdrawals are then given by $L_U - w$. Since nothing is withdrawn from insured liabilities L_I, the total liabilities thus become $L - w$. Writing $A(w, \gamma)$ for the total assets (with recognized losses) as a function of the withdrawals w and the quantity of marketable securities sold γ, we can express the leverage ratio $\lambda = \lambda(w, \gamma)$ as

$$\lambda = \frac{\text{Assets}}{\text{Equity}} = \frac{A(w,\gamma)}{A(w,\gamma) - (L - w)}, \tag{1}$$

where

$$A(w,\gamma) = x + \gamma\bar{f}(\gamma) + [s-\gamma]^+ f(\gamma) + [h - (\gamma - s)^+](\mathbb{I}_{\{\gamma \le s\}} + f(\gamma)\mathbb{I}_{\{\gamma > s\}}) + \ell - w,$$

for the given values of x, s, h, and ℓ.

If the withdrawal requests are larger than the bank's liquid holdings, then it will need raise cash by selling its illiquid, but marketable, asset holdings. Specifically, the bank must sell $\gamma \in [0, s+h]$ so that $x + \gamma\bar{f}(\gamma) \ge w$, if possible.[5] Based on the given balance sheet, if $\gamma \le s$ then we assume that all liquidated assets were AfS assets. On the other hand, if $\gamma > s$ then the bank liquidates all AfS assets and must be liquidating HtM assets as well. Formally, if any HtM assets are to be liquidated, then that entire block of assets is immediately recognized as AfS and marked accordingly. Summarizing these two cases:

(i) if $\gamma \le s$ then the bank holds $x + \gamma\bar{f}(\gamma)$ in liquid assets, $(s-\gamma)f(\gamma)$ in AfS assets, and h in HtM assets;

(ii) if $\gamma > s$ then the bank holds $x + \gamma\bar{f}(\gamma)$ in liquid assets, $(s+h-\gamma)f(\gamma)$ in AfS assets, and 0 HtM assets.

However, in satisfying the withdrawals, the bank may fail due to having insufficient liquidity or insufficient equity. We call these cases illiquidity and insolvency respectively.

(i) **Illiquidity**: The bank cannot meet withdrawals: $w \ge x + [s+h]\bar{f}(s+h)$ or, equivalently, $\gamma = s+h$.

(ii) **Insolvency**: The bank has negative equity: $x + \gamma\bar{f}(\gamma) + [s-\gamma]^+ f(\gamma) + [h - (\gamma - s)^+](\mathbb{I}_{\{\gamma \le s\}} + f(\gamma)\mathbb{I}_{\{\gamma > s\}}) + \ell \le L$.

Remark 2.5. If there are no price impacts on the illiquid asset, i.e., $f \equiv p$, then insolvency can only occur at the moment that the HtM assets are re-marked as AfS assets.

[5] We impose a no short selling constraint so that $\gamma \le s + h$ throughout.

For our analysis, we shall need a final assumption on the behavior of the realized balance sheet. Recall that $\lambda_{\max} > 1$. As a function of the quantity sold, we require that the rate increase in the realized value of the (total) assets sold is always larger, by a factor of $\frac{\lambda_{\max}-1}{\lambda_{\max}} = 1 - \frac{1}{\lambda_{\max}} > 0$, than the corresponding rate of decrease in the market value of the remaining unsold assets. More precisely, we impose the following technical assumption on the inverse demand function f.

Assumption 2.6. *For the remainder of this work, we will assume that the mapping* $\gamma \in [0, s+h] \mapsto \gamma\bar{f}(\gamma) + (1 - \frac{1}{\lambda_{\max}})(s + h - \gamma)f(\gamma)$ *is strictly increasing.*

Lemma 2.7. *Suppose the inverse demand function* f *is differentiable on* $(0, s+h)$*. Then Assumption 2.6 holds provided the differential inequality*

$$f(\gamma) > (1 - \lambda_{\max})(s + h - \gamma)f'(\gamma)$$

is satisfied, for all $\gamma \in [0, s + h]$.

Proof. It suffices to check that $\gamma \mapsto \gamma\bar{f}(\gamma) + (1 - \frac{1}{\lambda_{\max}})(s + h - \gamma)f(\gamma)$ has a strictly positive derivative on $(0, s + h)$. Using the definition of $\bar{f}$, differentiating, and reorganising the terms, we see that this is equivalent to the stated differential equality. □

Remark 2.8. Under Assumption 2.6, we get that $\gamma \mapsto \gamma\bar{f}(\gamma)+(1-\frac{1}{\lambda_{\max}})(\bar{s}-\gamma)f(\gamma)$ is strictly increasing on $[0, \bar{s}]$, for any $\bar{s} \in (0, s + h]$. In particular, this holds for $\bar{s} \in \{s, s + h\}$ which we shall make use of in Section 3. At the same time, we stress that the map $\gamma \mapsto \gamma\bar{f}(\gamma) + (\bar{s} - \gamma)f(\gamma)$ is instead non-increasing on $[0, \bar{s}]$, for any $\bar{s} \in (0, s + h]$, as one can readily verify by, e.g., arguing as in Lemma 2.7.

We conclude this section by highlighting two common examples of inverse demand functions and outlining the parameter choices for which our assumptions are satisfied.

Example 2.9. Take $f(\gamma) := p(1 - b\gamma)$ as in, e.g., Greenwood et al. (2015). Then $\bar{f}(\gamma) = p(1 - \frac{b}{2}\gamma)$. Naturally, $b \leq 1/(s + h)$ and hence Assumptions 2.3 is satisfied. By Lemma 2.7, Assumption 2.6 holds if and only if either $b < 1/(s + h)$ for $\lambda_{\max} \in (1, 2)$ or $b < 1/[(\lambda_{\max} - 1)(s + h)]$ for $\lambda_{\max} \geq 2$.

Example 2.10. Take $f(\gamma) = p\exp(-b\gamma)$ as in, e.g., Cifuentes et al. (2005). Then $\bar{f}(\gamma) = \frac{p(1-\exp(-b\gamma))}{b\gamma}$ for $\gamma > 0$ and $\bar{f}(0) = p$. Naturally, $b \geq 0$, so Assumption 2.3 holds. By Lemma 2.7, Assumption 2.6 holds if and only if $b < 1/[(\lambda_{\max} - 1)(s + h)]$.

3 A clearing problem for depositor runs

In the previous section, we fixed the granularity of the balance sheet and introduced the mechanisms behind our model of a bank run. The goal of the present section is now to (i) formalise this model as clearing problem, (ii) establish existence of equilibrium solutions, and (iii) provide a tractable algorithm for computing these. Throughout, we are working under the notation and assumptions stated in Section 2.

Our model for a (potential) bank run may be expressed as the solution to a clearing problem that is jointly in the equilibrium amount of withdrawals w^* and the equilibrium quantity sold γ^* out of the marketable securities. This may be formalised as the search for fixed points of the mapping $\Phi : [0, L_U] \times [0, s + h] \to [0, L_U] \times [0, s + h]$ defined by $\Phi = (\Phi_w, \Phi_\gamma)$, where

$$\Phi_w(\gamma^*) = L_U \wedge \Big[\lambda_{\max}L - (\lambda_{\max} - 1)(x + \gamma^*\bar{f}(\gamma^*) + [s - \gamma^*]^+ f(\gamma^*) + [h - (\gamma^* - s)^+](\mathbb{I}_{\{\gamma^* \leq s\}} + f(\gamma^*)\mathbb{I}_{\{\gamma^* > s\}}) + \ell)\Big]^+ \tag{2}$$

$$\Phi_\gamma(w^*,\gamma^*) = [s+h] \wedge \frac{(w^*-x)^+}{\bar{f}(\gamma^*)}. \tag{3}$$

For a given quantity sold, (2) returns the withdrawals required for the depositors to enforce their maximum acceptable leverage ratio. Given also the withdrawals, (3) then ensures that the proceeds from the quantity sold match the withdrawal requests. A pair $(w^*,\gamma^*) \in [0,L_U] \times [0,s+h]$ is therefore a clearing solution if and only if it is a fixed point of Φ, meaning that we have

$$(w^*,\gamma^*) = \Phi(w^*,\gamma^*) = (\Phi_w(\gamma^*)\ ,\ \Phi_\gamma(w^*,\gamma^*)), \tag{4}$$

provided also that the bank is solvent in this case, i.e., provided

$$x + \gamma^*\bar{f}(\gamma^*) + [s-\gamma^*]^+ f(\gamma^*) + [h-(\gamma^*-s)^+](\mathbb{I}_{\{\gamma^*\le s\}} + f(\gamma^*)\mathbb{I}_{\{\gamma^*>s\}}) + \ell > L. \tag{5}$$

If (w^*,γ^*) satisfies (4), but violates (5), then the bank is insolvent. In that case, the values (w^*,γ^*) correspond to the run having occurred and the bank only subsequently being declared insolvent. This is arguably more in line with the timeline of events in an actual run, but one can of course also look for the amount of liquidations γ that first induces technical insolvency by violating (5) during the run.

For clearing solutions corresponding to a run (i.e., $w^* > x$) without causing illiquidity (i.e., $\gamma^* < s+h$), we have $w^* = x + \gamma^*\bar{f}(\gamma^*)$ with all withdrawals being met, solvency issues aside. On the other hand, illiquidity corresponds to clearing solutions with a quantity sold $\gamma^* = s+h$ and withdrawals $w^* \ge x + (s+h)\bar{f}(s+h)$. When a bank is left illiquid, the value of w^* reflects the withdrawal requests and not the actualized withdrawals, as the bank would generally not be able to cover all requests.

Proposition 3.1 (Existence of clearing solutions)**.** *Consider the partial order of component-wise inequality. For this ordering, there exist minimal and maximal clearing solutions* $(w^\downarrow,\gamma^\downarrow) \le (w^\uparrow,\gamma^\uparrow)$.

Throughout, we work with the minimal solution, as this is the best case for the bank and represents the solution that a run would most naturally arrive at. We have the following (exhaustive) algorithm for finding the minimal clearing solution.

