EUROPEAN CENTRAL BANK

EUROSYSTEM

Working Paper Series

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Shock amplification in an interconnected financial system of banks and investment funds

No 2581 / August 2021

Abstract

This paper shows how the combined endogenous reaction of banks and investment funds to an exogenous shock can amplify or dampen losses to the financial system compared to results from single-sector stress testing models. We build a new model of contagion propagation using a very large and granular data set for the euro area. Based on the economic shock caused by the Covid-19 outbreak, we model three sources of exogenous shocks: a default shock, a market shock and a redemption shock. Our contagion mechanism operates through a dual channel of liquidity and solvency risk. The joint modelling of banks and funds provides new insights for the assessment of financial stability risks. Our analysis reveals that adding the fund sector to our model for banks leads to additional losses through fire sales and a further depletion of banks' capital ratios by around one percentage point.

Keywords: Fire sales, liquidity, overlapping portfolios, price impact, stress testing
JEL: D85, G01, G21, G23, L14

Non-technical summary

Since the global financial crisis, the role of non-bank financial institutions has been increasing. This growth particularly stands out for investment funds, whose total assets under management have reached almost half of the total size of the banking sector in the euro area, highlighting the ever-growing importance of investment funds in financing today's economy. These changes in the financial system call for the extension of the scope of financial stability analysis and given the highly interconnected nature of financial institutions, it has as well become crucial to improve modelling capabilities for the joint analysis of different financial sectors within a single framework. Hence, such exercises need not only the possibly simplified replication of sectoral stress tests but also the ability to allow for institution-level contagion even across different sectors. Moreover, such joint models of banks and non-banks can also inform policymakers in developing the non-bank macroprudential toolkit of the future.

In this paper, we introduce a new model for the joint stress testing of banks and investment funds in the euro area, developed at the ECB jointly with national central banks. The model relies to a large extent on granular datasets covering banks' and funds' balance sheet information with, in particular, their securities holdings, fund holdings and loan portfolios at counterparty- or aggregate level. This granular dataset makes it possible to shed light on previously not well known interconnections among the two different financial sectors, i.e. banks and funds holding shares of investment funds or funds holding securities issued by banks, securities jointly held by both banks and funds or loans to investment funds. Equipped with this dataset, we are not only able to initially shock the sectors following existing practices in sector-level stress testing exercises conditional on a set of macro-financial variables but we also propose additional liquidity-driven endogenous reactions for both sectors.

Our initial shock consists of redemptions from investment funds, increased PDs for non-financial corporations (NFC) combined with stochastic NFC defaults as well as an instantaneous stock market shock. After initial losses from these changes in our system, endogenous reactions activate, i.e. reactions from within our modelled system, under which we assume certain short-term funding withdrawal within the banking system, access of solvent but illiquid banks to short-term funding in the interbank market, possible redemptions from investment funds driven by liquidity needs of banks and funds and fire sales of marketable securities at discounted prices. All these reactions lead to additional sizeable losses within the financial system, which are not captured by stress tests that adhere to the common assumption of a static balance sheet.

Applying a Covid-19 shock scenario, developed for the ECB Vulnerability Analysis exercise in 2020, combined with end-2019 balance sheet information as a starting point, we find over a two quarter horizon that the presence of funds together with banks in our modelled financial system increases average bank capital depletion by one percentage point. This effect is largely due to asset fire sales of overlapping portfolios. Fire sale losses are clearly driven by funds' need to meet exogenous redemptions and these endogenous losses are especially high for investment funds, who have no access to central bank funding as is the case for banks.

Our findings show that the joint modelling of several financial sectors delivers clear added value to the analytical capabilities of central banks and confirms the need for increasing the coverage of financial sectors as part of our system-wide stress testing endeavors.

1 Introduction

The landscape of the financial system in the euro area has changed significantly since the global financial crisis and the subsequent more onerous bank regulatory requirements. Banks' dominant position as liquidity provider waned in relative terms, while other types of financial institutions gained momentum. Especially, investment funds have recently attracted significant attention from supervisory authorities due to the remarkable growth of the sector. Over the last ten years their assets under management more than doubled in the euro area, reaching EUR 14 trillion in 2019. This compares to total assets of around EUR 31 trillion for euro area credit institutions, which however has remained broadly stable since 2008. This has also resulted in a stronger role of capital market-based financing of the real economy, e.g. via fund purchases of debt securities from non-financial corporations, as opposed to traditional bank financing.

This changing structure of the financial system raises new questions and new challenges for the analysis of financial stability, calling for more holistic risk assessments covering the entire financial system and for a better understanding of the complex interactions between different financial intermediaries, including not only banks but also non-bank financial institutions. The enlarged role of non-bank financial institutions in financial intermediation in the euro area therefore necessitates an improved stress test modelling capacity to enable broad-based assessments of financial stability risks and the financial sector's resilience to their materialisation. Such System-Wide Stress Test models should ideally capture a broad set of financial institutions (banks and non-banks) and cater for the most important sources and channels of contagion between different types of financial institutions.

The present paper is a first step in this direction with a stress testing model that covers banks interacting with investment funds in a coherent framework using very granular data. Such a tool can be used for system-wide stress testing analysis to provide holistic assessments of financial sector resilience. To our knowledge, this is the first attempt in a stress testing context to model jointly banks and investment funds and their interconnections with firm level data. We demonstrate that accounting for the firm-by-firm interactions between banks and funds allows to capture important amplification effects that one might overlook when carrying stress test for banks or funds in isolation. Going forward, the intention is to extend this modelling framework also to other financial intermediaries, such as insurance corporations, hedge funds, money market funds, pension funds and CCPs, with the ultimate goal to develop a tool for policy analysis and system-wide stress testing using different macro-financial shock scenarios.

The literature on the joint modelling of several financial sectors under stress with realistic institutional features is rather scarce. We can divide it into three strands: theoretical work, models that use aggregated data and approaches built on granular data. A theoretical contagion model for banks and funds with asset fire sales as the main channel establish Calimani, Hałaj, and Żochowski (2019). The modelled agents optimize their balance sheet structure while trying to comply with solvency and liquidity requirements. Both demand and supply endogenously determine asset prices. However, the model operates on a small scale of few banks and asset managers and the agents' properties are randomly drawn. Aikman et al. (2019) propose a representative agent equilibrium model for banks, investment funds, hedge funds, pension funds and insurers. Using aggregate balance sheet information, they model the secondary market for tradable securities, the repo market as well as the interest rate swap market. Mirza, Moccero, Palligkinis, and Pancaro (2020) employ aggregate information on the banking sector as well as several types of funds to model a shock propagation due to overlapping portfolios and asset fire sales.

Hałaj (2018) proposes an agent-based model with a focus on funding risk, where banks and funds interact through a number of different channels. The model uses granular data that is matched to real banking data, but the underlying interbank network is simulated. Chrétien et al. (2020) use the most granular data for banks, funds and insurance companies located in France to model a shock propagating through the system via bilateral holdings. The information on all sectors' asset holdings is available at ISIN level. This allows a detailed modelling of overlapping portfolios, which the authors augment with bilateral large credit exposures used for the simulation of traditional default cascades. This approach does not include asset fire sales but it accounts for changes due to price depreciation on the balance sheet of financial institutions. Caccioli, Ferrara, and Amanah (2020) focus on fire sale modelling for banks, investment funds and insurers in the UK financial system using granular, ISIN-level information on holdings of modelled sectors. Fricke and Fricke (2021) look more specifically at the vulnerability of the mutual fund sector in the US, finding it to be relatively stable but making the case for an integration with banks. Farmer, Kleinnijenhuis, Nahai-Williamson, and Wetzer (2020) model a financial system with banks, hedge funds and asset managers through which a shock from an EBA stress test scenario propagates. They model overlapping portfolios through tradable assets, bilateral repurchase agreements and unsecured interbank loans. Banks also face several regulatory constraints. This paper is the closest to our approach in terms of the methodology; however, it uses less granular, publicly available data for banks and only stylised information for funds. Roncoroni, Battiston, Escobar Farfàn, and Martinez Jaramillo (2021) extend a network valuation of assets approach with common asset contagion and endogenous recovery rate for banks and investment funds, this is then applied to a climate risk scenario.

In addition to the joint modelling of several financial sectors, another stream of literature, focussing on multiple interacting constraints, is relevant for our contribution. In this context, Hałaj and Laliotis (2017) emphasize that liquidity crises may pave the way for solvency crises or amplify the effect that solvency stress may have on liquidity. Their objective is to evaluate the augmentation effects of funding shocks through fire sales, interbank linkages, overlapping asset portfolios and cross-holding of debt channels, and measure deteriorating funding conditions of banks due to their solvency issues or the availability of unencumbered collateral. The authors propose to couple liquidity and solvency crisis, as one normally precedes the other. Hesse et al. (2012) present another framework to link liquidity and solvency risks. Their model allows for a simulation of a growth in funding costs due to changes in solvency, a simulation of a funding market closure depending on the level of capitalization, and an examination of the impact of concentration risk on funding. Cont, Kotlicki, and Valderrama (2019) also advise the joint stress testing of solvency and liquidity for banks. Their approach is based on a three-period model using the relations between solvency and liquidity shocks, to create a comprehensive framework for liquidity and solvency risk.

Our modelled financial system consists of two sectors: banks and open-ended investment funds domiciled in the euro area. To study the interaction between these agents, we take three major steps. First, we merge several databases to identify exposures on the asset and liability side of individual institutions. Second, we propose an accurately modelled shock to the real economy that causes initial losses for banks and funds. The basis for this in our paper is the economic shock caused by the coronavirus (COVID-19) outbreak, which is based on an adverse scenario path as documented in the ECB macroeconomic projections. This shock scenario was also used in the Vulnerability Analysis of banks directly supervised within the ECB Single Supervisory Mechanism. Finally, we model the agents' behaviour in response to the shock, which can either amplify or dampen damages to the financial system. Here, the shock propagates through a number of different channels: banks lend to funds, while funds deposit their cash with banks (liquidity providing); banks and funds hold securities issued by the other sector (securities cross-holdings); and finally, agents in both sectors invest in similar assets (overlapping portfolios).

The model dynamics develop as follows. A macro-financial shock is translated into increased probabilities of defaults (PDs) of firms and households, which affect banks' impairment calculations. Moreover, the increased firm-level PDs enter a stochastic simulation algorithm that estimates correlated corporate defaults (firm default shock). A stock market shock from the macro-financial scenario also hits the balance sheets of banks and funds which write down losses on their loans and securities. Worsened fund performance materialises as a result of scenario-induced asset price changes (market price shock) and firm defaults. This activates the liquidity channel of contagion. Funds, in shortage of cash to repay redeeming investors, sell some of their assets (redemption shock). Banks with solvency issues withdraw short-term funding from other banks while financially sound banks withdraw short-term funding from distressed and defaulted banks as a precautionary measure. To obtain liquidity, banks are assumed to have access to a central bank facility up to the level of their available high-quality liquid assets (HQLA)[1]. Banks that are not in solvency distress can also borrow in the unsecured inter-bank market, and, if this does not provide sufficient liquidity, banks can redeem their investment fund shares and sell non-eligible assets pro rata at a price discount. This price discount is determined endogenously as a function of the volumes sold. Finally, banks that cannot get above their minimum liquidity thresholds, default due to illiquidity issues. Investment funds are assumed to default only when their net asset value (NAV) reaches zero. In this case, not only investors suffer losses; banks also write down losses on their loans to defaulting funds.

In our framework, two particular mechanisms are worth highlighting. First, we model correlated defaults of firms using their shocked probability of default and simulated stock prices in a stochastic manner. Second, we calibrate market price impact functions using granular data on market volumes and prices at security level. We then aggregate the obtained parameters to sufficiently granular buckets such as to cover the universe of se-

[1]Central banks are not modelled explicitly and HQLA is equivalent to cash in our model.

curities employed in our model. To our knowledge, this paper is the first multi-country study of a financial network consisting of banks and funds across euro area countries that uses granular data. Moreover, it relies on a realistic macro-financial shock scenario. Additionally, the paper proposes a mechanism for generating correlated firm defaults in line with the literature on market-based finance, which is applied to tens of thousands of non-financial corporations (NFCs) in the system. Lastly, the paper employs calibrated price impact functions for marketable assets using real market information at the ISIN level. Our results confirm that focusing only on the banking sector in stress testing exercises might underestimate the systemic effects of exogenous real economy shocks in the financial system. The contribution of investment funds to system-level losses is significant. In addtion, overlapping portfolios and asset fire sales seem to be the main channel for shock propagation from one sector to another.

This paper proceeds as follows. Section 2 sets the stage, introducing a financial system of banks and funds, their regulation and the parts of their balance sheets relevant from our model's point of view. Section 3 describes the datasets used for the analysis as well as the main empirical facts for the banking and fund sector, and their interconnections. Section 4 presents the realistic shocks to the system coming from satellite models. Section 5 lays down the details of the model and its dynamics. Section 6 discusses results of the model and some interesting counterfactual simulations. Section 7 draws conclusions.

Appendix A, on mathematical and balance sheet notations, describes important standards, to which we adhere throughout the paper. Appendices B to G provide more information on the data used in our model, further methodological details and the results from an experiment where funds are deactivated in our model.