Proposition 3.2 (Clearing algorithm)**.** *The minimal clearing solution* $(w^\downarrow,\gamma^\downarrow)$ *is determined by the following six step algorithm:*

1. **No sales:** *If either* $L_U \le x$ *or* $\lambda_{\max}L - (\lambda_{\max}-1)(x+sp+h+\ell) \le x$, *then* $\gamma^\downarrow = 0$ *and* $w^\downarrow = L_U \wedge [\lambda_{\max}L - (\lambda_{\max}-1)(x+sp+h+\ell)]^+$. *Else, continue to next step.*

2. **Run without re-marking HtM I:** *If*

$$\begin{aligned}&L - x - (1-\tfrac{1}{\lambda_{\max}})(h+\ell) \in [(1-\tfrac{1}{\lambda_{\max}})sp, s\bar{f}(s)], \quad \textit{and}\\ &L_U \ge \lambda_{\max}L - (\lambda_{\max}-1)(x+\gamma^*\bar{f}(\gamma^*)+(s-\gamma^*)f(\gamma^*)+h+\ell), \quad \textit{for}\\ &\gamma^*\bar{f}(\gamma^*) + (1-\tfrac{1}{\lambda_{\max}})(s-\gamma^*)f(\gamma^*) = L - x - (1-\tfrac{1}{\lambda_{\max}})(h+\ell), \quad \gamma^* \in [0,s],\end{aligned}$$

then $\gamma^\downarrow = \gamma^*$ *and* $w^\downarrow = x + \gamma^*\bar{f}(\gamma^*) \in (x, L_U)$. *Else, continue to next step.*

3. **Run without re-marking HtM II:** *If* $L_U \in (x, x+s\bar{f}(s)]$ *and* $L_I \ge (1-\frac{1}{\lambda_{\max}})[(s-\gamma^*)f(\gamma^*)+h+\ell]$ *for* $\gamma^* \in [0,s]$ *solving* $\gamma^*\bar{f}(\gamma^*) = L_U - x$, *then* $\gamma^\downarrow = \gamma^*$ *and* $w^\downarrow = L_U$. *Else, continue to next step.*

4. **Re-marking HtM I:** *If*

$$\begin{aligned}&L - x - (1-\tfrac{1}{\lambda_{\max}})\ell \in [s\bar{f}(s) + (1-\tfrac{1}{\lambda_{\max}})hf(s), (s+h)\bar{f}(s+h)], \quad \textit{and}\\ &L_U \ge \lambda_{\max}L - (\lambda_{\max}-1)(x+\gamma^*\bar{f}(\gamma^*)+(s+h-\gamma^*)f(\gamma^*)+\ell), \quad \textit{for}\end{aligned}$$

$$\gamma^* \bar{f}(\gamma^*) + (1 - \tfrac{1}{\lambda_{\max}})(s + h - \gamma^*) f(\gamma^*) = L - x - (1 - \tfrac{1}{\lambda_{\max}})\ell, \quad \gamma^* \in [s, s+h],$$

then $\gamma^{\downarrow} = \gamma^*$ *and* $w^{\downarrow} = x + \gamma^* \bar{f}(\gamma^*) \in (x, L_U)$. *Else, continue to next step.*

5. **Re-marking HtM II:** *If* $L_U \in (x, x + (s+h)\bar{f}(s+h)]$ *and* $L_I \geq (1 - \frac{1}{\lambda_{\max}})[(s + h - \gamma^*) f(\gamma^*) + \ell]$ *for* $\gamma^* \in [s, s+h]$ *solving* $\gamma^* \bar{f}(\gamma^*) = L_U - x$, *then* $\gamma^{\downarrow} = \gamma^*$ *and* $w^{\downarrow} = L_U$. *Else, continue to next step.*

6. **Illiquidity:** *If it gets to this final step, then* $\gamma^{\downarrow} = s + h$ *and depending on whether*

$$\lambda_{\max} L - (\lambda_{\max} - 1)(x + (s+h)\bar{f}(s+h) + \ell) \geq L_U \quad \text{and} \quad L_U - x \geq (s+h)\bar{f}(s+h), \quad \text{or}$$
$$\lambda_{\max} L - (\lambda_{\max} - 1)(x + (s+h)\bar{f}(s+h) + \ell) < L_U \quad \text{and} \quad L \geq x + (s+h)\bar{f}(s+h) + (1 - \tfrac{1}{\lambda_{\max}})\ell,$$

we either have $w^{\downarrow} = L_U$ *or* $w^{\downarrow} = \lambda_{\max} L - (\lambda_{\max} - 1)(x + (s+h)\bar{f}(s+h) + \ell) \in (x, L_U)$, *respectively.*

When the algorithm terminates, one must additionally confirm that the candidate clearing solution leaves the bank solvent, i.e., that (5) *is satisfied. If the algorithm terminates before Step 6, but* (5) *is violated, then the bank is liquid but insolvent. If the algorithm terminates in Step 6 and* (5) *is violated, then the bank is both illiquid and insolvent.*

4 Case studies based on Silicon Valley Bank

The case of SVB's default is an insightful example of bank balance sheet vulnerabilities leading to bank runs and can be analysed using our framework. In the next sections we describe how different elements of the SVB default story correspond to features of our model (subsection 4.1). Moreover, we run simulations to illustrate some key drivers of the bank runs and to show effectiveness of certain policy interventions, fostering insured deposit base and a prudent allocation of assets between HtM and AfS portfolios. A time series of SVB's balance sheet prior to the default in March 2023 allows us to analyse how the bank's vulnerabilities evolved. Extrapolating from this statement, the model can be used then as a monitoring tool of bank balance sheet vulnerabilities.

4.1 Balance sheet dynamics of SVB

In a nutshell, as a report from the Federal Reserve Board shows (FRB, 2023), Silicon Valley Bank mismanaged its balance sheet growth caused by funding inflow from the technology and venture capital sectors. Notably, it was partly supported by a period of exceptionally low interest rates after the 2020 COVID-19 crisis. SVB invested those inflows of deposits in longer-term securities, i.e., held-to-maturity (HtM), government or agency-issued mortgage-backed securities. These securities are low risk from a credit perspective and provide a predictable return based on the interest rate at the time of purchase. However, in the changing monetary policy regime, the asset portfolios were not effectively managed from the interest-rate risk perspective. Notably, SVB was actively removing hedges as rates were rising. At the same time, SVB failed to manage the risks of its highly concentrated liabilities, which proved much more unstable than anticipated.

Changing market conditions led firms with cash constraints – and also those supported by flighty venture capital funding – to start withdrawing their deposits. The velocity of outflows was quickly accelerated as social networks reinforced a run dynamic. SVB reached a point in March 2023 when it was forced to announce a restructuring of its balance sheet, including a completed sale of USD 21 billion of AfS securities for a USD 1.8 billion after-tax loss. Notably, the HtM accounting regime was constraining the bank from further raising cash as dipping into HtM securities would result in a reclassification of the whole HtM portfolio and booking unrealised HtM losses in SVB's profit and loss accounts.

These several factors, i.e., ailing management and governance, fragile business model, and changing market conditions combined to lead to a detrimental bank run. Which of them were the most influential and which could be immunized to avert the collapse? Our model can be used to help address these questions. Given how parsimonious our framework is, we can use publicly available information about SVB to calibrate all crucial parameters of the model. Table 1 collects a time series of data characterizing the evolution of SVB. Between Q1 2020 and Q1 2022, i.e., one year before the collapse, total deposits grew more than threefold, from USD 56 billion to USD 181 billion. Only a small fraction of the funding base was insured deposits (USD 9 billion out of the USD 181 billion in Q1 2022). The absorbed funding was mostly invested into HtM securities (increase from USD 10 billion to USD 101 billion). When expectations about interest rate increases built, and eventually interest rated started to rapidly raise, the market value of the securities was gradually declining. However, thanks to the accounting treatment regarding how their value would be reflected in the financial results, this was only reflected in a build-up of the unrealized losses (increase from a gain of USD 0.8 billion to a loss of USD -15 billion in Q2 2022). This meant that even though the reported leverage ratio was hovering around 7.0 and 8.0 (measured by a ratio of total assets to Tier 1 capital), a leverage ratio factoring in the unrealized losses from HtM securities, and also from AfS portfolios, soared to almost 40.0. The described collection of balance sheet parameters of SVB is the main data source for the calibration of our model to run simulations to identify some tipping point parameters in the unwinding of a bank run on SVB.

		In USD billion										*Ratio*	
		Total deposits	Other funding	Insured deposits	Capital	Total assets	Cash	AfS	HtM	Unrealised Gains/Losses (HtM)	Unrealised Gains/Losses (AfS)	Tier 1 lev. ratio	Lev. ratio implied by Unrealised Gains/Losses
2020	q1	56	8.9	5	10.1	75	8	20	10	0.8	1.6	6.4	6.0
	q2	70	7.9	5	12.1	90	10	25	10	0.8	1.6	6.4	6.2
	q3	80	6.5	5	13.5	100	12	28	12	0.8	1.6	6.4	6.3
	q4	95	8.8	5	16.2	120	13	35	15	0.8	1.6	6.4	6.5
2021	q1	110	11.7	5	18.3	140	16	30	40	0.0	0.0	6.6	7.6
	q2	130	18.3	6	21.7	170	18	25	60	0.0	0.0	6.8	7.8
	q3	152	10.0	7	23.0	185	21	25	80	-0.5	0.0	7.0	8.2
	q4	172	16.9	8	26.1	215	23	27	103	-1.0	0.0	7.2	8.6
2022	q1	181	17.3	9	26.7	225	22	27	101	-7.5	-1.5	7.4	12.7
	q2	170	20.0	10	25.0	215	20	27	98	-11.5	-2.0	7.6	18.7
	q3	162	28.5	10	24.5	215	19	27	95	-16	-3.0	7.8	39.2
	q4	160	31.0	10	24.0	215	17	27	93	-15	-3.0	8.0	35.9

Table 1: Balance sheet evolution of the SVB
Note: Numbers shown starting from the beginning of 2020 when the dynamics of assets and liabilities started to materially change. Unrealised Gains/Losses on neither HtM nor AfS portfolios are included into CET1, with the treatment of the AfS part following SVB's choice allowed by FED's enhanced prudential regulatory (EPR) framework; "Lev. ratio implied by Unrealized Gains/Losses" = [Total assets]/([Capital]-[Unrealised Gains/Losses (HtM)]-[Unrealised Gains/Losses (AfS)]); "Other funding" = calibrated such that balance sheet identity is preserved and leverage ratio reported by SVB ([Tier 1 ratio]) equals to the calculated leverage ratio (i.e., [Total assets]/[Capital]), "AfS" = securities in available for sale accounting portfolios; "HtM" = securities in held-to-maturity accounting portfolios
Source: SVB financial reports and FRB (2023)

4.2 Drivers of bank runs and policy implications

Based on the calibrated model we run simulations to show how the maximum leverage ratios accepted by depositors, fire-sale price impact, unrealised losses and uncertainty of bank asset valuations impact bank run risk. We also show effectiveness of some policy interventions, related to the allocation of liabilities between insured and uninsured funding and of assets between available for sale and held for trading securities. Except where otherwise mentioned, for each of the following case studies, we consider $\lambda_{\max} = 7.5$ and $f(\gamma) = 1 - 0.0005\gamma$, which means that USD 10 billion of sold securities would have 50 bp impact on their market prices (from an initial price of $p = 1$).

Maximum acceptable leverage ratio. By looking at the funding withdrawals (Figure 2) and asset liquidations in equilibrium, we can analyse the evolution of vulnerabilities in SVB's balance sheet. We do not observe the maximum accepted leverage ratio of the depositors. However, we can infer sensible values considering a relatively stable period before 2021 when policy interest rates were low and the expectations of the hikes were still moderate. During this period SVB's balance sheet had not yet begun to balloon and its leverage ratio was hovering between approximately 6.5 to 8.0, computed as total assets divided by Tier 1 capital. This implies that, at least in the stable environment, the maximum accepted leverage ratio of the depositors was not below that level. Based on the clearing algorithm in Proposition 3.2, and later Case 1 of Proposition 5.2 for a bank optimising its HtM designation, we can infer theoretical upper bounds above which there is no run risk in our model. From a stress testing perspective, we will therefore focus on a range of leverage ratios between 6.5 and 8.5. We note that, in Section 5.2, we use our model to back out levels of $\lambda_{\max}$ that would imply the 'no-sales' case of the run equilibrium, since these acceptable leverage ratios would be consistent with the fact that a run only happened in 2023. During the analysed period 2020-2022, these implied values of $\lambda_{\max}$ stay between 6.5 and 8.0.

Until Q1 2021, funding withdrawals implied by the model are very limited and can be fully covered by cash holdings of SVB, depicted by the green bars, except for $\lambda_{\max} = 6.5$ implying partial liquidation of AfS portfolios. Only after, we can see rising equilibrium levels of funding withdrawals. As of 2022, the model indicated that runs following a more risk averse depositors (i.e., with acceptable leverage ratio 6.5 or even 7.0 for Q4 2022) could deprive SVB of available liquid resources and lead to dipping into HtM portfolios.

Fire-sale price impact. The other significant parameter of the model is the fire-sale impact of securities liquidations. This parameter is difficult to pin down (see Sydow et al. (2021)) and we conduct sensitivity analysis of our results to the price impact function. Figure 3 shows the amount of liquidated assets under various regimes of the price impact functions. Clearly, the more sensitive the valuation of assets to the sold volumes, the larger the needs to liquidate securities to restore liquidity.

Unrealised losses in HtM portfolios. The accounting of losses in securities portfolios masked the actual vulnerabilities stemming from the securities repricing pressure in the rising interest rate environment. However, the trigger for SVB's bankruptcy was related to investors' expectations about growing hidden losses. In hindsight, knowing the estimates of the unrealised losses (FRB, 2023), we can analyse how vulnerable the balance sheet of SVB was by assuming that the unrealised losses were to be reflected in capital and computing the implied withdrawals and liquidations in our simple model. To achieve that, we subtracted the estimated unrealised losses from AfS and HtM portfolios of SVB before running the simulations. Figure 4 shows that already at the the beginning of 2022 financial conditions of SVB became conducive to bankruptcy. In Q1, SVB would stay solvent but may already be considered illiquid and as of the subsequent periods, assuming a higher sensitivity of asset values in fire sales, the bank could be considered both illiquid and insolvent. The outcomes of the simulations indicate that, given the mounting unrealised losses, the financial conditions of SVB would deteriorate sharply.

We can also align the vulnerabilities that picked up in 2022 with SVB's outlook for income presented in earnings reports. SVB was revising its net interest income upwards in Q4 2021 to 50% in Q1 2022 (30% growth year on year) that was attracting investors. However, Q2 2022 brought a downward revision of the net interest income to 40%, coinciding with the aggressive monetary tightening policy of the Federal Reserve. The reversal of the projected trend and related volatility made the financial situation of SVB very uncertain which is reflected in a sharp increase of the run risk, as measured by our model in Figure 4. Admittedly, supervisors did not act preemptively nor in a timely manner to prevent the ultimate run from happening.

Share of insured deposits. The model allows us to test some policy interventions that may reduce vulnerabilities in the balance sheets of banks. The most straightforward one in the case of SVB, advocated by researchers and policy makers in the wake of SVB meltdown, was to foster diversified or insured funding sources. We can directly test the impact of the latter. To this end, we assume that a certain fraction of the uninsured deposits of SVB would be moved from uninsured to the insured category and, consequently, limiting the scope of the run *ex ante*. The Figure 5 shows results of the simulations. They indicate that reducing the volume of uninsured deposits by as much as 95% can eliminate the conditions for a run with adverse impact. However, limiting the size of uninsured funding by half, can already limit the size of financial losses, even though solvency default might not have been avoided. The banks' balance sheet was not sound enough to withstand the unhedged losses confirming that the adopted business model was flawed.

Allocation of securities to AfS and HtM porfolios. Since accounting of securities held by SVB was blamed for the collapse of the bank we can use the model to test whether a different allocation of securities across AfS and HtM portfolio could increase SVB's balance sheet soundness. To this end, we ran a simulation assuming that SVB held more AfS securities. Practically, we reallocated a fraction of HtM portfolio to AfS assuming that the same amount of unrealised losses be incurred. We experimented with fractions ranging from 20% to 80%. Fig. 6 shows equilibrium withdrawal of deposits across time and different accounting structure of securities portfolios. The main finding is the following: the reallocation *per se* would not save the bank but rather delay a meltdown. In the critical period, i.e., Q1 2022, depicted by the results of our model, holding significantly more AfS securities would allow the bank to raise liquidity following some depositors' decision to withdraw but not yet utilizing resources locked in the HtM portfolios. Notably, this result sheds light on the importance on interest rate risk management, including effective hedging, since the fair value depreciation reflecting market conditions is independent of accounting standards. HtM accounting hid the losses but did not mitigate them; only proper risk management could have been effective.

5 On the balance between HtM and AfS

Until now, we have taken for granted the bank's balance sheet composition. In this section, we endogenize the bank's decision to designate part of the marketable securities as HtM. Importantly, we keep all other aspects of the balance sheet fixed, so that it is only the allocation of the marketable securities between the AfS and HtM categories that the bank can vary.

There are simple and sound reasons for banks to rely on the HtM rules. Their business models (at least in the case of the more traditional universal banks) rely on maturity transformation, i.e., they invest in long-term projects financed by short-term funding. However, investment in long-term projects may be achieved via non-marketable loans or via bonds and equities, which are more liquid, frequently exchange-traded. At the same time, liquidity needs lead banks to also hold bonds to be able to quickly raise cash, and the holding period of those bonds may be short. Since these instruments are marked-to-market, they create volatility in the banks' profit and loss accounts. To decrease the variability of income from bonds and other securities, which are intentionally held for long-term investment purposes (i.e., held-to-maturity) regulators introduced accounting rules that allow banks to recognize these bonds at amortised costs. As discussed in the introduction, this has been utilised extensively by banks in recent years, especially since the onset of increasing interest rates.

In the opposite direction, there are also natural pressures to not rely too heavily on HtM. First of all, it is a real commitment to hold the assets to maturity and hence implies a loss of flexibility. Moreover, whenever a bank looks to sell even a fraction of those assets, they would need to derecognize the whole HtM portfolio, thus forcing them to acknowledge unrealized losses while also signalling the inability to

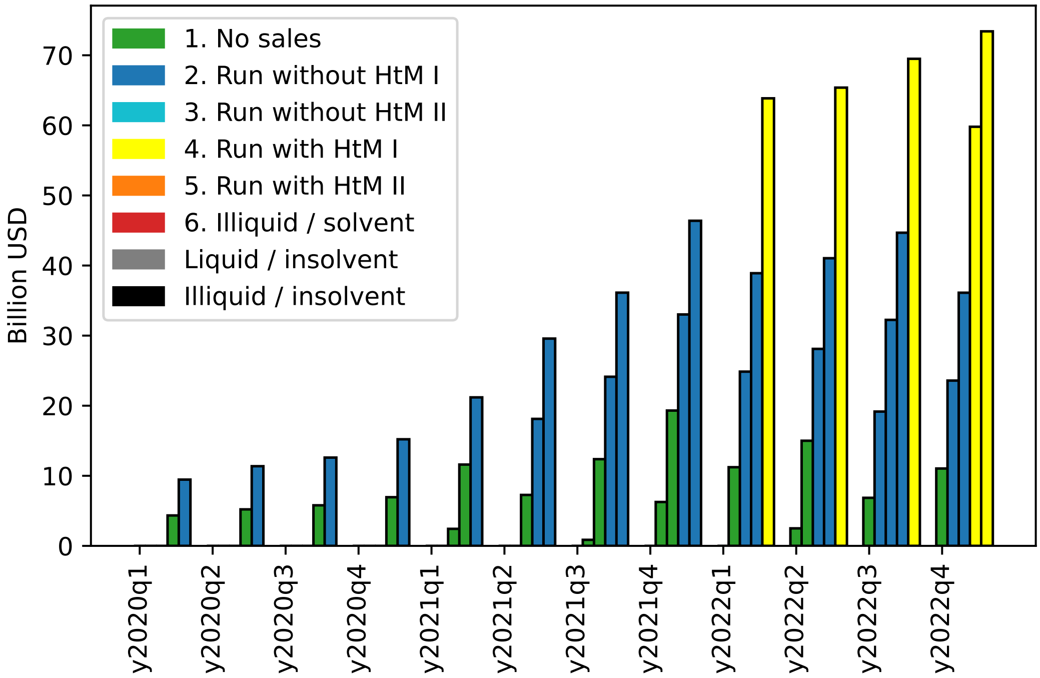

Figure 2: The figure shows equilibrium withdrawal of funding (in USD billion) from SVB for balance sheets observed between Q1 2020 and Q4 2022 and for various calibrations of maximum acceptable leverage ratios $\lambda_{\max}$. For each period there is a group of bars, each of them corresponding to one value of $\lambda_{\max}$ from the set $\{6.5, 7.0, 7.5, 8.0, 8.5\}$. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Price impact elasticity $b = 0.0005$. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

stay true to their commitment. Consequently, banks tend to aim for some optimal level that should ideally leave enough AfS securities to cover liquidity needs under almost all foreseeable scenarios.

5.1 Maximising the HtM designation subject to price shocks

Working with the balance sheet from Figure 1, consider a bank with total assets A of which $\bar{A} := A - x - \ell$ are held in marketable securities that may be designated as some combination of AfS and HtM. To account for the impact of a possible future devaluation of the marketable securities, we introduce a simple one-period model, wherein the HtM and AfS allocations are decided at time 0, before a price shock then arrives at time 1. Given the new price, a potential run is resolved within the equilibrium formulation from Section 3. As is implicit in the word shock, we stress that we shall only consider downside risk.

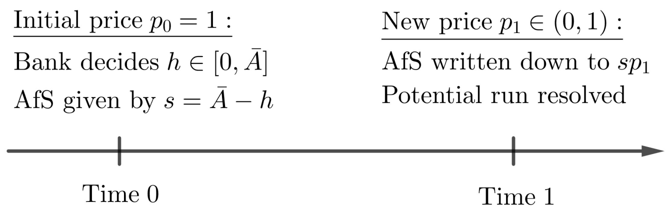

Figure 7: Graphical depiction of the one-period model. Given the price shock, the bank suffers realised losses on the AfS assets, while the value of the HtM assets is unchanged. The unrealized losses can only be an issue if the state of the bank is such that a run forces it to remark the HtM assets.