2 Sectors

Our system consists of two types of agents: banks and investments funds. The banking system is the standard financial intermediary for the real economy, and banks' behaviour in a network setup has been extensively studied and used in stress testing exercises. For instance, Cont and Schaanning (2019) examine the transmission of a shock through a banking system due to common portfolio holdings. They analyse spillovers across portfolios due to deleveraging of banks in stress scenarios and measure the loss that distressed liquidations would impose on other institutions. Covi, Gorpe, and Kok (2021) and Montagna, Torri, and Covi (2020) also study the interconnectedness of the banking system based on bilateral linkages. They measure the systemic importance of banks and their degree of fragility, at the same time providing insights to the channels of contagion. On the contrary, including funds in a network model is a recent innovation in the literature, and modelling decisions are less clear-cut.

2.1 Banking sector

Our banking system comprises banking groups that operate as autonomous entities; we discuss their consolidation level in Section 3. Each banking group is characterised by its individual balance sheet, and its behavior is driven by a need to satisfy its contractual obligations (liquidity needs) and regulatory constraints (solvency ratio).[2]

Below we summarise risks stemming from the banking sector that are relevant for our model. Most of them have already been extensively studied from a financial stability perspective and employed in stress testing exercises.

- *Direct contagion via credit risk and bank defaults*: risks that propagate from the liability side of defaulting entities (Eisenberg and Noe, 2001) and through expected losses to other financial corporations and the real economy.
- *Solvency-liquidity feedback loop*: amplification of a bank's first-round losses if a run on its short-term debt takes place. Lenders reevaluate individual bank's asset value after the initial shock and decide whether or not, and on which conditions, to roll over the short-term funding (Pederson and Brunnermeier, 2009; Georgescu, 2015).
- *Direct contagion via market risk*: defaulting entities' issued bond and equity prices drop, causing immediate losses to the holders of those securities. An initial stock market shock is also introduced.
- *Indirect contagion via market risk*: price impact from asset fire sales when banks are suffering from losses and liquidity shortage in the short-term and, thus, start selling (risk-bearing) assets. This leads, in turn, to mark-to-market losses for other financial corporations with overlapping asset portfolios.

Figure 1 shows an aggregate balance sheet representation of euro area banks. In addition to the already introduced items, a bank i also has a portfolio of loans on the asset side that add up to $\sum_j l_{i,j}$. Index j denotes borrowers in the system that can be other financial institutions, NFCs or households. These loans also appear on the liability side of both other banks and investment funds with $\sum_j l_{j,i}$ representing the sum of the debt of entity i. Similarly, issued tradable securities are represented both on the asset and liability side of banks. Note that our granular databases (large exposure and securities holdings statistics) do not perfectly match aggregate banking sector statistics given that adhere to different reporting standards.

Securities	$\sum_j h_{i,j}$	4.4
Loans	$\sum_j l_{i,j}$	19.3
Other assets		7.5
Total		**31.2**

(a) Assets

Capital and reserves	k_i	2.6
Securities	$\sum_j h_{j,i}$	3.6
Deposits	$\sum_j l_{j,i}$	18.9
Other liabilities		6.1
Total		**31.2**

(b) Liabilities

Figure 1: Banks' modelled balance sheet (numbers given in trillions of euros).
Source: ECB COREP and FINREP data.

Now, we turn to the regulatory constraints for the banking sector implemented in our model. The first constraint regards solvency capital requirements. We distinguish

[2]In our model, a bank is not optimizing its risk exposure amounts (REA), e.g. by deleveraging.

two different solvency thresholds for each bank i: a distress threshold τ_i^{dis} and a lower default threshold τ_i^{def}. First, we define a uniform minimum CET1 capital requirement as $\chi^{\text{MC}} = 4.5\%$. On top of this, we add the bank-specific Pillar 2 requirement χ^{P2R}, determined by the Supervisory Review and Evaluation Process (SREP) and the shortfall of additional Tier-1 and Tier-2 capital $\chi^{\text{sf AT1/T2}}$, which are also part of the CET1 requirements from 2020 onwards[3], hence the default threshold in percentages is:

$$\chi^{\text{def}} = \chi^{\text{MC}} + \chi^{\text{P2R}} + \chi^{\text{sf AT1/T2}}$$

For the distress threshold, we also take into account the combined buffer requirement (CBR), which is the sum of the uniform capital conservation buffer (CCoB), the countercyclical capital buffer (CCyB) and the maximum of the structural risk related macroprudential capital requirements: systemic risk buffer (SyRB), buffer for global systemically important institutions (G-SII) and buffer for other systemically important institutions (O-SII):

$$\chi^{\text{CBR}} = \chi^{\text{CCOB}} + \chi^{\text{CCyB}} + \max\{\chi^{\text{SyRB}}, \chi^{\text{O-SII}}, \chi^{\text{G-SII}}\}.$$

Hence, the distress threshold is given by

$$\chi^{\text{dis}} = \chi^{\text{def}} + \chi^{\text{CBR}}.$$

Finally, we obtain the default and distress thresholds in monetary units for each bank i by multiplying the relevant capital requirements by the total risk exposure amount (REA):

$$\tau_i^{\text{def}} = \chi^{\text{def}} \cdot \text{REA}_i, \quad \tau_i^{\text{dis}} = \chi^{\text{dis}} \cdot \text{REA}_i \,. \tag{1}$$

Additionally, we also consider the prudential liquidity regulation, the goal of which is to ensure that banks are liquid enough at all times. We take into account the LCR constraint, which promotes the short-term resilience to liquidity shocks. It does so by requiring banks to hold an adequate stock of unencumbered High Quality Liquid Assets (HQLA) to meet their liquidity needs for a 30-calendar-day liquidity stress scenario. We denote this as

$$\text{LCR}_i = \frac{c_i^{\text{HQLA}}}{c_i^{\text{out 30}}} \geq 1, \tag{2}$$

where c_i^{HQLA} denotes the stock of HQLA and $c_i^{\text{out 30}}$ is the net cash outflows over the next 30 calendar days under a stress scenario, and thus becomes the value of liquidity requirements.[4]

While minimum capital requirements must always be satisfied, it is accepted for the LCR to fall below 100% during crisis times (Basel Committee on Banking Supervision, 2020), although it may entail additional supervisory activities. To be on the safe side and keep consistency with the regulation, we use the parameter $\lambda^{\text{LCR}} = 1$, which determines

[3]For more information see the 2020 SSM SREP Methodology Booklet.

[4]The net cash outflows is computed as the difference between expected stressed outflows and stressed inflows, both being obtained from the application of stress rates. The value of outflows is not updated though in the model. Also, we use reported HQLA but without an application of central bank haircuts as applied in monetary operations.

the portion of LCR that banks actually try to maintain during stress periods, i.e. they make sure that $c_i^{\text{HQLA}} \geq \lambda^{\text{LCR}} \cdot c_i^{\text{out 30}}$. We will assume that banks make adjustments to restore the full LCR constraint only at a lower frequency. Based on this, we can immediately write the liquidity distress threshold as

$$\tau_i^{c,\text{dis}} = \lambda^{\text{LCR}} \cdot c_i^{\text{out 30}}. \tag{3}$$

The liquidity default threshold is $\tau_i^{c,\text{def}} = 0$. Throughout the paper, we will denote the pool of HQLA as cash-equivalent liquidity for bank i as c_i. This assumes that banks have access to central bank repo financing and that they are able to exchange their liquid assets into cash within a short time horizon.

Importantly, we do not model any policy response, such as capital relief measures given our exogenous shock scenario, as we want to measure the potential scenario impact without supervisory actions.

Overall, we can represent the state of each bank in a liquidity-solvency space as illustrated in Figure 2.

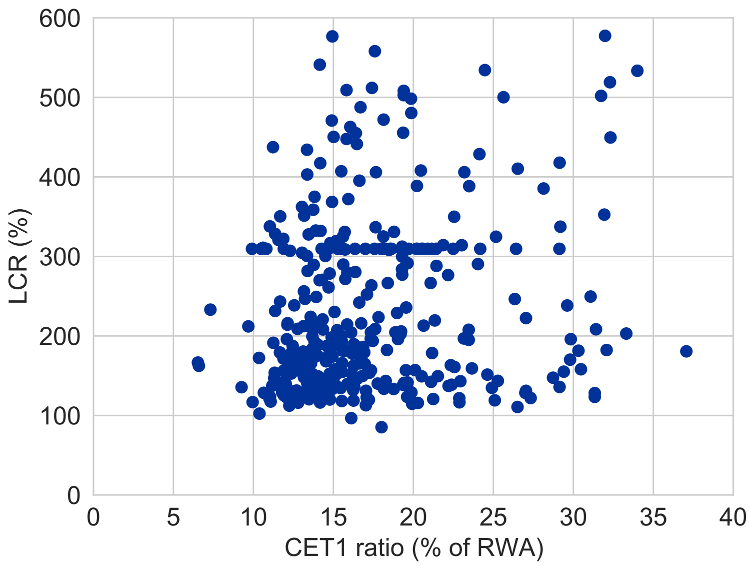

Figure 2: Liquidity-solvency relationship for all banks in the system (LCR in percent and CET1 as a percentage of RWA).
Source: COREP data and authors' calculations.

2.2 Investment funds sector

In contrast to the banking sector, there is no clear consensus on the contribution of the investment fund sector to financial instability. However, the funds' recent expansion, in conjunction with evidence revealing feedback loops in market-based finance, renders the joint analysis of banks and investment funds within one consistent framework important. Moreover, although investment funds are mainly domiciled in international financial centers and a few large jurisdictions, their widely diversified portfolios in geographical terms imply that their actions could significantly impact countries where domestic non-banks represent only a small share of the market.

Part of the literature analyses the investment fund sector individually. For instance, Baranova, Douglas, and Silvestri (2019) focus on modelling amplification dynamics arising from fire-sales in the corporate bond market. In particular, they assess the degree to which redemptions by open-end investment funds, following an initial drop in asset prices, may have a destabilising impact on market functioning. They find that the magnitude of amplification depends on the origin of the initial price shock, as well as the degree to which a given type of shock affects other asset classes and different investor types. Moreover, the redemptions from open-end corporate bond funds accompanied by regulatory constraints could lead to material drops in the asset prices. Bouveret (2017) documents liquidity stress tests for investment funds performed in various IMF FSAP studies. It discusses the links between the banking and investment fund sectors, as well as a proposal to link both liquidity stress tests. More recently, ESMA (2019b) went further in defining stress testing possibilities for investment funds in response to a redemption shock. They review the various methods available and test a large sample of UCITS-regulated bond funds. Goldstein, Jiang, and Ng (2017) also explore flow patterns in corporate bond funds. They show that outflows are more sensitive to poor performance than inflows to good performance, especially under conditions of high market illiquidity. These results imply the fragility in the corporate bond market. Fricke and Wilke (2020) quantifies vulnerabilities within the fund sector and spillover effects for the wider financial system. The authors show that German funds' fire sales in response to a pronounced decline in stock and bond market prices might trigger sizeable second round losses, which propagate through the German fund sector via funds' direct (funds holding each others' fund shares) and indirect (funds holding the same assets) interconnections.

We focus our analysis on open-ended investment funds, which may pose a large threat to the stability of the financial system due to significant liquidity and maturity transformation that they undertake (Gourdel, Maqui, and Sydow, 2019). Indeed, investment funds are funded by shares that are purchased by investors and that can be redeemed (sold back to funds) by investors upon request, making fund shares very liquid. On the other hand, funds invest in assets of longer maturities and lower liquidity such as corporate bonds. Investment funds are exposed to different risk factors, depending on their type: maturity and liquidity mismatch, concentration, leverage. In spite of individual differences in investment strategies and, thus, exposure to different risk factors, investment funds are in general interconnected with one another and with the rest of the financial system, on both sides of their balance sheet. Therefore, faced with market stress, they can transmit shocks to the financial system and generate feedback loops. We model the following transmission channels:

- *The asset liquidation channel*: funds are forced to liquidate securities when they face a significant redemption shock.
- *The direct exposure channel of fund shares*: counterparties holding fund shares would be affected by a decrease in Net Asset Value (NAV).
- *The direct exposure channel of loans*: banks that finance funds would suffer losses in

case of a fund default.[5]

Figure 3 shows the aggregated balance sheet of euro area investment funds that are covered by our model. The proportions of the different items on the balance sheet reflect the fact that investment funds are for the vast majority equity financed vehicles. Therefore, losses on funds' portfolios are directly transmitted to investors through mark-to-market accounting and do not lead to defaults on fixed commitments (i.e. debt liabilities). In fact, investment funds that do not employ leverage cannot really default. When they do in our model, the criterion used is that a fund's NAV reaches zero (Chrétien et al., 2020). We synthetically define the total net assets (TNA) of a fund i as a capital-like measure resembling the difference of financial assets and liabilities to banks:

$$k_i = \sum_j h^{\text{trd}}_{i,j} + \sum_j h^{\text{red}}_{i,j} + c_i - \sum_j l_{j,i}, \tag{4}$$

where the first two terms are the fund's tradable and redeemable assets, the third term is cash; from these, we deduct the total of all liabilities to banks. In this sense, we are able to identify, in theory, funds that are in solvency default, i.e. using a default threshold $\tau^{\text{def}}_i = 0$:

$$k_i < 0 = \tau^{\text{def}}_i. \tag{5}$$

The liquidity default threshold is similarly given by $\tau^{c,\text{def}}_i = 0$, leading to the liquidity default condition:

$$c_i < 0 = \tau^{c,\text{def}}_i, \tag{6}$$

where c_i denotes cash of investment funds. Note that this difference compared to banks does not cause ambiguity because funds with cash shortage will sell their tradable assets or redeem other fund shares to obtain liquidity. Also, note that we did not define distress thresholds for the investment fund sector due to a lack of corresponding regulation. Furthermore, since funds should be able to convert their assets into cash, a liquidity-induced default would not happen to them in our model – only if they have sold all of their assets but this is clearly preceded by the solvency default of funds as can be seen from equation (4) and the solvency default condition $k_i < 0$. Finally, as can be seen in Figure 3, fund defaults appear very unlikely as the value of their assets is far higher than the loans they received.