As discussed in the introduction, the empirical findings of Kim et al. (2023) show that, faced with

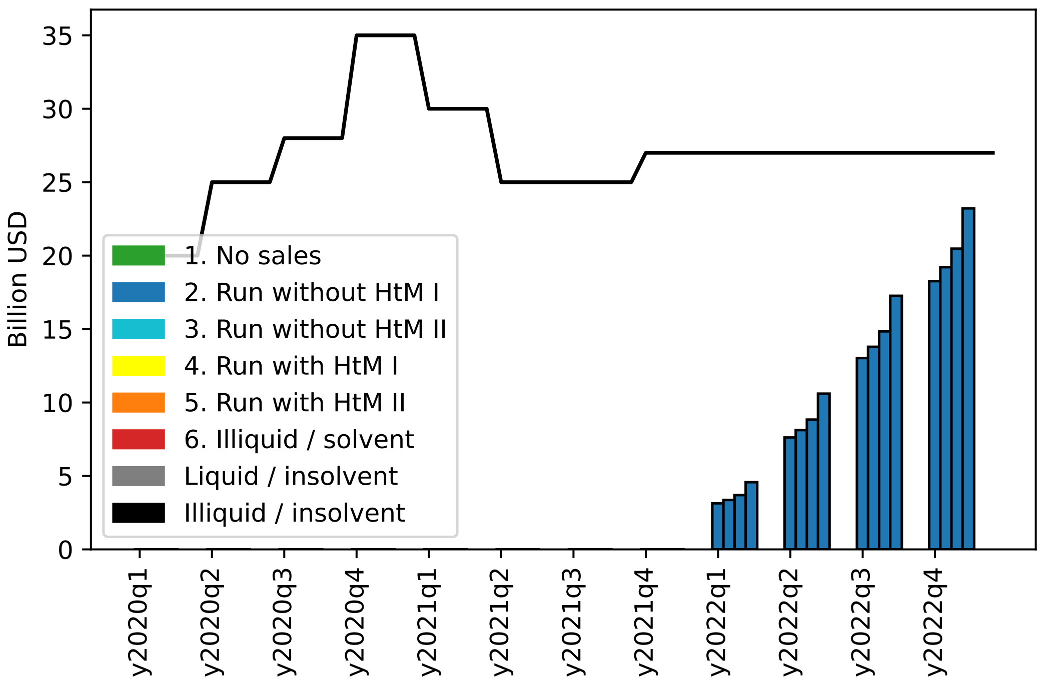

Figure 3: The figure shows equilibrium liquidation of securities by SVB (in USD billion) for balance sheets observed between Q1 2020 and Q4 2022 and for various parameterisations of price impact functions. For each period there is a group of bars, each of them corresponding to one parameter of the linear impact function (b) from the set $\{0.0001, 0.0002, 0.001, 0.002\}$. For instance, 0.0001 corresponds to 10 bp impact on asset prices when USD 10 billion of securities are liquidated, like in Greenwood et al. (2015). Max acceptable leverage ratio = 7.5. Black line indicated the total volume of securities in the AfS portfolio. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

the expectation of negative shocks, banks have looked to maximize their HtM holdings. If there were no potential costs associated with this, the bank would choose to designate all of the marketable securities $\bar{A}$ as HtM, since this insulates the bank from price fluctuations up until maturity and makes the balance sheet look as strong as possible. In our model, this would amount to the cases where there are no risk of having to sell marketable securities in a run, independently of the amount of HtM $h \in [0, \bar{A}]$ fixed at time 0 and the price shock arriving at time 1.

As soon as a potential run could necessitate the sale of marketable securities, care must be taken if the bank aims to act responsibly when deciding on its HtM designation. First of all, as discussed in the introduction, regulations call for the bank's intent and ability to hold the HtM assets to maturity. This implies that the bank should seek to avoid having to remark the HtM assets for a reasonable range of price shocks. Secondly, since the entire holdings must be remarked at once, remarking the HtM assets causes a shock to the balance sheet (from unrealized losses) that may add further fuel to a run in our model. More generally, this is also a bad signal to the outside world and may remove the flexibility to recognize those same assets as HtM in the future. Thus, the bank management has incentives to avoid remarking of HtM assets for what it considers to be likely values of the price shock.

The above considerations lead us to formulate the following optimisation problem based on the one-period model of Figure 7: the bank maximises the amount of HtM assets that it holds, at time 0, subject to having enough AfS to cover liquidity needs in a potential run at time 1, for any price shock within some given range. That is, the bank solves for the maximum $h^* \in [0, \bar{A}]$ at time 0 such that there is no need to remark HtM assets at time 1 if the new price p_1 is above or equal to a given threshold price.

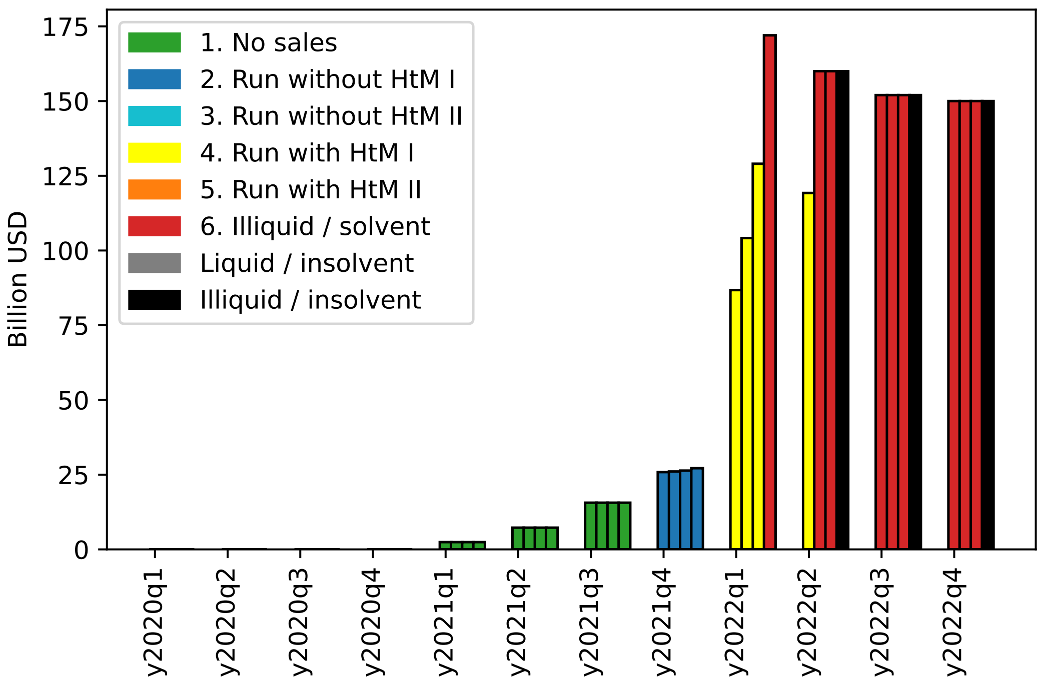

Figure 4: The figure shows equilibrium funding withdrawals from SVB (in USD billion) in a hypothetical scenario of unrealised losses in AfS and HtM portfolios being realized in the value of the securities portfolios and for balance sheets observed between Q1 2020, and Q4 2022 and for various parameterisations of price impact functions. For each period there is a group of bars, each of them corresponding to one parameter of the linear impact function (b) from the set $\{0.0001,\ 0.0002,\ 0.001,\ 0.002\}$. For instance, 0.0001 corresponds to 10 bp impact on asset prices when 10 USD billion of securities are liquidated, like in Greenwood et al. (2015). Max acceptable leverage ratio = 7.5. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

From here on, we let p_1 stand for this threshold price, as it is the only value of the price at time 1 that we need to consider (larger prices amount to less equilibrium sales). The optimisation problem can then be written as

$$h^* = \max\{h \in [0, \bar{A}] \mid \gamma^{\downarrow}(p_1, \lambda_{\max}) \leq \bar{A} - h\} \tag{6}$$

or, equivalently, $h^* = \bar{A} - s^*$ where

$$s^* = \min\{s \in [0, \bar{A}] \mid \gamma^{\downarrow}(p_1, \lambda_{\max}) \leq s\}. \tag{7}$$

Here we have made explicit the dependence of the (minimal) equilibrium liquidations $\gamma^{\downarrow}$ on the threshold price p_1 and the maximum accepted leverage ratio $\lambda_{\max}$. If the bank cannot satisfy the constraint, then it holds everything as available-for-sale, so we set $h^* = 0$ and $s^* = \bar{A}$ in that case.

Remark 5.1. Considered as an internal risk management problem, the threshold price may be seen as a reflection of the banks' risk tolerance, expressed in terms of accepted negative valuation limits. This could for instance be related to Value-at-Risk or Expected Shortfall limits.

For tractability, we will model the price impact by a linear inverse demand function. Given the values p_1 and $\lambda_{\max}$, the next result fully characterizes the bank's optimum behavior.

Proposition 5.2 (Maximal HtM designation)**.** *Assume $\lambda_{\max} > 2$ and let $\bar{l} := (\lambda_{\max} - 1)/\lambda_{\max}$. For a given threshold price $p_1 \in (0, 1)$, at time 1, we assume a linear inverse demand function, which takes the*

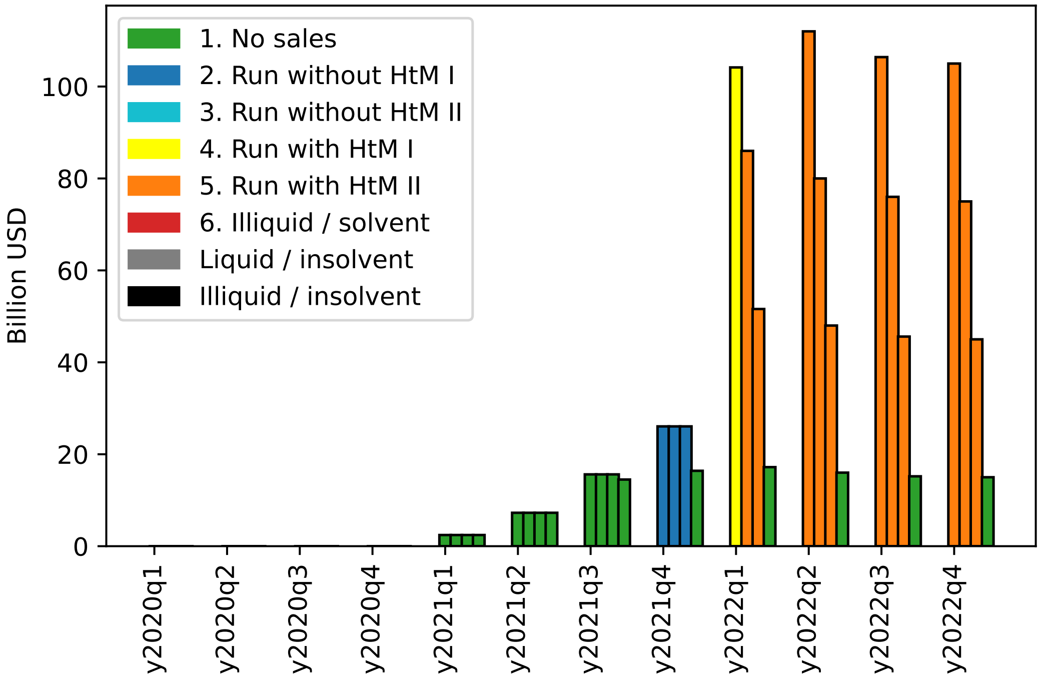

Figure 5: The figure shows equilibrium funding withdrawals from SVB (in USD billion) in a hypothetical scenario of unrealised losses in AfS and HtM portfolios being realized in the value of the securities portfolios and *ex ante* policy interventions limiting the size of the uninsured deposits. Results are shown for balance sheets observed between Q1 2020 and Q4 2022 and for various parameterisations of price impact functions. For each period there is a group of bars, each of them corresponding to one parameter of the reduction in the volume of uninsured deposits taken from the set {40%, 55%, 70%, 95%}. For instance, 40% means that 40% of uninsured deposits are moved to insured deposits category. Max acceptable leverage ratio = 7.5. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

form $f(\gamma) = p_1(1 - b\gamma)$ *with* $b < 1/[(\lambda_{\max} - 1)\bar{A}]$.

Case 1 *If* $L_U \leq x$ *or* $L \leq x + \bar{l}(\bar{A} + \ell)$*, then, by designating everything as HtM, the bank can insulate itself from the price shock and have no risk of a run on the marketable securities, so the bank will hold* $h^* := \bar{A}$ *and* $s^* := 0$.

Case 2 *If* $L_U > x$ *and* $L > x + \bar{l}(\bar{A} + \ell)$*, then the bank chooses to hold* $h^* := \bar{A} - s^*$ *as HtM, where* $s^* = s^*(p_1, \lambda_{\max})$ *is the minimal amount of AfS given by*

$$s^*(p_1, \lambda_{\max}) = \min\{s_{\mathrm{PW}}(p_1, \lambda_{\max}), s_{\mathrm{FW}}(p_1, \lambda_{\max}), \bar{A}\},$$

for the values

$$s_{\mathrm{PW}}(p_1, \lambda_{\max}) := \begin{cases} \dfrac{p_1 - \bar{l} - M}{bp_1} & \text{if } \dfrac{p_1 - \bar{l} - M}{bp_1} \leq \bar{s} \text{ and } p_1 \geq \bar{l} + b[L - x - \bar{l}(\bar{A} + \ell)] + C \\ +\infty & \text{else} \end{cases}$$

$$s_{\mathrm{FW}}(p_1, \lambda_{\max}) := \begin{cases} \max\{s_1, s_2\}\ , & \text{if } \max\{s_1, s_2\} \leq \bar{A} \text{ and } p_1 > 2b(L_U - x) \\ +\infty & \text{else} \end{cases}$$

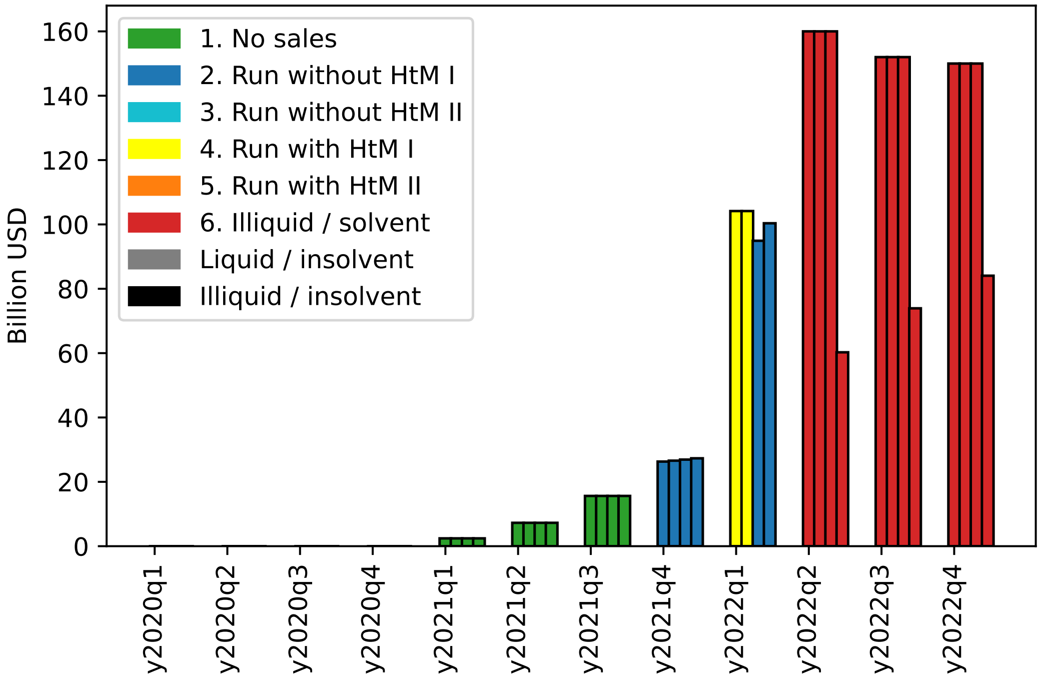

Figure 6: The figure shows equilibrium funding withdrawals from SVB (in USD billion) in a hypothetical scenario of unrealised losses in AfS and HtM portfolios being realized in the value of the securities portfolios and *ex ante* different allocation of securities to AfS and HtM accounting portfolios. Results are shown for balance sheets observed between Q1 2020, and Q4 2022 and for various parameterisations of price impact functions. For each period there is a group of bars, each of them corresponding to one parameter of the percentage reduction in the volume of HtM securities and allocating to the AfS portfolio. Reduction parameters are selected from the set $\{20\%, 40\%, 60\%, 80\%\}$. For instance, 40% means that 40% of HtM securities are moved to the AfS category. Max acceptable leverage ratio = 7.5. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

with

$$s_1 := \frac{1-\sqrt{1-2b(L_U-x)/p_1}}{b}, \quad s_2 := \frac{\bar{A}+\ell-p_1 s_1(1-bs_1)-L_I/\bar{l}}{1-p_1(1-bs_1)}$$
$$M := \sqrt{(p_1-\bar{l})^2-2p_1 b[L-x-\bar{l}(\bar{A}+\ell)]},$$
$$C := \sqrt{b\big(L-x-\bar{l}[\bar{A}+\ell]\big)\big(2\bar{l}+b(L-x-\bar{l}[\bar{A}+\ell])\big)},$$

where, defining

$$G(s) := \lambda_{\max}L-(\lambda_{\max}-1)[x+\bar{\gamma}(s)\bar{f}(\bar{\gamma}(s))+(s-\bar{\gamma}(s))f(\bar{\gamma}(s))+\bar{A}+\ell-s] \quad \text{and}$$
$$\bar{\gamma}(s) := \frac{p_1[(\lambda_{\max}-1)bs-1]+\sqrt{p_1^2[(\lambda_{\max}-1)bs-1]^2+4\lambda_{\max}^2 p_1 b(\bar{l}-\frac{1}{2})(L-x-\bar{l}[\bar{A}+\ell-s(1-p_1)])}}{p_1 b(\lambda_{\max}-2)},$$

we have $\bar{s} := 0$ *if* $L_U \leq G(0)$ *or* $\bar{s} := \bar{A}$ *if* $L_U \geq G(\bar{A})$ *while* $\bar{s}$ *is the (unique) solution to* $G(s) = L_U$ *on* $(0, \bar{A})$ *if* $G(0) < L_U < G(\bar{A})$.

5.2 Implications for SVB's implied risk tolerance

We conclude our discussion of the optimal HtM holdings implied by Proposition 5.2 by comparing it to the observed HtM holdings of SVB. In doing so, we can infer the size of the p_1 shock that is implied by SVB's allocation of assets between AfS and HtM accounting portfolios. This implied shock p_1 can provide a simple metric to capture the deposit withdrawal risk of SVB or, conversely, the risk tolerance of SVB. Notably, this shock p_1 crucially depends on the level of $\lambda_{\max}$. In the following numerical calibration, we consider varying levels of $\lambda_{\max}$ to investigate the riskiness of SVB's asset allocation.

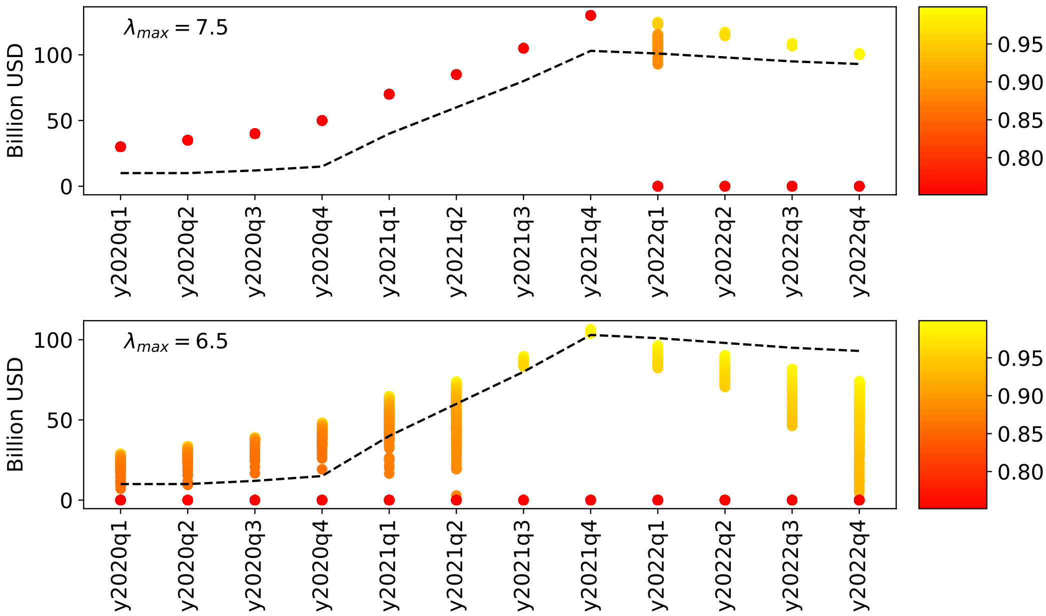

Figure 8: The figure shows theoretically optimal HtM portfolios of SVB for maximum accepted leverage ratios of 7.5 (upper pane) and 6.5 (lower pane). Optimized volume of HtM is represented by colored circles, each of which corresponds to a price shock p_1 with values indicated in the colorbar to the right. The dashed black line indicates the observed volumes of HtM portfolios.

Starting with the baseline calibration of $\lambda_{\max} = 7.5$, we can observe that the optimum over- or undershoots the historical HtM holdings of SVB, as illustrated in Figure 8, upper pane. In all quarters up to Q4 2021, the optimal HtM is uniformly the maximum admissible value, i.e., $\bar{A}$, as SVB falls into Case 1 of Proposition 5.2. However, beginning in 2022, except for incredibly mild shocks, the model indicates that SVB would keep all securities holdings in the AfS book to maximize available liquidity. We view this latter signal—the observed HtM being larger than the optimal HtM—as a warning sign to the health of SVB, since it indicates that they are highly susceptible to deposit withdrawal risks. We wish to note that the observed HtM may be larger than desired by SVB, as the bank needs to decide its HtM over time whereas our optimization allows for complete rebalancing of the assets $\bar{A}$ at all times.