Under UCITS III, leverage can be used for investment purposes, with no need to match specific assets, but it is limited.[6] The Alternative Investment Fund Managers (AIFM) directive on the other hand does not include leverage limits, but corresponding funds are usually only moderately leveraged, with the exception of hedge funds.[7] Therefore, in-

[5]Funds are mainly equity-financed and do not borrow large amounts from banks. Hence, fund defaults are a very rare phenomenon.

[6]They can borrow up to 10% of their net asset value (NAV) on a temporary basis (financial leverage). While it is not allowed to short stocks, the same can be achieved using derivatives, up to 100% NAV (synthetic leverage). However, this form of leverage is also constrained directly or indirectly under the UCITS Directive.

[7]For more details, see ESMA (2019a).

Deposits and claims		0.9
Securities	$\sum_j h_{i,j}$	12
Non-financial assets		0.4
Remaining assets		0.8
Total		**14.1**

(a) Assets

Investment funds shares	k_i	12.9
Loans	$\sum_j l_{j,i}$	0.5
Remaining liabilities		0.7
Total		**14.1**

(b) Liabilities

Figure 3: Investment fund's modelled balance sheet (in trillions of euros).
Securities are a combination of equities, debt securities and fund shares. Cash is included in the 'Deposits and claims' category.
Source: ECB Investment Fund Statistics, Lipper IM and authors' calculations.

vestment funds modelled in our framework exhibit a low leverage.

Open-end funds are governed by dynamics that require a number of assumptions, in particular regarding the timing and volume of redemptions. As detailed afterwards, redemptions can either be endogenous, i.e. from other explicitly modelled financial institutions, or exogenous. In both cases, we need to re-concile these redemptions with the data on modelled sectors and entities to ensure accounting consistency. More precisely, to close the system or consider redemptions only from modelled agents, then net redemptions/sales from funds must match a change in holdings by investors. To meet redemptions, managers will use cash buffers.

3 Data

The specificity and uniqueness of our study is that we construct a database using several granular data sets for individual banks and funds. The banking sector includes significant and less significant banks domiciled in the euro area while the investment fund sector consists of open-ended investment funds domiciled in the euro area.

3.1 Banking sector

To model the banking sector, we exploit several data sources, which provide very granular information on exposures of banks towards individual entities in the system covering all the sectors of the economy. A bank's portfolio is firstly built by relying on the COREP (Common Reporting) dataset based on the European Banking Authority's supervisory reporting framework, which includes data on bilateral exposures from the Large Exposures framework[8]. The residual fraction of banks' loan portfolios, that do not satisfy the reporting thresholds defined in the Large Exposures framework, is included in the form of resid-

[8]The Large Exposures reporting framework provides counterparty level information on bilateral exposure euro area credit institution towards the real economy. Introduce in 2014, the framework requires institutions to report all exposures exceeding either 300M€ or 10% of eligible capital towards either a single counterparty or group of connected entities. For each exposure, both the original amount, as well as a net amount after credit risk mitigation and exemptions are reported. The latter is used as a proxy for the loss given default (LGD) of the exposure (see Appendix B.1 for more details)

ual exposures nodes constructed at a country-sector level. More specifically, we compute the exposures towards such nodes as the difference between aggregate exposures from FINREP (Financial Reporting) and the sum of reported granular exposures. Additionally, we employ Securities Holdings Statistics by banking group (SHS-G[9]) to enrich the portfolio information of banks, bringing in information on securities held by credit institutions at ISIN-level.

From COREP, we also retrieve information on the banks' capitalisation, which we complement by data on regulatory constraints as well as capital and liquidity buffers provided by national authorities across the ESCB in order to define bank-level default- and distress-thresholds, as outlined in equation 1.

The combination of the aforementioned data sources, as described in Appendix B, allows to reconstruct the network of bank to bank exposures as well as bank to non-banks exposures: this defines the basic system of entities, through which a macro-financial shock propagates on the basis of credit, liquidity, and market risk channels from one entity to another. At this stage, our model does not consider hedging positions in derivatives of banks and funds, which might be a relevant factor for future analysis.

Figures 4 and 20 provide a broad overview of the dataset,showing distributions of our granular loan database were both the lender and the borrower are known at entity level.

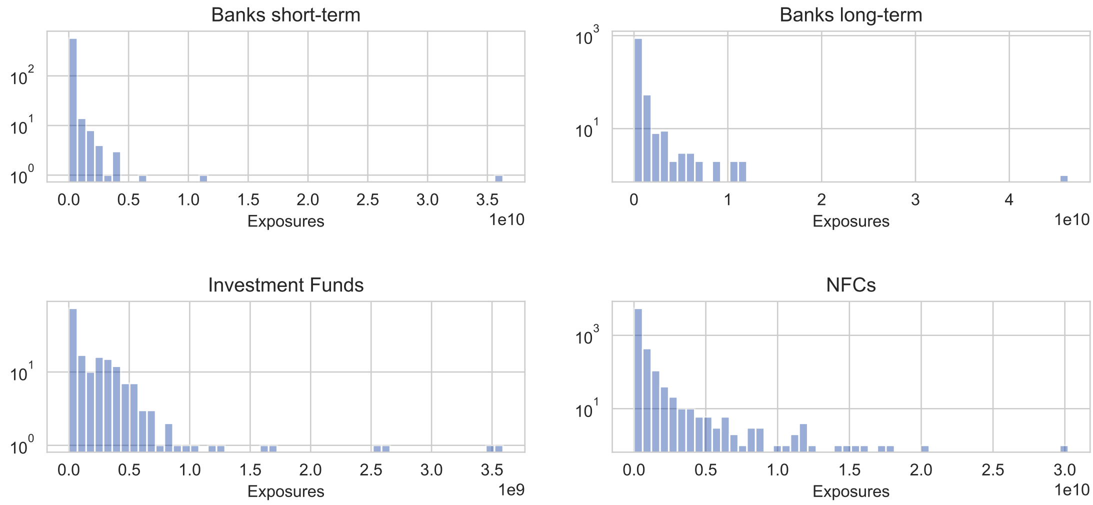

Figure 4: Histograms of banks' granular exposures towards other banks, investment funds and NFCs as of Q4 2019 (in EUR).
Short-term bank exposures have a residual maturity of less than 30 days; long-term bank exposures cover the remainder.
Source: Authors' calculations.

[9]Security Holding Statistics by Banking Group covers all significant banking groups under direct ECB (about 120 groups) supervision, including holdings of all subsidiaries and branches within and outside euro area. Each institution reports granularly its portfolio holdings at individual ISIN level, including market value and nominal value held, and whether the amounts are held to maturity or in the trading book.

3.2 Investment fund sector

The investment funds sector can be generally broken down into six main categories: equity, bond, mixed, money-market, real estate and hedge funds. Due to data constraints, our model includes three fund categories: equity, bond and mixed funds. These funds represent more than 80% of the assets under management of the euro area. Equity funds are the biggest category in our sample, covering slightly more than half of the assets under management (AUM); bond funds represent one third of the sample's AUM and mixed funds are the smallest subgroup with only about 13% of total AUM. While benefiting from the extensive supervisory data for the banking sector, we have to use available market data for the investment fund sector on securities portfolios, cash holdings, net asset value and flows. At the fund level, we obtain detailed balance sheet information and portfolio composition using private vendor data from Lipper IM by Refinitiv. Moreover, we evaluate the coverage of Lipper IM data using aggregated fund sector statistics from the ECB databases Quarterly Sectoral Accounts (QSA) and Investment Funds Balance Sheet Statistics (IVF). Compared to the country-level aggregates, the coverage by Lipper IM varies significantly across countries, as Table 2 and Appendix C report.[10]

As the names suggest, the equity and bonds funds in our dataset invest almost solely in equity and bonds respectively, whereas the mixed funds hold a mixture of the two asset types (see Figure 5). Since market liquidity of fund shares is high, investors can easily buy or redeem fund shares. Therefore, funds flows are very volatile. Figure 6 shows the evolution of flows across funds broken down by fund category. We notice a fat tail in the flows: the median is always very close to 0% and the range between the 25th and 75th percentiles is narrow but the 10th/90th percentiles exceed minus/plus 5% of NAV and the 5th/95th percentiles often exceed minus/plus 10% of NAV.

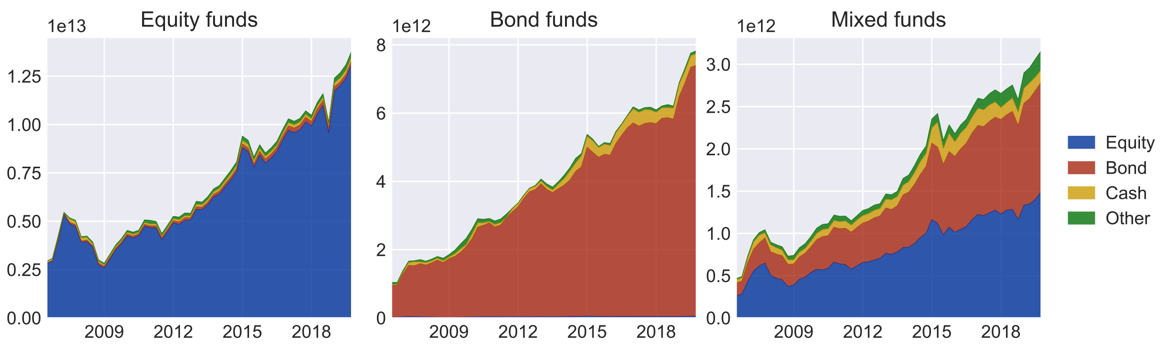

Figure 5: Equity and bond holdings by fund type (in euros with scale multiplier on the top left of the charts).
The holdings are summed over all funds in our sample, without restriction of domicile.
Source: Lipper IM and authors' calculations.

[10] When necessary for modelling purposes, we aggregate all assets held by the fund sector in each country, for which information at the fund level is not available, and attribute this outcome to one aggregate fund per country-fund class.

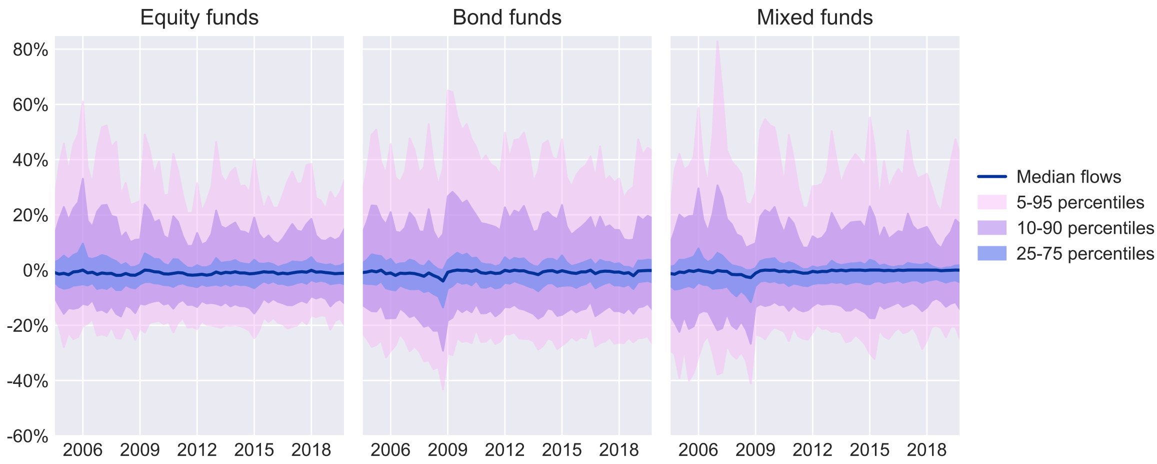

Figure 6: Distribution of monthly fund-level flows as a percentage of assets.
Source: Lipper IM and authors' calculations.

3.3 Identification of entities and group consolidation

In order to generate with the available data a network of linked nodes, the first step of our data management process requires the preparation of auxiliary databases. Such data sources are used to gather all the entity identification codes, which allow to uniquely recognise each counterparty reported by funds and banks.

We rely on three databases for this step: GLEIF (Global Legal Entity Identifier Foundation), RIAD (Register of Institutions and Affiliates Data) and CSDB (Centralised Securities Database). These databases provide the LEI (Legal Entity Identifier), RIAD and ISIN codes, which allow us to identify each entity and the securities it issued.

We adopt a process of recursive data matching based on different keys (LEI, RIAD and ISIN codes, entity name, etc.) that returns cleaned data for all the reported entities observed. Afterwards, we are able to define a unique identifier for the network that collects all the same entities under the same node.

Finally, through the GLEIF and RIAD data, we are able to assign each entity to its own group using information about the ultimate parent. This process is core as we aim at consolidating exposures by groups of entities. The only caveat in this last step is that it depends on the sectors of the firms composing one group. If we observe companies belonging to more than one sector within the same group, we will slice it into as many subgroups as the number of detected sectors. The reasons for this separation are the behavioural assumptions and dynamics embedded in our model for each sector. This procedure allows us to consolidate exposures only towards entities of the same group and sector.

3.4 Evidence on the interconnection between sectors

The banking and investment fund sectors are closely connected in many ways: banks lend to funds while funds deposit their cash at banks - thus, also providing banks with liquidity; banks and funds hold each other's securities; banks and funds hold similar portfolios. These links may serve as contagion channels. Empirical findings presented in this section aim at pointing out which channels will play a bigger role in our analysis.