This picture changes materially when considering a lower maximum leverage $\lambda_{\max} = 6.5$. Firstly, the lower pane of Figure 8 shows that, for balance sheet structures until Q2 2021, asset shock values consistent with the observed HtM holdings can now be found. This implied shock p_1 hovers around a sensible value of 0.9. Then, beginning especially in 2022, the theoretical optimal HtM portfolios are always lower than the observed holdings for all admissible values of the shock. This can be interpreted as a stark warning sign that the level of HtM securities held by SVB was not commensurate with the funding risk of the bank.

In addition to the fixed maximal leverage $\lambda_{\max}$ as investigated above, in Figure 9 we plot the smallest values of the maximum leverage ratio accepted by depositors so that SVB requires no funding risk with

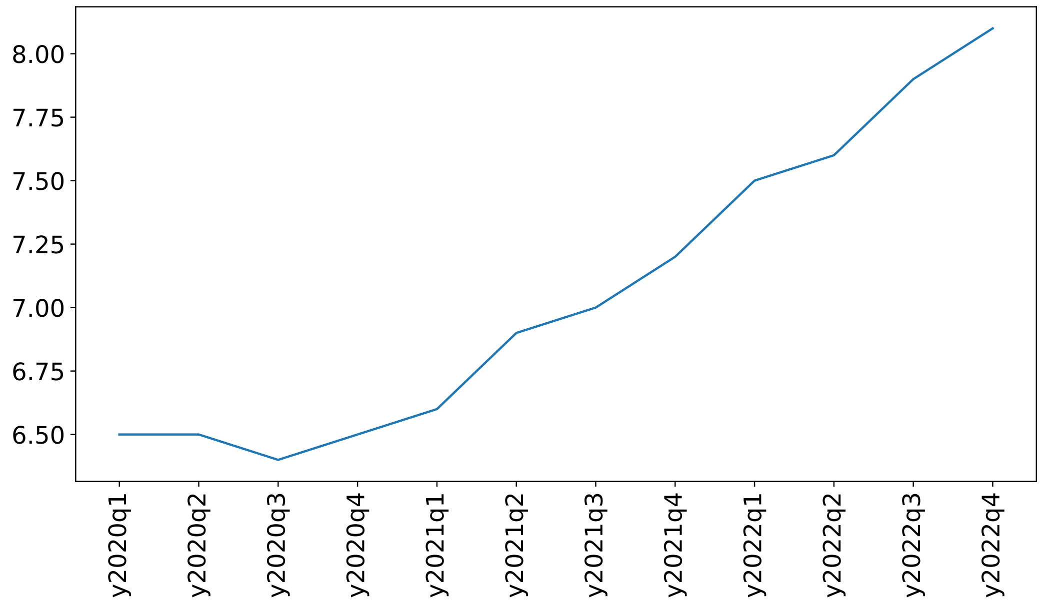

Figure 9: Smallest value of $\lambda_{\max}$ that, for a balance sheet of SVB with all securities held in the HtM portfolio $(= s + h)$, implies no selling of securities in equilibrium.

a 100% HtM portfolio. That is, the smallest $\lambda_{\max}$ so that SVB experiences Case 1 of Proposition 5.2. We highlight that, beginning in Q3 2020, there is a nearly linear growth in this smallest leverage from approximately 6.5 to over 8.0. This smallest leverage ratio to obtain Case 1 of Proposition 5.2 provides another measure of funding risk as the higher this minimum leverage, the more likely a (partial) run will occur with the potential need to remark HtM assets.

6 Conclusions

Despite a better understanding of bank runs and policies to mitigate them, run risks remain a significant concern. This was forcefully demonstrated by the March 2023 collapse of SVB and its serious repercussions across the broader financial markets.

In this work, we have built a model that can help explain the mechanics of bank runs in relation to structural vulnerabilities of the balance sheet, i.e., a large share of uninsured liabilities, accounting rules that may 'hide' revaluation risk of bank assets, and insufficient liquidity buffers to cover funding withdrawals, especially in the presence of fire-sale risk. The parsimonious setup of the model makes it straightforward to devise simple indicators of banks' exposure to run risks, as quantified by equilibrium withdrawals of funding that are easily computed via our explicit algorithm.

By inputting publicly available balance sheet data, the model allows us to analyze the build-up of run risk at SVB in the years before its collapse. Firstly, we demonstrate that, during 2022, its balance sheet composition and growth were creating conditions for an imminent bank run. Whether this applies already at the beginning of 2022 or later in the year, and whether it comes with illiquidity or insolvency implications, depends on investors' perception about unrealized losses materialising or not and on the volume of cash that could be raised by liquidating securities portfolios, i.e., on the sensitivity of market prices to the volume of off-loaded securities. Secondly, we show that SVB's choice to park most of deposits in HtM asset portfolios violated prudent risk management principles, as the bank lost its ability to mobilise liquidity necessary to prevent a rational run by depositors. Finally, we are able to monitor when and how liquidity problems transform from manageable, meaning that they are avoidable by investing new deposit volumes into AfS assets, to a state where default is imminent.

In related recent work, the empirical findings of Granja (2023) and Kim et al. (2023) have cast doubt on whether—or at least to what extent—banks take seriously the intent and ability to hold HtM assets until they mature. Our conclusions reveal that, under reasonable assumptions, such negligence was indeed foreboding of SVB's failure when one factors in a simple model of depositor runs—with the implied outcomes being detrimental already throughout 2022. Notably, additional case studies of other banks in other jurisdictions could lead to further insights.

Looking ahead, the severity of SVB's collapse and the ensuing crisis has spurred a serious public debate about the future of the HtM framework and possible regulatory responses. Not least, there have been calls to abandon it, but Kim et al. (2023) note that this may not be feasible in view of enduring support for the original motivations behind the rules, particularly in relation to banks' economic hedge of interest rate risk through its deposit franchise in scenarios that do not lead to a run. Similarly, Granja (2023) stresses the need to carefully consider trade-offs, noting in particular the aforementioned hedge and the concern that mark-to-market valuations may have been a key propagator of contagion during the Global Financial Crisis. Thus, Kim et al. (2023) mention increased enforcement of the existing GAAP restrictions on HtM designations and, in parallel, Granja (2023) suggest that a relevant measure may be to enforce more thorough scrutiny and evaluation of the reasonableness of banks' claims about their ability to hold the assets to maturity. Moreover, we note that, in case of doubt about effectiveness of economic hedges of the interest rate risk, there are other means to protect valuation of interest-rate-sensitive portfolios using interest rate derivatives (EBA, 2023). Additionally, there are supervisory tools to assess banks' total balance sheet sensitivity to interest rate shocks, for instance the EBA technical standards to "evaluate if there is a large decline in the net interest income or in the economic value of equity that could trigger supervisory measures".[6]

By endogenizing banks' HtM designation within our bank run model, we are able to derive a simple notion of the reasonableness of their choice, or desire, to hold given levels, in a way that is easy to assess quantitatively. Specifically, for a given balance sheet, we characterize the bank's maximal HtM designation such that it is able to safely hold on to these assets in a potential depositor run driven by a negative price shock of some specific size. For a bank's HtM designation to be reasonable, it should be in line with a plausible range of price shocks. By calibrating to the balance sheets of SVB over time, we obtain clear warning signs about the reasonableness of its HtM designations already from Q3 2021 or Q1 2022, depending on what we assume about the depositors' propensity to run. Notably, these indications appear before the size of unrealized losses at SVB ballooned. This suggests that our approach may be relevant as a simple way for regulators and supervisors to monitor bank's reasonable usage of the HtM framework when accounting for its implications on funding and liquidity risks.

A Proofs of main results

A.1 Proof of Proposition 3.1

One readily confirms that the two mappings (2) and (3) are non-decreasing in $(w, \gamma) \in [0, L_U] \times [0, s+h]$. As the domain of Φ defined by (2)–(3) is a complete lattice, the claim therefore follows from Tarski's fixed point theorem.

[6]See https://www.eba.europa.eu/activities/single-rulebook/regulatory-activities/supervisory-review-and-evaluation-process-srep-1

A.2 Proof of Proposition 3.2

By Proposition 3.1 there exists a minimal clearing solution $(w^*, \gamma^*) \in [0, L_U] \times [0, s+h]$, provided the bank is solvent. The left-hand side of the solvency condition (5) reads as

$$x + \gamma \bar{f}(\gamma) + (s - \gamma) f(\gamma) + h + \ell, \quad \text{for} \quad \gamma \in [0, s], \quad \text{and}$$

$$x + \gamma \bar{f}(\gamma) + (s + h - \gamma) f(\gamma) + \ell, \quad \text{for} \quad \gamma \in (s, s+h].$$

Following Remark 2.8, these are both non-increasing functions of γ on the respective domains. Moreover, at $\gamma = 1$, there is a jump of size $(f(s) - 1)h \leq 0$, since $f(s) \leq 1$ by Assumption 2.3. Consequently, if the bank was already insolvent at some level of liquidations γ, it is also insolvent for all larger values. It therefore suffices to check for solvency at the termination of the algorithm, since the algorithm is increasing in the value of γ^*.

By construction, we must have that either $\gamma^* = 0$ (no sales), $\gamma^* \in (0, s]$ (run without re-marking of HtM), $\gamma^* \in (s, s+h)$ (run with re-marking of HtM), or $\gamma^* = s + h$ (illiquidity). By studying these case-by-case, we will be able to conclude that our clearing solution is indeed realized by one of the steps presented in Proposition 3.1. By proceeding in increasing order with respect to the values of (γ^*, w^*), we arrive at the minimal solution.