Figure 7 suggests that banks' exposure through lending to funds is small. This is not surprising since the majority of funds in our sample are UCITS that have a 10% limit on borrowing. Lipper IM provides information only on the total amount of cash held by fund. Therefore, we assume that funds keep cash at their custodian banks, as known from Lipper IM. This assumption might significantly overstate the degree of interlinkages between custodian banks and investment funds via cash holdings, including potential spillovers from funds to custodian banks.[11] However, in the absence of more granular data sources, this assumption is required in order to make the model internally consistent. Moreover, as funds have a limited amount of cash, they will tend to sell large amounts of their securities. This figure also shows that banks are mostly subject to credit risk losses, especially via exposures that are not available at granular level. Loans to individual firms represent only 21% of banks' total assets. Hence, for loans to households, NFCs, FCs and residual sectors (e.g. governments) we use information aggregated at country-sector level, as described in Section 3.1. Similarly, information on granular securities holdings in banks' balance sheets is also limited, covering 7% of total assets. The banks' missing assets in Figure 7 are in line with the share of 'Other assets' (around 23%) shown in Figure 1. This means that our constructed dataset, combining granular and aggregate country-sector data, can replicate the overall EA statistics.

Figure 8 shows that, within the group of banks and funds, funds hold much more individual securities in their portfolios than banks. This will help us explain the fire sale losses in the different simulation exercises.

Figure 9 shows pairwise cosine similarities of the 100 largest banks and funds ordered by their size of tradable and redeemable portfolios. This analysis suggests that there is no systematic sector-specific difference in the portfolios of entities in these two sectors. In fact, there are banks and funds with very similar portfolios. This suggests that fire sale losses will matter a lot in a joint model of banks and investment funds.

In Appendix D, we show network charts representing our dataset aggregated at sector- and at entity-level. A visual inspection of the two different aggregation levels can give an indication about the details that are not captured in a model that omits granular data, such as representative agent models.

3.5 The macro-financial scenario

In this paper, we analyse the impact of the economic shock caused by the COVID-19 outbreak. This involves two comprehensive scenarios of different severity as set out in the June 2020 ECB staff macroeconomic projections, which were also used in the vulnerability analysis of banks directly supervised within the Single Supervisory Mechanism.[12]

The first scenario foresees a real gross domestic product (GDP) decline of 8.7% in 2020. The second scenario is more severe and projects a real GDP decline of 12.6% in 2020. We report results only for the more severe scenario.

Each scenario comprises almost 20 different macro-financial variables that are linked

[11] For example, equity, bond and mixed funds domiciled in Luxembourg hold only around half of their cash at their custodian bank.

[12] For more information, see: `https://www.bankingsupervision.europa.eu/press/pr/date/2020/html/ssm.pr200728~7df9502348.en.html`

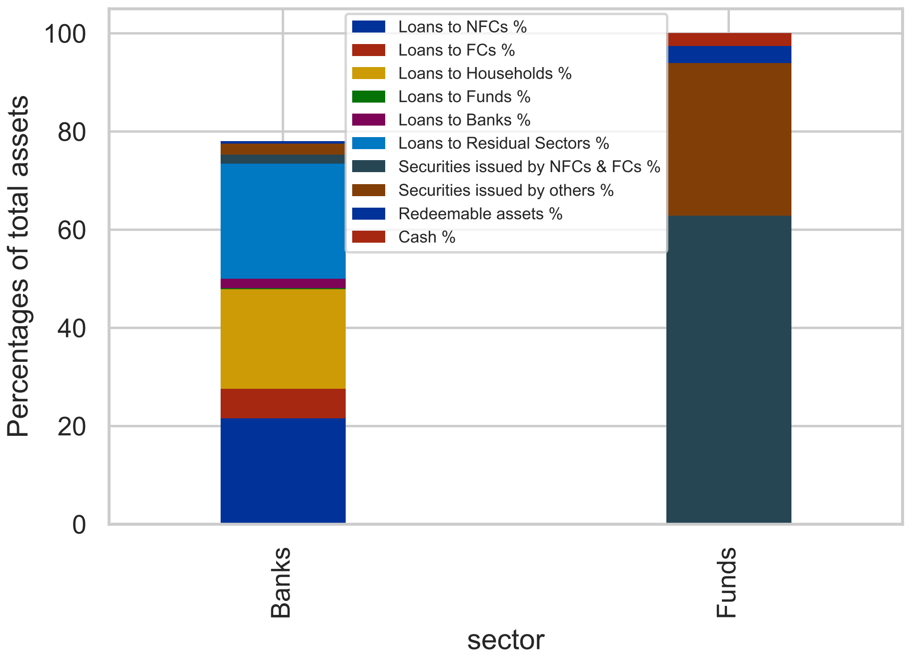

Figure 7: Sector-level coverage of assets stressed in our model.
This figure does not show a representative balance sheet of the EA banking and investment fund sector but rather the share of asset categories that we are able to shock in our model. Thus, for banks we miss eligible liquid assets (those are replaced by HQLA but not stressed) and securities, which are not captured at ISIN-level in the database. Redeemable assets are fund shares.
Source: FINREP, COREP, Large Exposure reporting, SHS-G, Lipper IM and authors' calculations.

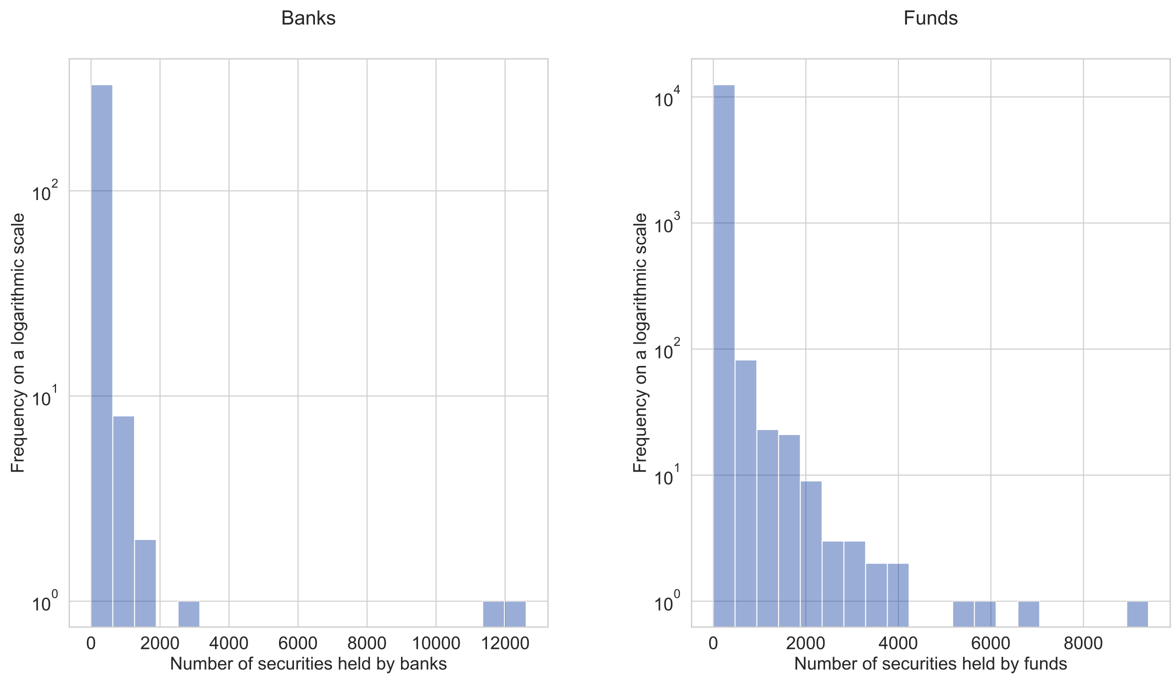

Figure 8: Distribution of number of securities held by banks and funds.
Source: SHS-G, Lipper IM and authors' calculations.

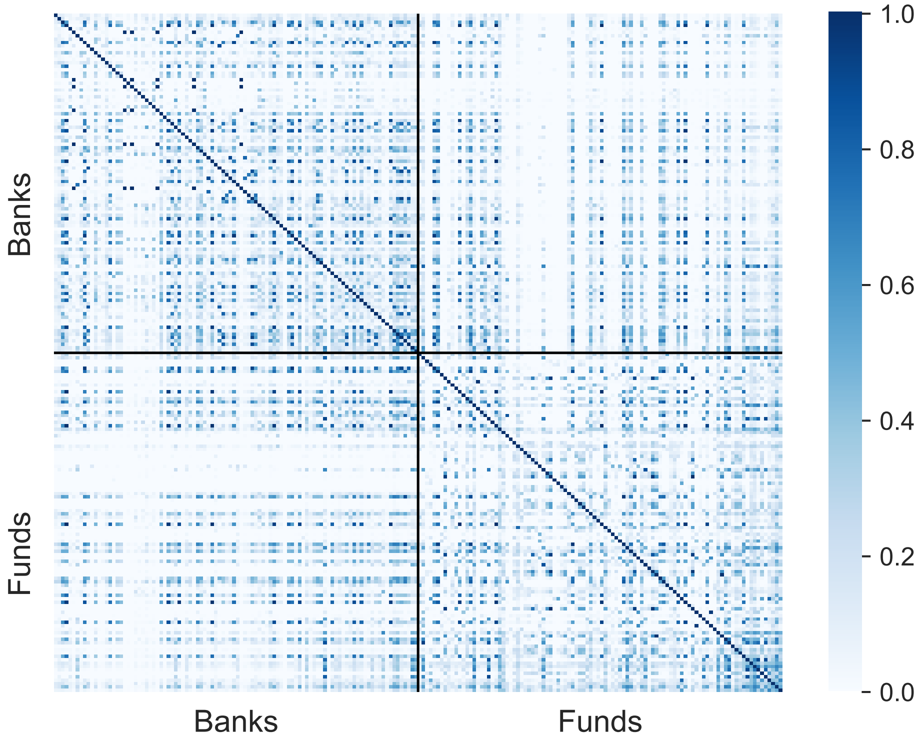

Figure 9: Pairwise cosine similarities of portfolios of the 100 largest banks and 100 largest funds.
Cosine similarity is a measure of similarity between two non-zero vectors of an inner product space. We account for both tradable and redeemable (investment fund shares) portfolios. One point in the heatmap shows the portfolio similarity of two respective entities, the value of which can be read from the colour bar on the right hand-side.
Source: Authors' calculations.

via ECB satellite models into bank- or fund-specific risk drivers.[13] Each scenario materialises in Q1 2020, and we assume that the financial system reacts in the following quarter in line with our intra-quarter model dynamics described in the following sections.

4 Initial exogenous shocks

A key contribution of our work compared to previous literature is that we translate the macro-financial scenario into a series of scenario-consistent initial shocks that unfold via different risk channels, instead of focusing on a single channel that is unlikely to exist in isolation in reality. In particular, we implement firm default as well as market price and redemption risk channels, as described in Sections 4.1, 4.2, 4.3, respectively.

4.1 Defaults of corporations

The first exogenous shock to banks and funds comes from *defaults* of non-financial and financial corporations[14] (for easier understanding we refer to them as NFCs throughout

[13]For more information, see: https://www.ecb.europa.eu/pub/projections/html/ecb.projections202006_eurosystemstaff~7628a8cf43.en.html
[14]These financial corporations are neither banks nor investment funds.

the paper). We restrict this shock to NFC defaults because we assume that the initial shock comes from outside the financial system of banks and investment funds. We randomly sample a vector of default events using individual firm default probabilities and firm-by-firm pairwise asset correlations as described in Appendix E.4. This methodology relies on the simulation of one-quarter ahead geometric Brownian motion paths for the market values of all entities' assets, which include the correlations in their noise component. Depending on each PD we can define a threshold for each path such that the lower bound satisfies the number of defaults predicted by the one year ahead PDs. We use the macro-financial scenario in order to infer deviations from the baseline values of the geometric Brownian motion parameters. For more details, see Section 5.1 as well. This is the only stochastic initial shock applied in the framework.

4.2 Stock market shock

The second exogenous shock is a market shock designed as a change in stock prices. This shock comes directly from the macro-financial scenario and is a key driver of the overall modelled economic downturn. For any company a, the new stock price $P_{a,t+1}$ is computed such that a change in the aggregate over all companies equals the change in the stock price index prescribed by the scenario. In more detail, suppose that the stock price index in a given country changes by a ratio γ between t and $t+1$. For simplicity we assume that the stock price index is a price-weighted average of stock prices of all listed companies in the country. We denote by I the set of all corporations, whose shares issued are covered by our data. Thus, the value of the index is proportional to the total market capitalisation of these corporations, given by $\sum_{a\in I} V_{a,t}$, where $V_{a,t}$ is the market value of a company a at t. Therefore, we have

$$\gamma = \frac{\sum_{a\in I} V_{a,t+1}}{\sum_{a\in I} V_{a,t}} . \tag{7}$$

The market capitalisation of a company is given as the number of shares multiplied by the stock price. This means that any change of the stock price of a company a impacts proportionally the value of V_a. To take into account a given change in the index, a simple solution is then to set $p_{a,t+1} = \gamma \cdot p_{a,t}$ for all $a \in I$.

4.3 Redemption shock

The third and last initial shock in our model represent exogenous redemptions on investment funds' portfolios, similar to Baranova et al. (2017) and ESMA (2019b). A key advantage of our framework is that we use fund-specific characteristics to determine fund flows. Crucially, we avoid the caveat of aggregating, as done in Aikman et al. (2019), where a global redemption shock of 4.2% is applied. Our granular approach is more suitable to capture within-sector heterogeneity and reproduce distributions of flows with a large inter-fund variance at each quarter, as observed empirically (see Figure 6).

We use the Bayesian model averaging (BMA) approach of Gourdel, Maqui, and Sydow (2019) and compute the impact of the macro-financial scenario on the funds' valuation in our framework. Then, we obtain scenario-conditional redemptions by focusing on valuation-adjusted flows, initially computed at the country level, which we combine with the holdings of every fund to get fund-level flows. These exogenous redemptions are simply deducted from the cash amounts of funds when applying the shock: $c_{i,t+1} = c_{i,t} - r_{i,t}$, where $r_{i,t}$ is the fund-level exogenous redemption. In particular, we assume that exogenous redemptions are coming from the rest of the world, i.e. from those agents, which do not redeem endogenously in the model. Therefore, given a fund i, we deduct bank and fund investors' holdings (which are available at granular level in our system) $\sum_j h^{\text{red}}_{j,i}$ from the capital of the fund and the amount of ouflow $r_{i,t}$ is proportional to the amount held by external investors:

$$r_{i,t} = \rho_i \cdot \left(\sum_j h^{\text{trd}}_{i,j} + \sum_j h^{\text{red}}_{i,j} + c_i - \sum_j l_{j,i} - \sum_j h^{\text{red}}_{j,i} \right), \tag{8}$$

where ρ_i is a result of the BMA regressions.

5 Model dynamics

At the core of our model, we operate an iterative evolution of the financial system that takes into account the quarterly path of the macro-financial scenario. The state of the financial system at the beginning of our simulation is altered by the initial scenario-induced exogenous shocks. This leads to firm defaults and market price and redemption shocks bring the portfolios of banks and funds into an unsustainable situation. Therefore, financial agents react to meet their regulatory and internal targets. Moreover, in our model this triggers higher frequency, intra-quarter dynamics that lie in the interaction of solvency, liquidity and market risk. The agents' behaviour affects their peers, which allows for either absorption or amplification of the initial imbalance of the system. In particular, amplifications may arise from defaults, liquidations or fire sales among financial institutions. Importantly, we also take into account that, given the initial shocks, banks' net operating income declines in the first quarter in line with the projections laid out in the 2020 vulnerability analysis of the Single Supervisory Mechanism under the Covid-19 severe scenario.[15] Figure 29 in Appendix F shows the importance of the income channel. To compute net operating income we consider interest income, interest expenses, expenses on share capital repayable on demand, dividend income, fee and commission income, fee and commission expenses, other operating income, other operating expenses and administrative expenses while excluding credit risk provisions. To calibrate the income channel, we use the average income projection from the VA exercise and distribute it across banks using average historical income values over the last three years.[16]

Figure 10 illustrates the ordering of events in the model. The first block contains the immediate reaction to the application of exogenous shocks from Section 4. The second block contains a simulation of intra-quarter endogenous reactions of the agents.

Our simulation framework combines six highly interrelated model dynamics. All these six channels of risk propagation are tailored to reflect realistic assumptions about the spread of financial contagion and are, therefore, increasing the likelihood of tail events.

[15]For more information, see: https://www.bankingsupervision.europa.eu/press/pr/date/2020/html/ssm.pr200728~7df9502348.en.html

[16]The average over the last three years reduces the idiosyncratic risk that a single bank gets punished with a low income reporting figure at our data cut-off.

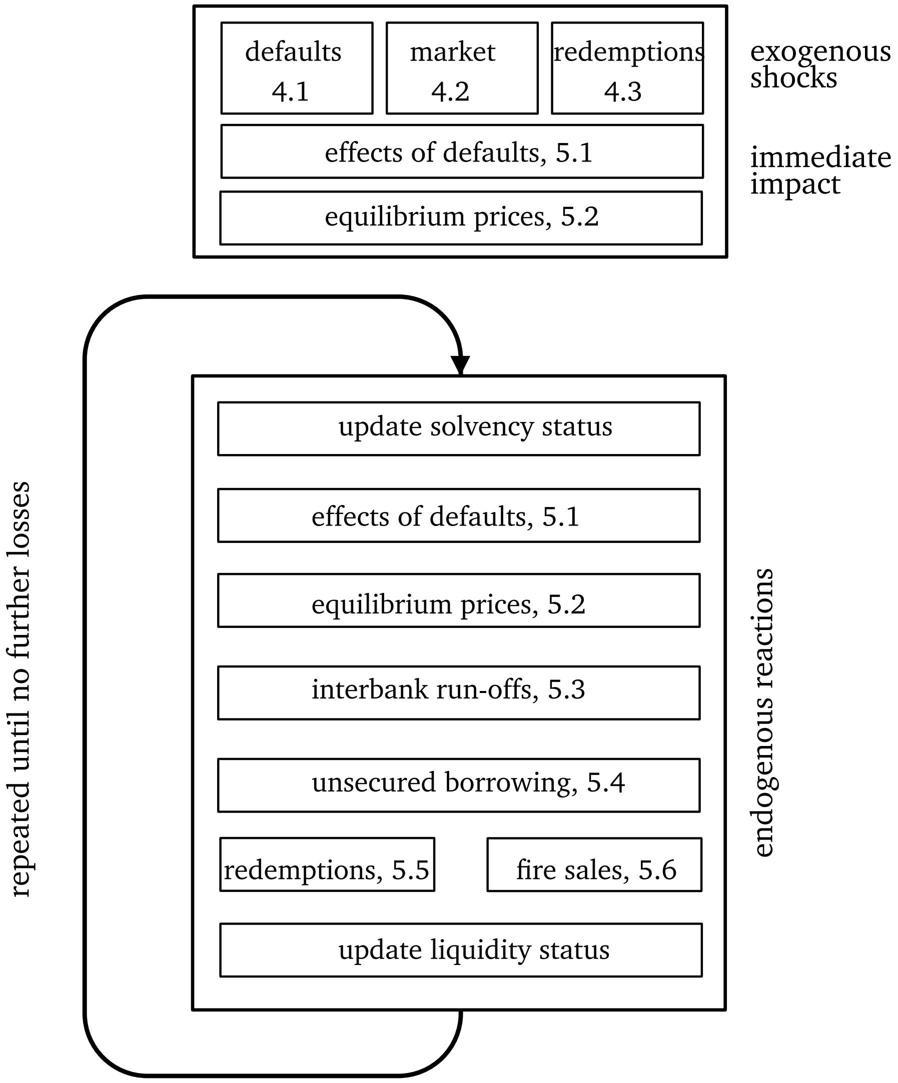

Figure 10: Organisation of simulation steps.
Source: ECB.

First, the exogenous stochastic defaults of NFCs lead to loan losses in bank portfolios and, by the depreciation of issued securities, to market losses for both banks and funds, as detailed in Subsection 5.1. Second, any change in market prices translates into a change in the valuation of investment funds, as described in Subsection 5.2. In consequence, banks start withdrawing short-term funding from each other depending on the distance-to-default of counterparties and their own liquidity needs, as shown in Subsection 5.3. On the other hand, banks with a liquidity surplus continue providing loans to other solvent banks in need of liquidity, as set out in Subsection 5.4. If this is not enough, the last resort for banks to obtain liquidity is selling non-eligible illiquid assets at discounted prices and redeem investment fund shares (funds can sell all kinds of assets), as described in Subsections 5.5 and 5.6. The latter mechanics lead to further price depreciation in marketable securities and, consequently, in redeemable securities as well.

5.1 Effects of defaults

The main driver of initial losses in the system are default events. We sample them at granular level for all available NFCs in the banks' loan portfolios following the initial exogenous shocks, as described in subsection 4.1. These sampled defaults are augmented with exposures, which are only available at aggregate (country-sector) level. Moreover, we model further endogenous bank and fund defaults that come from contagion within the financial system. We can quantify two types of losses following endogenous defaults: credit losses on the exposures of banks and market losses for both banks and funds on their holdings of securities issued by defaulted entities.

Suppose that previous defaults have already been accounted for. Then, at each further step, we consider a boolean vector $\vec{\theta}$ of new endogenous defaults such that, for every granular institution a in our sample,

$$\theta_a = \begin{cases} 1 & \text{if } a \text{ defaults between } t \text{ and } t+1 \\ 0 & \text{otherwise} \end{cases} . \tag{9}$$

We suppose that when an institution defaults it does so on both loans and issued securities (equities and bonds), subject to a certain LGD ratio. Moreover, the prices of equities and bonds issued by the defaulting institutions go to zero, meaning that from the point of view of a holder i (either a bank or a fund), if a defaults between t and $t+1$ then $h^{\text{trd}}_{i,a,t+1} = 0$, this loss is measured in market losses $m_{i,t}$ (see equation (17) below). We then obtain simulated loan losses $\hat{\ell}$ for every holding entity i,

$$\hat{\ell}_{i,t} = \sum_a \left(\theta_a \cdot \text{LGD}_{i,a} \cdot l_{i,a,t}\right) + \sum_a \left(\text{PD}_{i,a} \cdot \text{LGD}_{i,a} \cdot l_{i,a,t}\right), \tag{10}$$

where again $l_{i,a,t}$ is a loan exposure from bank i to a counterparty a at time t. The first sum accounts for granular exposures, the second sum accounts for aggregate exposures. For granular exposures, $\text{LGD}_{i,a}$ is the exposure-level loss-given default defined as the net exposure of bank i towards financial entity a after credit risk mitigation over the original gross exposure, for details see Appendix B.1. If we consider an aggregate country-sector exposure, we cannot sample individual default events. We rather calculate loan losses on the aggregate exposure for each bank using average PD estimates and LGDs that only depend on the country-sector composition as Appendix E.5 describes in detail. The above loan losses are not immediately absorbed by capital since banks need to maintain a stock of provisions[17] for expected losses: $\mathrm{ProvStock_{i,t}}$ for the existing non-defaulted portfolio of bank i (EBA, 2016). For the sake of simplicity, we only account these provisions at bank level. The stock of provisions is amortized by provisions held for newly defaulted loans:

$$\mathrm{ProvStock_{i,t+1}} = (1-\alpha_{\mathrm{i}}) \cdot \mathrm{ProvStock_{i,t}}, \tag{11}$$

where α_i is the ratio of the total defaulted exposure (both granular and aggregate) to the original gross exposure amount of bank i:

$$\alpha_i = \frac{\sum_a \left(\theta_a \cdot l_{i,a,t}\right) + \sum_a \left(\mathrm{PD}_{i,a} \cdot l_{i,a,t}\right)}{\sum_a l_{i,a,t}}. \tag{12}$$

Hence, since the ratio α_i of provisions is released after each step of registering new defaults, impairment losses on the loan portfolio becomes

$$\ell_{i,t} = \hat{\ell}_{i,t} - \alpha_i \mathrm{ProvStock_{i,t}}. \tag{13}$$

On top of credit losses, a re-adjustment of fund prices happens to reflect the market losses $\mathbf{m}$, as we describe in subsection 5.2. This implies a decrease in the price of redeemable assets, generating further losses denoted by the vector $\mathbf{m}_t$ (equation (17)), such that the total impact of this step on financial entities is given by $\mathbf{m}_t - \boldsymbol{\ell}_t$.

5.2 Equilibrium of prices with open-end funds

One issue that presents a technical complication in our framework is how the changing prices of tradable asset holdings affect funds' NAV and, thus, the amount that investors can redeem from the funds. The general idea is that given a change in prices of tradable securities, the impact on funds implies an indirect effect on their investors. This comes as a necessary addition to several of the other mechanisms presented below.

Investment funds frequently hold shares of one another, making the problem more complex. For example, if two funds i and j hold each other's shares, then the final value of their respective total net assets (TNA) has to take into account this mutual influence. One way to think about it is that, if the value of i decreases because of a shock on tradable assets, then it affects the portfolio of j. So, the TNA of j decreases, which in turn will impact the portfolio of i, etc. These granular, fund-level mechanics provide a clear value-added when compared to models that use only aggregate data, as, for instance, Aikman et al. (2019).

The framework of Chrétien et al. (2020) models how entities posting new information about their value propagates a shock to portfolios. This sequential approach mimics the process of mutual funds posting their NAV after closure of the market. However, we follow the more conservative approach of Gourdel and Sydow (2021), similar to Fricke and Wilke (2020), by calculating directly the equilibrium position of NAVs, i.e. the limit

[17]We use the stock of provisions for Stage 1 and Stage 2 assets in the model. See Figure 30 in Appendix F for a distribution chart.

to which this dynamic converges. The change in market prices will at once establish new TNAs, denoted as a vector $\mathbf{k}_t \in \mathbb{R}^F$, with F the number of funds. We assume that the TNA is the sum of cash, tradable and redeemable asset holdings, from which bank loans are deducted. Therefore, equation (4) can be written in vector format as

$$\mathbf{k}_t \;=\; \mathbf{H}_t^{\text{trd,f}} \cdot \mathbf{1}_{\text{trd}} + \mathbf{H}_t^{\text{red,f}} \cdot \mathbf{1}_{\text{red}} + \mathbf{c}_t - \mathbf{l}_t, \tag{14}$$

with

- $\mathbf{H}_t^{\text{trd,f}} \in \mathbb{R}^{F\times S}$ the matrix of funds' tradable security holdings, S the number of securities,
- $\mathbf{H}_t^{\text{red,f}} \in \mathbb{R}^{F\times F}$ the matrix of funds' redeemable security holdings,
- $\mathbf{1}_{\text{trd}} = (1,\ldots,1)^\top \in \mathbb{R}^S$ and $\mathbf{1}_{\text{red}} = (1,\ldots,1)^\top \in \mathbb{R}^F$ are column vectors of ones,
- $\mathbf{c}_t \in \mathbb{R}^F$ is the column vector of cash for all funds,
- $\mathbf{k}_t \in \mathbb{R}^F$ is the column vector of capital (TNA) for all funds,
- $\mathbf{l}_t \in \mathbb{R}^F$ is the column vector of bank loans for all funds.