Step 1 (No sales). Assume $\gamma^* = 0$. Then $w^* = \Phi_w(0) = L_U \wedge [\lambda_{\max} L - (\lambda_{\max} - 1)(x + sp + h + \ell)]^+$. This is a clearing solution if and only if $w^* \leq x$. This, in turn, holds if and only if $L_U \leq x$ or $\lambda_{\max} L - (\lambda_{\max} - 1)(x + sp + h + \ell) \leq x$. The latter holds if only if

$$L \leq \frac{x}{\lambda_{\max}} + \frac{\lambda_{\max} - 1}{\lambda_{\max}}(x + sp + h + \ell) = x + \frac{\lambda_{\max} - 1}{\lambda_{\max}}[sp + h + \ell].$$

Step 2 (Run without re-marking HtM I). Suppose $\gamma^* \in (0, s]$. Then $w^* \in (x, L_U]$ with $L_U > x$. For this step, assume $w^* \in (x, L_U)$. Since $\gamma^* \in (0, s]$, we can see that $w^* = \Phi_w(\gamma^*)$ holds if and only if

$$L_U \geq \lambda_{\max} L - (\lambda_{\max} - 1)(x + \gamma^* \bar{f}(\gamma^*) + (s - \gamma^*) f(\gamma^*) + h + \ell). \tag{8}$$

Note that w^* equals the right-hand side of (15). Moreover, γ^* must satisfy $\gamma^* \bar{f}(\gamma^*) = w^* - x$ and it is the unique such solution, since the left-hand side is strictly increasing in γ^* on $[0, s+h]$ (by Assumption 2.3). Inserting $w^* = x + \gamma^* \bar{f}(\gamma^*)$ in (15) and recalling that the right-hand side equals w^*, we obtain

$$w^* = L - (1 - \frac{1}{\lambda_{\max}})((s - \gamma^*) f(\gamma^*) + h + \ell).$$

Thus, the liquidation $\gamma^* \in (0, s]$ satisfies

$$\gamma^* \bar{f}(\gamma^*) + (1 - \frac{1}{\lambda_{\max}})(s - \gamma^*) f(\gamma^*) = L - x - (1 - \frac{1}{\lambda_{\max}})(h + \ell), \tag{9}$$

and it must be the unique solution to this equation on $(0, s]$, since the left-hand side is strictly increasing in γ^* on $[0, s]$ by Assumption 2.6 and Remark 2.8. This is possible if and only if

$$L - x - (1 - \frac{1}{\lambda_{\max}})(h + \ell) \in \big[(1 - \frac{1}{\lambda_{\max}})sp, s\bar{f}(s)\big]. \tag{10}$$

Consequently, we have a clearing solution if and only if both (10) and (15) hold with γ^* in (15) being the unique solution to (9).

Step 3 (Run without re-marking HtM I). Now assume $\gamma^* \in (0, s]$ and $w^* = L_U$. Then γ^* satisfies $\gamma^* \bar{f}(\gamma^*) = w^* - x = L_U - x$. As the left-hand side is strictly increasing in γ^* on $[0, s+h]$, we have a unique solution which is in $(0, s]$ if and only if $L_U \in (x, x + s\bar{f}(s)]$. With $\gamma^* \leq s$, we have $w^* = \Phi_w(\gamma^*) = L_U$ if and only if $L_U \leq \lambda_{\max} L - (\lambda_{\max} - 1)(x + \gamma^* \bar{f}(\gamma^*) + (s - \gamma^*) f(\gamma^*) + h + \ell)$. Writing $L = L_I + L_U$, this re-arranges to

$$L_I \geq (1 - \frac{1}{\lambda_{\max}})[(s - \gamma^*) f(\gamma^*) + h + \ell]. \tag{11}$$

Consequently, (γ^*, w^*) is a clearing solution if and only if $L_U \in (x, x + s\bar{f}(s)]$ and (11) holds for the unique solution $\gamma^* \in (0, s]$ of $\gamma^* \bar{f}(\gamma^*) = L_U - x$.

Step 4 (Re-marking HtM I). Suppose $\gamma^* \in (s, s+h)$. Then $w^* \in (x, L_U]$. For this step we assume $w^* \in (x, L_U)$. We have $w^* = \Phi_w^*(\gamma^*)$ if and only if

$$L_U \geq \lambda_{\max} L - (\lambda_{\max} - 1)(x + \gamma^* \bar{f}(\gamma^*) + (s + h - \gamma^*) f(\gamma^*) + \ell), \tag{12}$$

and w^* is then given by the right-hand side of (12). Noting that γ^* must be the unique solution of $\gamma^* \bar{f}(\gamma^*) = w^* - x$ (where the left-hand side is strictly increasing in γ^* on $[0, s + h]$), we can insert this in (12) and solve for

$$w^* = L - (1 - \frac{1}{\lambda_{\max}})((s + h - \gamma^*) f(\gamma^*) + \ell).$$

In turn, $\gamma^* \in (s, s + h)$ must solve

$$\gamma^* \bar{f}(\gamma^*) + (1 - \frac{1}{\lambda_{\max}})(s + h - \gamma^*) f(\gamma^*) = L - x - (1 - \frac{1}{\lambda_{\max}})\ell, \tag{13}$$

and it must the unique such solution since the left-hand side is strictly increasing on $[0, s + h]$ by Assumption 2.6. This is feasible if and only if

$$L - x - (1 - \frac{1}{\lambda_{\max}})\ell \in \big[s\bar{f}(s) + (1 - \frac{1}{\lambda_{\max}})h f(s), (s + h)\bar{f}(s + h)\big]. \tag{14}$$

In conclusion, we have a clearing solution if and only if (14) and (12) hold, when γ^* in (12) is given by the unique solution to (13).

Step 5 (Re-marking HtM II). Now assume $\gamma^* \in (s, s+h)$ and $w^* = L_U$. Then $\gamma^* \bar{f}(\gamma^*) = L_U - x$, which is possible if and only if $L_U \in (x, x + (s + h)\bar{f}(s + h))$. Moreover, we see that $\Phi_w(\gamma^*) = L_U$ holds if and only if

$$L_I \geq (1 - \frac{1}{\lambda_{\max}})[(s + h - \gamma^*) f(\gamma^*) + \ell]. \tag{15}$$

We thus have a clearing solution if and only if $L_U \in (x, x + (s + h)\bar{f}(s + h))$ and the unique solution γ^* to $\gamma^* \bar{f}(\gamma^*) = L_U - x$ satisfies (15).

Step 6 (Illiquidity). Finally, assume $\gamma^* = s + h$. Then

$$w^* = \Phi_w(s + h) = L_U \wedge [\lambda_{\max} L - (\lambda_{\max} - 1)(x + (s + h)\bar{f}(s + h) + \ell)].$$

This is a clearing solution if and only if $w^* - x \geq (s + h)\bar{f}(s + h)$. Given the expression for w^*, this holds if and only if either

$$\lambda_{\max} L - (\lambda_{\max} - 1)(x + (s + h)\bar{f}(s + h) + \ell) \geq L_U \quad \text{and} \quad L_U \geq x + (s + h)\bar{f}(s + h), \quad \text{or}$$
$$\lambda_{\max} L - (\lambda_{\max} - 1)(x + (s + h)\bar{f}(s + h) + \ell) < L_U \quad \text{and} \quad L \geq x + (s + h)\bar{f}(s + h) + (1 - \frac{1}{\lambda_{\max}})\ell,$$

since the last inequality is equivalent to

$$\lambda_{\max}L - (\lambda_{\max} - 1)\big(x + (s+h)\bar{f}(s+h) + \ell\big) \geq x + (s+h)\bar{f}(s+h).$$

This completes the proof.

A.3 Proof of Proposition 5.2

Suppose first that $L_U \leq x$ or $L \leq x + \bar{l}(\bar{A} + \ell)$. In the first situation, there is no risk of a run on the marketable securities. If, instead, we are in the other situation, then a run is possible depending on the HtM versus AfS designation. However, by taking $s^* = 0$, we have $\lambda_{\max}L - (\lambda_{\max} - 1)(x + s^* p_1 + h + \ell) = \lambda_{\max}L - (\lambda_{\max} - 1)(x + \bar{A} + \ell) \leq x$ at time 1, no matter what p_1 is. With this choice, Proposition 3.2 therefore gives that the clearing solution is of the 'No sales' type, for any $p_1 \in (0,1)$, and hence $s^* = 0$ is the minimizer of (7), as claimed.

From here on, suppose instead that $L_U > x$ and $L > x + \bar{l}(\bar{A} + \ell)$. Since $sp_1 + h \leq \bar{A}$, for any choice of $s \in [0, \bar{A}]$, it follows that, at time 1, we have $\lambda_{\max}L - (\lambda_{\max} - 1)(x + sp_1 + h + \ell) < x$ (along with $L_U > x$), so, by Proposition 3.2, we cannot have a 'No sales' (minimal) clearing solution. Thus, we can proceed by identifying the feasible regions yielding that the (minimal) clearing solution belongs to either of the two 'Run without re-marking HtM' scenarios in Proposition 3.2. We refer to these regions as partial or full withdrawals, and we denote the minimal attainable AfS over each by, respectively, s_{PW} or s_{FW} (assigning the value $+\infty$ if the region is empty).

We begin by characterizing the partial withdrawal region. Since $p_1 \leq 1$, and since we assume $L > x + \bar{l}(\bar{A} + \ell)$, by Proposition 3.2 partial withdrawals are feasible for $s \in [0, \bar{A}]$ if and only if

$$L_U \geq \lambda_{\max}L - (\lambda_{\max} - 1)\big(x + \bar{\gamma}(s)\bar{f}(\bar{\gamma}(s)) + (s - \bar{\gamma}(s))f(\bar{\gamma}(s)) + \bar{A} - s + \ell\big), \tag{16}$$

$$L - x - \bar{l}(\bar{A} - s + \ell) \leq s\bar{f}(s), \quad \text{and} \tag{17}$$

$$\bar{\gamma}(s)\bar{f}(\bar{\gamma}(s)) + \bar{l}(s - \bar{\gamma}(s))f(\bar{\gamma}(s)) = L - x - \bar{l}(\bar{A} - s + \ell), \tag{18}$$

for some $\bar{\gamma}(s) \in (0, s]$. Using $\bar{f}(s) = p_1(1 - bs/2)$ in (17), we obtain the quadratic expression

$$-\frac{b}{2}p_1 s^2 + (p_1 - \bar{l})s \geq L - x - \bar{l}(\bar{A} + \ell). \tag{19}$$

As the right-hand side is strictly positive, we can confirm that this holds for $s \geq 0$ if and only if

$$\frac{p_1 - \bar{l} - M}{bp_1} \leq s \leq \frac{p_1 - \bar{l} + M}{bp_1} \quad \text{with} \quad M := \sqrt{(p_1 - \bar{l})^2 - 2bp_1\big(L - x - \bar{l}(\bar{A} + \ell)\big)} \tag{20}$$

and

$$p_1 > \bar{l} + \sqrt{2bp_1\big(L - x - \bar{l}(\bar{A} + \ell)\big)}.$$

The latter can be seen to hold if and only if

$$p_1 \geq \bar{l} + b\big(L - x - \bar{l}(\bar{A} + \ell)\big) + C \tag{21}$$

for C as in the statement of the proposition. Now, for any s in the above range (recalling also that $\bar{l}sp_1 \leq L - x - \bar{l}(\bar{A} - s + \ell)$), by Step 2 in the proof of Proposition 3.2, there is a unique $\bar{\gamma}(s) \in (0, s]$ satisfying (18). Inserting the expressions for f and $\bar{f}$ in (18), we obtain a quadratic equation

$$\bar{\gamma}^2 p_1(\bar{l} - \frac{1}{2})b + \bar{\gamma}p_1[1 - \bar{l}(sb + 1)] = L - x - \bar{l}[\bar{A} + \ell - s(1 - p_1)],$$

in $\bar{\gamma}$. Knowing that each s in the above range corresponds to a unique $\bar{\gamma} \in [0, s]$, we can solve the above equation to yield the expression for $s \mapsto \bar{\gamma}(s)$ in the statement of the proposition. Further, we can observe that this map is strictly increasing in s, as follows directly from $f(\bar{\gamma}(s)) < 1$ and the fact that, for fixed s, the left-hand side of (18) is strictly increasing in the value of $\bar{\gamma}(s)$ on $[0, s]$. This in turn ensures that the right-hand side of (16) is strictly increasing in s. Let $G(s)$ denote the right-hand side of (16). If $G(\bar{A}) \leq L_U$, then all $s \in [0, \bar{A}]$ satisfy (16), and we set $\bar{s} := \bar{A}$. If $G(0) \geq L_U$, then at most $s = 0$ can satisfy (16), and we set $\bar{s} := 0$. If $G(0) < L_U < G(\bar{A})$, then we can define $\bar{s} \in (0, \bar{A})$ to be the unique value for which there is equality in (16). With these definitions, we have that (16) holds for a given $s \in (0, \bar{A}]$ if and only if $s \leq \bar{s}$. Noting that (20)–(21) enforces $s > 0$, we conclude that the feasible region for partial withdrawals is given by the constraints (20)–(21) and $s \leq \bar{s}$. If this region is non-empty (in particular implying $\bar{s} \in (0, \bar{A}]$), then clearly the minimal s over the region is $s = (p_1 - \bar{l} - M)/bp_1$.