We show in Appendix E.2 the derivation of the solution:

$$\mathbf{k}_t \;=\; \left(\mathbf{I}_F - \mathbf{H}_{t-1}^{\text{red,f}} \cdot \frac{1}{\mathbf{k}_{t-1}^\top}\right)^{-1} \left(\mathbf{H}_{t-1}^{\text{trd,f}} \cdot \mathbf{1}_{\text{trd}} + \mathbf{c}_t - \mathbf{l}_t\right) \;. \tag{15}$$

The existence of the inverse matrix is an immediate consequence of its Neumann series representation. We then use that $\mathbf{p}_t^*/\mathbf{p}_{t-1}^* = \mathbf{k}_t/\mathbf{k}_{t-1}$, where $\mathbf{p}_t^* \in \mathbb{R}^F$ is the column vector of funds' NAVs, to update the value of redeemable securities, such that:

$$\mathbf{H}_t^{\text{red}} = \mathbf{H}_{t-1}^{\text{red}} \cdot \left(\frac{\mathbf{p}_t^*}{\mathbf{p}_{t-1}^*}\right)^\top . \tag{16}$$

Note that these price changes have an impact on the market losses of the entities:

$$m_{i,t} = \sum_{\phi} \left(h_{i,\phi,t} - h_{i,\phi,t-1}\right), \tag{17}$$

which are absorbed by the capital in case of banks (for funds, our capital-like measure is synthetic and endogenous):

$$k_{i,t} = k_{i,t-1} + m_{i,t}. \tag{18}$$

Finally, the credit and market losses diminish capital figures as follows:

$$\mathbf{k}_t = \mathbf{k}_{t-1} - \boldsymbol{\ell}_t + \mathbf{m}_t. \tag{19}$$

5.3 Interbank run-offs

Following initial losses due to credit and market risk, banks may experience run-offs on their short-term liabilities.[18] We model liquidity withdrawals for that part of the banks' liabilities, for which data from the Large Exposure Statistics is available and, thus, the

[18]Funds do not experience run-offs on their short-term liabilities in the current implementation of the model.

counterparty is known. The banks enter this modelling block with liquidity position $\mathbf{c}_t$ defined in Section 2.1, which is adjusted when their short-term interbank exposures change (and therefore other institutions' liabilities).

Let us denote the withdrawal matrix of banks by $\mathbf{W}$. Its elements $w_{i,j}$ denote the withdrawal of short-term assets of bank i from bank j. The amount of liquidity the banks withdraw is then given by $\mathbf{w}^{\text{in}} = \left\{\sum_j w_{i,j}\right\}_i$, while the amount of withdrawn liquidity from banks is $\mathbf{w}^{\text{out}} = \left\{\sum_j w_{j,i}\right\}_i$.

In the first step, defaulted or distressed banks withdraw liquidity from all their counterparties. At the same time, all banks withdraw their short-term deposits from distressed or defaulted banks as a precautionary measure.

Therefore, the initial ($t = 0$) withdrawal of i from j is given by

$$w_{i,j,0} = \begin{cases} l^S_{i,j}, & \text{if either } i \text{ or } j \text{ is defaulted or distressed} \\ 0, & \text{otherwise.} \end{cases} \tag{20}$$

In the second step, we aim at finding the equilibrium of withdrawals by assuming that banks try to close their gap of remaining liquidity needs. The initial liquidity need is given by the difference of the liquidity distress threshold and actual liquidity holdings $\tau^{c,\text{dis}}_{i,t} - c_{i,t}$, which after the initial witdrawals becomes

$$g_{i,t} = \left(\tau^{c,\text{dis}}_{i,t} - c_{i,t} + \sum_j w_{i,j,t} - \sum_j w_{j,i,t}\right)^+, \tag{21}$$

where we take into account withdrawn short-term exposures and outflows to other entities. At this stage, banks have residual short-term assets given by $\text{res}_{i,t} = \sum_j l^S_{i,j} - \sum_j w_{i,j,t}$ and further additional withdrawals will be proportional to the liquidity needs:

$$w^{\text{add}}_{i,j} = \min\left(\frac{g_{i,t}}{\text{res}_{\text{i,t}}}, 1\right)\left(l^S_{i,j} - w_{i,j,t}\right). \tag{22}$$

After one round the withdrawal matrix is updated:

$$w_{i,j,t} = w_{i,j,t-1} + w^{\text{add}}_{i,j} \tag{23}$$

where t only denotes the rounds of iterations. Note that in this way both $g_{i,t}$ and $\text{res}_{\text{i,t}}$ are recalculated in each step and the iteration is assured to converge ($w^{\text{add}}_{i,j} = 0$) by the fact that $\text{res}_{\text{i,t}}$ is strictly decreasing and bounded from below by zero.

Once the iteration has converged, we update the liquidity position and short-term exposures:

$$c_{i,t} = c_{i,t-1} + \sum_j w_{i,j} - \sum_j w_{j,i} \tag{24}$$

$$l^S_{i,j,t} = l^S_{i,j,t-1} - w_{i,j} \tag{25}$$

where $\{w_{i,j}\}$ is the final withdrawal matrix. However, it is possible that banks will have remaining liquidity needs that they attempt to satisfy as described below.

5.4 Unsecured borrowing

As additional mitigating mechanism in case of liquidity needs, we include the possibility for certain banks to access new unsecured borrowing. As an example, Cont, Kotlicki, and Valderrama (2019) suggest that banks can borrow unless they are subject to a recent credit downgrade. However, the amount a bank can borrow is constrained by the bank's distance to a downgrade. In a similar fashion, we consider that all banks that are not in solvency distress at that point of the simulation have access to short-term loans, with a similar constraint based on a distance to distress. Importantly, this funding source would still be accessible to banks that are in liquidity distress but well capitalised. Thus, the amount that a bank i borrows on the interbank market is capped by

$$u_i = \min\left((\tau_i^{c,\text{dis}} - c_i)^+, \beta\left(k_i - \tau_i^{\text{dis}}\right)^+\right) \tag{26}$$

with $\beta \in \mathbb{R}_+$. The term after the comma in equation (26) expresses that banks, which are closer to their solvency distress threshold, can get less liquidity in the interbank market. Moreover, we suppose that any bank i with a liquidity surplus above its cash target is willing to lend, with lending capacity of j

$$v_j = (c_j - \tau_j^{c,\text{dis}})^+. \tag{27}$$

There is no profit maximization at this step.[19] We match entities with a heuristic recursive algorithm described in E.1. Compared to other models that use only aggregate data this is a clear modelling advantage as such mechanics would otherwise be ignored.

5.5 Endogenous redemptions

As the last opportunity to close a remaining liquidity gap, financial institutions can one after another redeem their investment funds holdings and sell assets at a discount. Suppose a bank has a liquidity gap amounting to

$$g_i = (\tau_i^{c,\text{dis}} - c_i)^+. \tag{28}$$

As for funds, there is no positive value for a distress threshold related to cash. Regarding the cash target, we assume for each fund i a target of cash holdings c_i^{TG} determined after exogenous redemptions: $c_i^{\text{TG}} = \frac{c_{i,0}}{k_{i,0}} \cdot k_{i,t}$, i.e. the cash ratio of a fund is assumed to be kept constant.[20] Therefore, the liquidity gap for funds is defined by

$$g_j = (c_j^{\text{TG}} - c_j)^+. \tag{29}$$

To close its liquidity gap, an entity starts redeeming funds shares and selling tradable asset with the amounts proportional to the shares of available redeemable and tradable

[19] However imperfect, this mechanism, combined with the assumption that big lenders are most likely to step in, is consistent with the line of reasoning given by Giannetti and Saidi (2019). Their logic is that big banks are supposed to internalize negative spillovers and therefore provide liquidity to distressed entities.

[20] Such fund behaviour was also observed in some euro area countries during the March 2020 market turmoil. In order to meet redemptions, funds started selling assets and the ratio of cash to NAV got even slightly higher than before stress - evidence for the build-up of precautionary liquidity buffers.

assets on its balance sheet. These amounts are determined on a pro-rata basis at fund level. Let ϕ be a fund. The redemption $r_{i,\phi}$ by investor i, and the total of redemptions claimed are computed as follows (similarly to tradable assets in subsection 5.6):

$$r_{i,\phi} = \frac{h^{\text{red}}_{i,\phi}}{\sum_{\varphi}(h^{\text{trd}}_{i,\varphi} + h^{\text{red}}_{i,\varphi})} g_i \quad \text{and} \quad R_\phi = \sum_i r_{i,\phi}. \tag{30}$$

Redemptions do not depend on the fund performances but rather on decisions made by other agents regarding their own portfolio. We assume that the redeemed amount $r_{i,\phi}$ is directly added to the cash of i:

$$c_{i,t} = c_{i,t-1} + \sum_{\phi} r_{i,\phi} \tag{31}$$

because it is a short-term expected inflow, but is not debited immediately from ϕ, which has to address the redemption in the following period when the amount of total claims R_ϕ is deducted from its cash.

5.6 Fire sales

Along with redemptions, banks and funds start proportionally selling tradable assets as the last possibility to close liquidity gaps, as defined in equations (28) and (29). In this modelling block, banks are willing to sell only their non-eligible securities since they have access to central bank funding using their high quality liquid assets (HQLA). We assume that there is no endogenous price impact for HQLA.[21] By contrast, funds sell all kinds of securities holdings as they do not have access to central bank funding. Thus, they do not discriminate between eligible and non-eligible types of assets and sell all of their securities holdings.

Let $\mathbf{H}^{\text{trd}}_t = \left\{h^{\text{trd}}_{i,\phi,t}\right\}_{i,\phi}$ denote the portfolio matrix of tradable securities at time t, by market values, where i is the holder and ϕ is the security. Similarly, $\mathbf{H}^{\text{red}}_t$ is the portfolio matrix of redeemable holdings. Moreover, $\mathbf{p}_t = (p_{\phi,t})_\phi$ denotes the vector of prices of tradable securities.

We assume that fire sales are applied to cover liquidity shortfalls proportional to the share of tradable securities in the securities holdings portfolio. Empirical evidence shows that following a proportional approach to selling (also called 'slicing approach') is a sensible assumption to capture average behavior, see e.g. Coval and Stafford (2007) and Jotikasthira, Lundblad, and Ramadorai (2012). Furthermore, a very recent assessment by ESMA on the behaviour of funds with large corporate debt exposures during the COVID-19 market turmoil provides another evidence that supports the choice of the slicing approach.[22]

Then, starting from time t, the fire sale algorithm proceeds as follows:

[21] An alternative approach could be the reconstruction of HQLA from granular securities within our simulation, which would allow for a price impact on the amount of available HQLA. However, this not only means higher computational costs but also difficulties to assess at which point a bank would turn to the central bank to exchange specific assets for cash.

[22] See ESMA (2020).

(i) Determine the supply value $S_{\phi,t}$ that will be sold of each security (at the final prices). Based on the slicing hypothesis, the sale is done pro rata for the value of each security in the initial portfolio, meaning that entity-level and aggregated supply of ϕ are

$$s_{i,\phi} = \frac{h^{\text{trd}}_{i,\phi}}{\sum_{\varphi}(h^{\text{trd}}_{i,\varphi} + h^{\text{red}}_{i,\varphi})} g_i \quad \text{and} \quad S_{\phi} = \sum_i s_{i,\phi} \tag{32}$$

respectively, $s_{i,\phi}$ is the value that i wants to recover from ϕ. Note that the choice of the liquidation approach, here the slicing approach, may be a crucial driver for the magnitude of the shock transmission between sectors. For example, under the waterfall approach (selling the most liquid assets first), the magnitude of the shock transmission may be considerably reduced due to a reduced price impact.[23]

(ii) Determine the new vector of prices $\mathbf{p}_{t+1} = (p_{\phi,t+1})_{\phi}$ using the total amounts sold and the price impact method from Fukker, Kaijser, Mingarelli, and Sydow (2021) (see also E.3 for details on calibration):

$$p_{\phi,t+1} = p_{\phi,t}(1 - B_{\phi}(1 - \exp(-S_{\phi}\lambda_{\phi}/B_{\phi}))) \tag{33}$$

and update the value of tradable portfolios

$$\mathbf{H}^{\text{trd}}_{t+1} = \mathbf{H}^{\text{trd}}_{t} \left(\frac{\mathbf{p}_{t+1}}{\mathbf{p}_{t}}\right)^{\mathsf{T}}. \tag{34}$$

(iii) Find the new NAV vector $\mathbf{p}^*_{t+1} = (p^*_{i,t})_{i\in\text{InvF}}$ of funds, as explained in 5.2, and update the value of redeemable portfolios:

$$\mathbf{H}^{\text{red}}_{t+1} = \mathbf{H}^{\text{red}}_{t} \cdot \left(\frac{\mathbf{p}^*_{t+1}}{\mathbf{p}^*_{t}}\right)^{\mathsf{T}}. \tag{35}$$

(iv) Update internal accounting variables of entities to reflect changes of portfolio values. Let i be a financial institution and ϕ a security. When we account for the change in REA, it is the change from $h_{i,\phi,t}$ to $h_{i,\phi,t+1}$ that matters. However, when we want to account for losses only and see the impact on the total capital we need to disentangle what is converted as cash from actual losses that stem from the decrease in prices.

Cash holdings are updated with the amounts received after the iteration has converged:

$$c_{i,t+1} = c_{i,t} + \sum_{\phi} s_{i,\phi}. \tag{36}$$

Note that S_{ϕ} increases due to the price declines and the iteration terminates thanks to the finite amount of assets that can be sold and the introduction of a lower boundary for security prices (see Appendix E.3 for more details on the boundaries). Assuming that the whole residual liquidity need is recovered by i we have a change in capital due to the

[23]For more details, see, for example, ESMA (2019b) and IMF (2015). As background, adding an assumption of coordination among market participants could further reduce the observed price impact.

price impact given by $m_{i,t+1} = \sum_{\phi} \left(h_{i,\phi,t+1} - h_{i,\phi,t}\right)$. More generally, using the change in prices we get

$$m_{i,t+1} = \sum_{\phi} \frac{h_{i,\phi,t}}{p_{\phi,t}} \left(p_{\phi,t+1} - p_{\phi,t}\right) \, . \tag{37}$$

For banks, capital is also updated by

$$k_{i,t+1} = k_{i,t} - m_{i,t}. \tag{38}$$

6 Results

In this section, we report results of our model following deterministic (market shock and exogenous redemptions) and stochastic (NFC defaults) shocks to the system conditional on the Covid-19 scenario explained in section 3.5. We take a system-wide point of view and look at the amount of losses generated by the above exogenous shocks and endogenous reactions separately. Our results confirm that funds play a role not only as standalone entities with very large balance sheets, but also because they interact with banks and can amplify financial stress negatively affecting the entire system.

6.1 Contagion and its channels

Figure 11 shows a single realisation of the joint losses for banks and funds based on the exogenous shocks, as described in section 4, and only one Monte Carlo simulation. 'Q1E' labels losses from these initial exogenous shocks materializing at the end of the first quarter, which are coming from granular NFC defaults as well as expected losses on aggregate exposures (labelled "Defaults" in Figure 11) and exogenous market losses. The latter are either coming from the market scenario, affecting equity prices, or from the price drop of exogenously defaulting NFCs issuing securities (labelled "Market" in Figure 11), as outlined as well in Section 4. As a second round effect, the NAV of funds holding those securities drops as well (see also Figure 26 in Appendix F). 'Q2R'x' denotes additional endogenous losses generated in the interaction round 'x' of our algorithm that converges after 10 iterations in this particular example. Market losses, in this example, are lower than the default-induced losses in the exogenous block. The figure highlights that losses decrease with each iteration, ensuring the algorithm's convergence. Indeed, since liquidity needs are decreasing as a result of the events illustrated in Figure 10, associated losses from fire sales tend to go to zero over time. Since market losses are decreasing, possible new defaults turn up also for a limited number of iterations. Cliff effects could also take place though, e.g. when a small amount of additional losses hits some large entities resulting in further large losses to the system. However, we have not seen such situations throughout the whole range of Monte Carlo simulations.

Moving forward, Figure 12 shows the distribution of losses for both banks and funds for 10000 Monte Carlo simulations. The figure demonstrates that funds and banks experience losses of similar size. Moreover, our algorithm converges in at most thirteen but in general ten iterations. Again, exogenous market losses in 'Q1E' cover both the market scenario, affecting equity prices and a subsequent price drop of exogenously defaulting NFCs issuing securities and, as a second round effect, the NAV of funds holding those

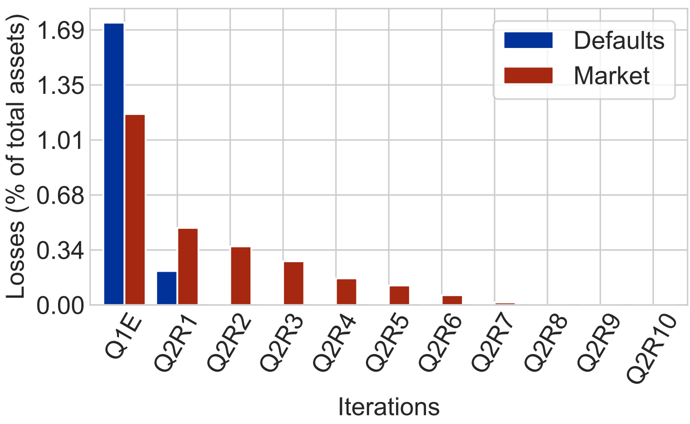

Figure 11: Aggregate losses for the whole system for one simulation (in percentage of total assets in the system).
'Q1E' shows the reaction following the initial exogenous shocks in the first quarter. 'Q2R1' to 'Q2R10' represent the iterations in the second quarter until convergence of the algorithm. In 'Q1E, 'Defaults' refer to NFC defaults and 'Market' to exogenous market losses both from the market scenario and from the price drop of exogenously defaulting NFCs. From 'Q2R1' onward bank and fund defaults as well as market losses are model-driven.
Source: Authors' calculations.

securities drops as well. Therefore, our model includes already some endogeneity at the point when the exogenous shocks materialise, as illustrated as well in the upper block of Figure 10. This is also the driver for the observed variance in the exogenous market losses in Figure 12 (in "Q1E"), despite the fact that the market shock from the scenario is fully deterministic. Figure 12 also shows that in our model funds engage in asset fire sales on a much larger scale than the banks that can withdraw liquidity from the inter-bank market in the first place. This is in line with the fact that funds do not have access to central bank funding and their cash holdings are too low (see Figure 7) to meet exogenous redemptions. Moreover, banks' liquid assets are extremely high (see LCR values in Figure 2); the calculation of which assumes though that short-term liabilities are not rolled over.[24] Looking at the variance of losses across simulations, we observe that it is extremely low demonstrating that liquidity shortfalls are nearly deterministic. This means that the set of defaulting institutions is not changing much across the simulations and largely driven by solvency defaults. Thus, bank liquidity withdrawals do not have a high variance and fund redemptions are either deterministic, as given by the scenario, or stochastic due to redemptions of banks. As we pointed out before, since bank liquidity withdrawals are quasi-deterministic, their redemptions from funds will also have a very low variance (see also Figure 16 below).

In Figures 13 and 14, we distinguish between exogenous and endogenous losses for banks and funds, respectively. As mentioned before, additionally to stochastic NFC defaults, we see a rather small variance in the case of exogenous market losses due to funds being exposed to defaulting NFCs and indirect price changes of defaulting entities' issued

[24] In our model, short-term liabilities for banks are covered by large exposures and large liabilities (see Appendices B.1 and B.2 for more details).

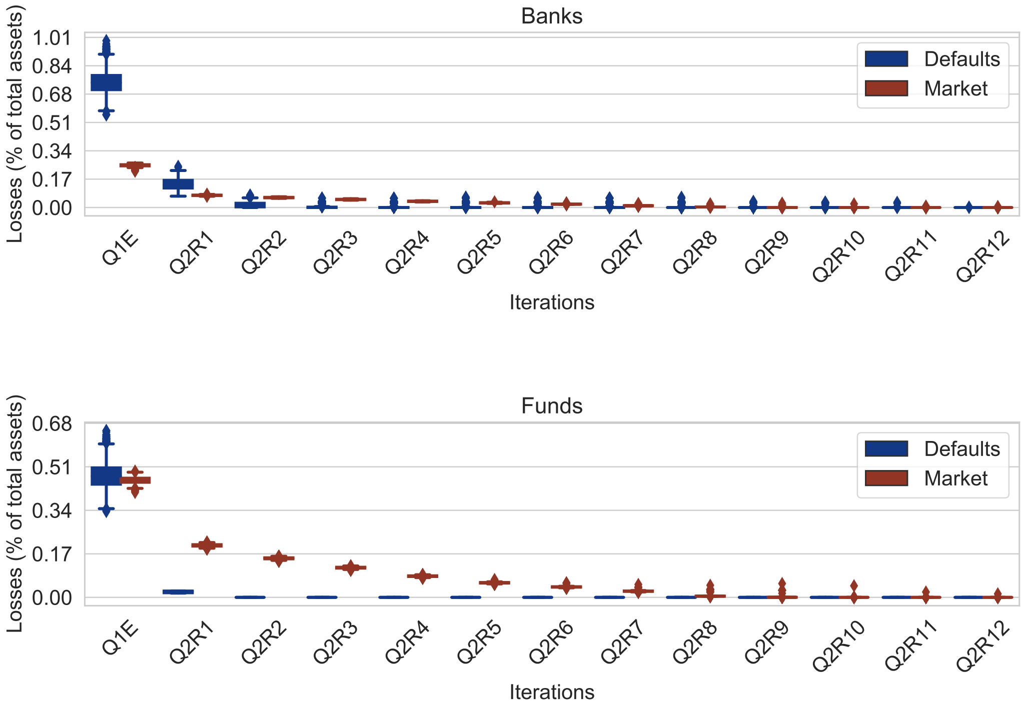

Figure 12: Distribution of losses for banks and funds based on 10000 Monte Carlo simulations (in percentage of total assets in the system).
'Q1E' shows the reaction following the initial exogenous shocks in the first quarter. 'Q2R1' to 'Q2R12' represent the iterations in the second quarter until convergence of the algorithm. In 'Q1E, 'Defaults' refer to NFC defaults and 'Market' to exogenous market losses both from the market scenario and from the price drop of exogenously defaulting NFCs. From 'Q2R1' onward bank and fund defaults as well as market losses are model-driven. Candlesticks represent the 25th and 75th percentiles of the distribution of Monte Carlo simulations.
Source: Authors' calculations.

securities. Also, losses from exogenous defaults, in percentage of total assets of the respective sector, are approximately six times larger for modelled investment funds than for banks. Hence, funds are more exposed to NFC defaults than banks. Regarding exogenous market losses for the two sectors, for banks these losses are rather limited compared to the ones from defaults. However, for investment funds they are in a similar ballpark than the losses stemming from exogenous defaults. Moreover, these results show that the funds' role in financial intermediation has increased substantially even though they are not directly giving out loans.

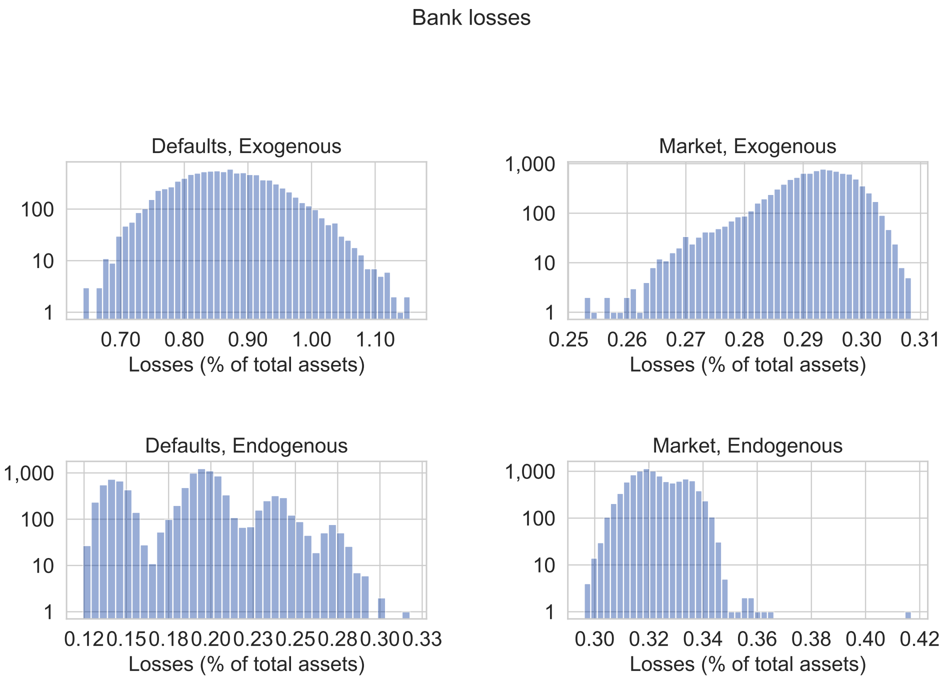

Figure 13: Histogram of losses from exogenous shocks and from endogenous reactions (contagion) for banks based on 10000 Monte Carlo simulations (in percentage of total banking sector assets).
'Defaults, Exogenous' refer to NFC defaults. 'Market, Exogenous' refers to exogenous market losses both from the market scenario and from the price drop of exogenously defaulting NFCs issuing securities. 'Endogenous' losses are model-driven.
Source: Authors' calculations.

Endogenous market losses result from the banks' and funds' reaction to shocks. The bottom rows of Figures 13 and 14 show an interesting multi-modal property for the distribution of these losses. Since there are only a few banks or funds that are vulnerable to our shocks, they act as gates letting losses spread within the system. Due to the stochastic property of liquidity shortfalls (see also Figure 16 below) we see that banks' liquidity withdrawal and fund redemptions are also random but centered around the most likely amounts: these can be thought of as local expected amounts of shortfalls across the simulations. Since there are institutions, which are more likely to default across the simulations, with a fixed set of exposures, their losses will also be fixed conditional on the set of entities defaulting. This is especially striking for endogenous default losses of funds in Figure 14 where the amount of losses is quasi-deterministic having only five possible values across the simulations. This is due to the fact that only a small number of funds default on their exposures. These funds are defaulting because they have loans from banks and their synthetic capital-like measure can fall below zero (equation (4)).