Next, we turn to full withdrawals. Given $s \in [0, \bar{A}]$, let $\gamma^* \in [0, \bar{A}]$ denote the corresponding quantity sold (in the minimal clearing solution). Since $L_U > x$, by Poposition 3.2 full withdrawals are feasible if and only if $\gamma^* \bar{f}(\gamma^*) = L_U - x$ with

$$s\bar{f}(s) \geq L_U - x \quad \text{and} \quad L_I \geq \bar{l}\big((s - \gamma^*(s))f(\gamma^*(s)) + \bar{A} - s + \ell\big) \tag{22}$$

Writing out $\bar{f}(\gamma) = p_1(1 - b\gamma/2)$, the above equality yields a quadratic equation

$$-p_1 \frac{b}{2}(\gamma^*)^2 + p_1\gamma^* = L_U - x$$

in γ^*. Since $L_U > x$, this has positive solutions $\gamma^* > 0$ if and only if $p_1 > 2b(L_U - x)$. We cannot have $\gamma^* \geq 1/b$, as our assumptions enforce $b < 1/\bar{A}$, so we would have $\gamma^* > \bar{A}$. Solving for γ^* thus gives that the only value allowing for $\gamma^* \in [0, \bar{A}]$ is

$$\gamma^* = \frac{1 - \sqrt{1 - 2b(L_U - x)/p_1}}{b} \quad \text{with} \quad p_1 > 2b(L_U - x). \tag{23}$$

It furthermore follows from these observations that $s\bar{f}(s) \geq L_U - x$ holds for a given $s \in [0, \bar{A}]$ if and only if $s \geq \gamma^*$ with γ^* given by (23). Since $\bar{f}(\gamma^*) < 1$, the second constraint in (22) holds if and only if

$$s \geq \frac{\bar{A} + \ell - \gamma^* p_1(1 - b\gamma^*) - L_I/\bar{l}}{1 - p_1 + p_1 b\gamma^*}, \tag{24}$$

where we have written out the expressions for f and $\bar{f}$. Set $s_1 := \gamma^*$ and let s_2 denote the right-hand side of (24). For full withdrawals to be feasible, we need $p_1 > 2b(L_U - x)$ and $\max\{s_1, s_2\} \leq \bar{A}$. In that case, the feasible values are $s \in [0, \bar{A}]$ with $s \geq \max\{s_1, s_2\}$, so the minimum of (7) is achieved at $\max\{s_1, s_2\}$, as desired. This completes the proof.

B Additional Sensitivity Analysis

Proposition 3.2 shows that there are some critical parameters of the model that change the liquidity and solvency state of the bank. One of them is the share of uninsured deposits that may be subject to a run risk. The other one is the uncertainty of the value of the assets held by the bank that impacts depositors' beliefs about the pool of assets that can be used to raise cash and cover deposit withdrawals. Notably, the market value of the balance sheet, even independent of the unrealised loss aspect, can be lower than the book value.

As Fig. 10 shows, the level of uninsured deposits influences the liquidity and solvency state of a bank. Moreover, a small shift in the composition of liabilities may determine whether the banks is liquid and solvent, being able to raise cash to cover potential bank run and hold adequate level of capital to cover even unexpected losses, or may lose solvency despite retaining capacity to satisfy immediate deposit withdrawals. This sheds light on the importance of careful calibration of requirements regarding banks' composition of funding sources.

We illustrated the impact of the initial valuation on the run risk in Fig. 11. It is interesting to see changes in the initial valuation of the asset portfolios can create jumps in the vulnerability of the banks. The impact is most visible at the end of 2022 when low valuation of securities portfolios may create conditions for deposit runs leading to illiquidity. The state of the bank is very sensitive to the beliefs regarding the valuation. For instance, as of Q4 2022, depending whether depositors believe 7.5% or 10% lower valuation of assets, the bank may move from liquid state, even though requiring tapping liquidity from HtM portfolios, to illiquidity meaning that the bank does not have enough resources to satisfy depositors.

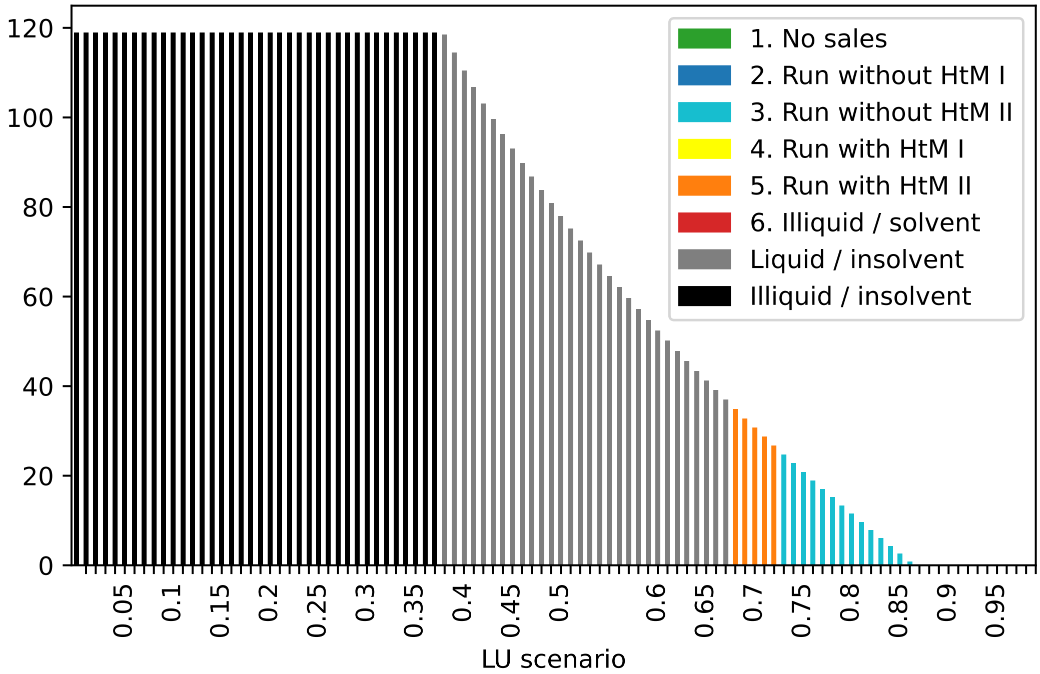

Figure 10: The figure shows equilibrium liquidation of assets (in USD billion) from SVB for balance sheets observed as of Q1 2022 for various calibrations of the share of uninsured deposits being converted into insured funding. Each bar corresponds to one value of a fraction of L_U, with the fractions varying from 0.0 to 1.0 (x-axis). For instance, a bar corresponding to 0.7 means that in case 70% of uninsured deposits are replaced by L_I category funding, the bank would solvent and liquid following funding withdrawals in equilibrium but would need to dip into HtM portfolios (orange bar) to generate enough cash. The max acceptable leverage ratio is set to 7.5. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

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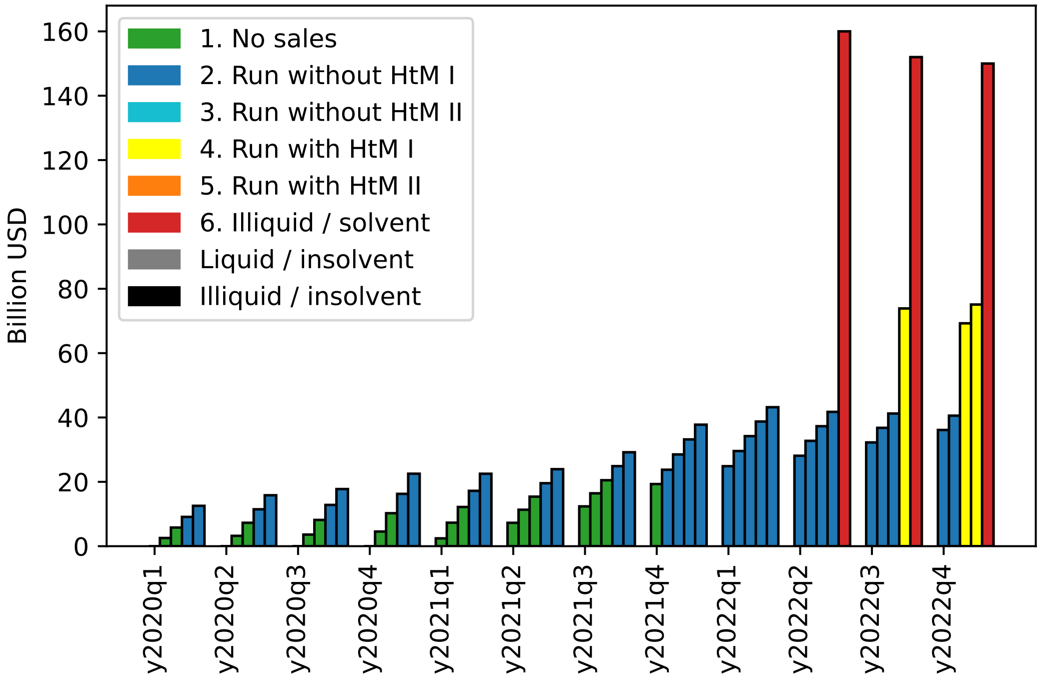

Figure 11: The figure shows equilibrium withdrawal of funding (in USD billion) from SVB for balance sheets observed between Q1 2020 and Q4 2022 and for various calibrations of initial valuation of marketable assets (i.e., parameter p). For each period there is a group of bars, each of them corresponding to one value of p from a set $\{1.000, \ ,0.975, \ ,0.950, \ 0.925, \ 0.900\}$ and max acceptable leverage ratio 7.5. Colored bars correspond to steps 1–6 of the algorithm in Prop. 3.2. Grey and black bars indicate insolvency differentiating liquidity and illiquidity state.

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