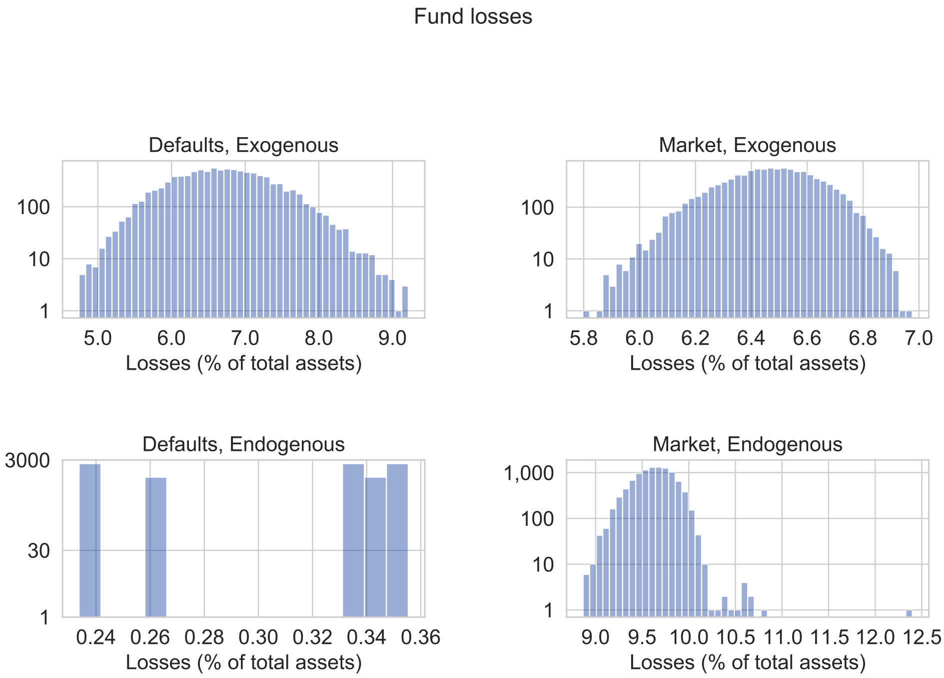

Figure 14: Histogram of losses from exogenous shocks and from endogenous reactions (contagion) for funds based on 10000 Monte Carlo simulations (in percentage of total investment fund sector assets).
'Defaults, Exogenous' refer to NFC defaults. 'Market, Exogenous' refers to exogenous market losses both from the market scenario and from the price drop of exogenously defaulting NFCs issuing securities. 'Endogenous' losses are model-driven.
Source: Authors' calculations.

Appendix F collects further simulation results and the underlying model mechanics. Focusing on the market and investment funds, we capture the different price impact across securities (see Figure 27). Importantly, despite the severity of the scenario, security prices remain above their calibrated floors (except for defaulted issuers; see Figure 28). With regard to the banking sector, we share details about the variables affecting our impairment calculations (see Figures 29 and 30).

In Figure 15 (left chart only), we also compare our model outputs with the results of the Vulnerability Analysis (VA) exercise conducted by the ECB Single Supervisory Mechanism focusing on banks' capital depletion. Careful comparison is needed as data, methodology and objectives are quite different between the two exercises. The VA exercise is partially based on an ad-hoc data collection for stress-testing purposes, it uses annual projections of variables for a three-year horizon and it is aimed at assessing the resilience of the banking system and its ability to continue funding the economy. Our model uses common reporting data (not specifically collected for stress testing exercises), follows a quarterly frequency with an exogenous shock in the first quarter and endogenous reactions starting in the beginning of the second quarter. It aims to understand better the contagion and amplification mechanism within and between the banking sector and the investment fund sector. Therefore, our model is more simplistic when stressing the banks compared to the VA.

Nevertheless, as the scenario used is consistent between the two exercises, it is interesting to confront our results for the credit risk losses with the VA; despite the difference in terms of data and methodology, this is where the results are the most comparable.[25] Moreover, for this comparison some simplifying assumptions are needed, i.e. we assume that the annual projection of the VA follows a linear path. Thus, dividing the annual result of the VA projection for the first year by a factor of two allows us to derive a two-quarter impact (corresponding to our exogenous shock in our first quarter and endogenous reaction in second quarter).[26]

Our credit risk losses (see chart on the left in Figure 15) are close to the VA results. One reason for the difference is data: we model correlated defaults using PDs from Moody's on granular exposures, which are based on individual market information and are, therefore, more point-in-time compared to PDs at the portfolio level used in the VA exercise. Then, in our methodology we highlight the amplification and contagion mechanisms thanks to the correlation of defaults related to the exogenous shock and through endogenous reaction. Those mechanisms are not included in the VA exercises as the stress is performed at the portfolio level without endogenous reactions. The impact of market losses (see middle chart in Figure 15) is very relevant in our framework, with a similar magnitude as for credit risk. In terms of percentage of REA, banks' median losses raise from 1.9 percentage points to 3.7 percentage points. Finally, adding the fund sector to the picture has a significant impact (see chart on the right in Figure 15) equal to about an additional one percentage point of REA on average. In total, adding to credit risk losses market losses and the investment fund sector leads in our model to a further decline in banks' capital ratios of almost three percentage points on average.

6.2 Financial flows and their relation to losses

Financial flows are the most important driver of behaviour for investment funds in our model. Thus, the initial exogenous redemption shock basically determines their endogenous behaviour. Given that the Covid-19 outbreak led to a severe market turmoil in 2020 Q1, which had also a strong effect on investment funds, we compare in Table 1 below our applied model-based initial redemptions (see Section 4.3) with the amount of redemptions observed in the market.[27] Our model-based redemption estimates are for equity funds somewhat larger than the ones observed in the market; while they are smaller for bond and mixed funds. These differences are driven by a combination of model, coefficient and residual uncertainty.

[25] In particular, market losses are calculated with a very different methodology in our model, employing granular holdings and endogenous reactions, whereas the VA applies exogenous shocks on the portfolio level.

[26] Future developments of our model will include the possibility of providing an annual impact, aligned with the EBA stress testing exercise.

[27] Model-based redemptions enter into the model as described in equation (8).

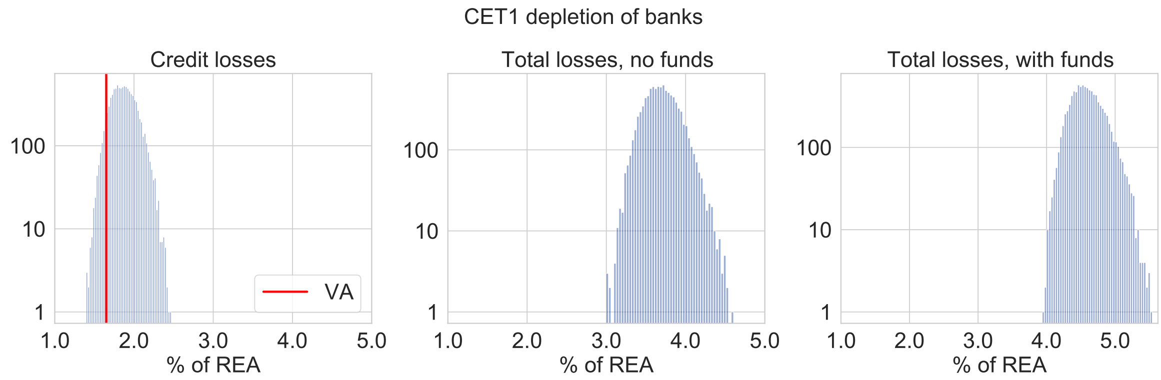

Figure 15: Distribution of average bank capital depletion along the 10000 Monte Carlo simulations. The red vertical line represents the average result from the VA exercise after two quarters. Further details on the VA comparison are provided in the text.
Source: Authors' calculations.

Net outflows	2020 Q1 model	2020 Q1 observed
Equity funds	-3.5%	-2.2%
Bond funds	-1.6%	-6%
Mixed funds	-1.1%	-3.7%

Table 1: Comparison of model based and observed fund ouflows.
Source: Authors' calculations and EPFR.

Within our partial-equilibrium model, our rules of behaviour make it possible to track the endogenous flows among the modelled sectors. These flows represent on the one hand short-term liquidity withdrawals within the banking sector and, on the other hand, the redemption of fund shares of both banks and funds to increase their liquidity buffer. Fire sales also serve to acquire liquidity but since a demand side is not directly modelled, we are not able to refer to it as financial flows. In fact, sold assets could be purchased either by entities with extremely high liquidity buffers for long-term investment purposes or by market makers in the financial market (such features will be covered by future extension of this model, together with the inclusion of additional financial sectors).

Figure 16 shows the histogram of aggregate outflows from banks and funds that result from their endogenous behaviour. Note that we assume short-term outflows only for the interbank market. While outflows from funds can be endogenously driven by banks and funds reacting to liquidity shortfalls, the outflows are limited as the funds get less liquidity-constrained. The reason for this is clear: funds start selling when they have liquidity shortfalls. At the same time, there is no close relation between the funds' liquidity and their losses. Their liquidity situation deteriorates only if the redemptions are large enough. However, this turns out not to be the case in our simulation, as confirmed by the amount of endogenous redemptions depicted in Figure 16. All this shows that funds do not face severe liquidity problems in the model. Thus, less fund-to-fund redemptions are needed. Moreover, we have seen in section 3 that banks have a limited amount of fund shares.

For banks, the opposite situation is the case as they are subject to a large amount of short-term liquidity withdrawals. This result is driven by the information contagion assumption we make, whereby short-term funding is withdrawn from and by distressed or defaulted banks. The amounts withdrawn from banks feature a multi-modal property. This shows that there are some sets of large banks, which are defaulting or getting into distress with a high likelihood, but separately. Since the exposures and, therefore, withdrawal amounts are deterministic, the smaller values around the peaks in the histogram are capturing smaller banks, which are defaulting or getting into distress with a lower probability, thus creating some uncertainty. In general, the stochasticity in bank defaults is also reflected in these liquidity withdrawals, which are a function of the set of defaulted or distressed entities.

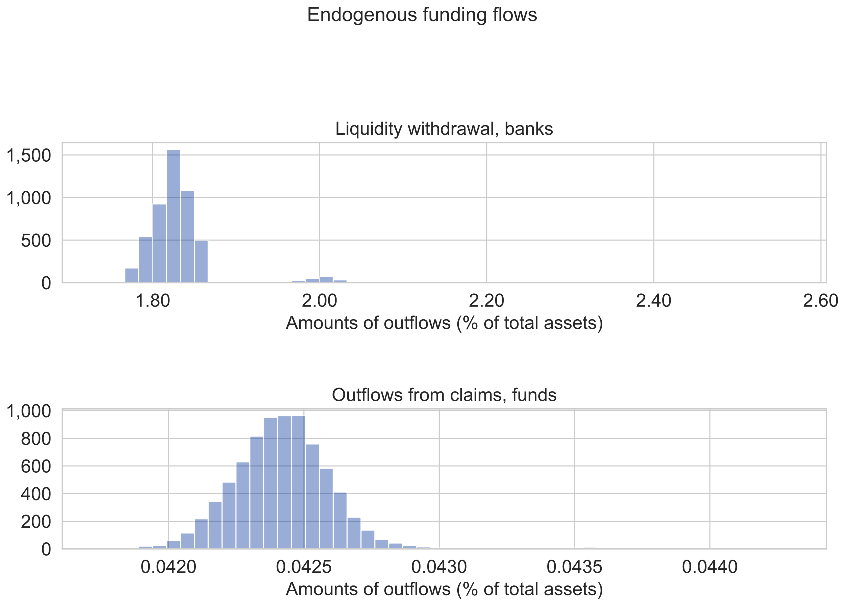

Figure 16: Histogram of financial flows following endogenous reactions based on 10000 Monte Carlo simulations (in percentage of total assets of the respective sector).
Source: Authors' calculations.

Figure 17 shows the total redeemed and withdrawn amount of liquidity ('Flows') in the endogenous loops, and the final losses from endogenous defaults and fire sales at the system level ('Losses'). Clusters for liquidity flows are visible, similar to Figure 16, reflecting the larger importance of stochastic bank defaults. It is interesting to note that higher amounts of outflows do not necessarily mean higher losses. An explanation for this is that losses from defaults are also driven by solvency and not only liquidity problems. Indeed, only entities that experience a liquidity shortage are able to sell securities, while insolvency issues cannot be alleviated in our model.

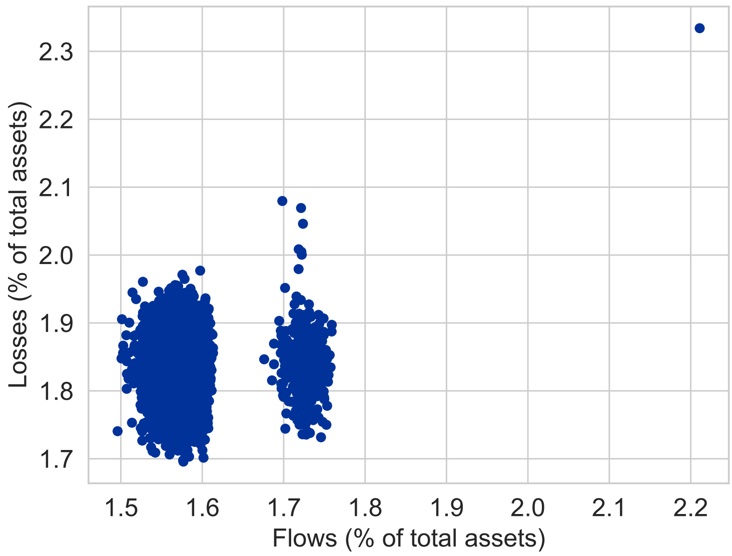

Figure 17: System-level flows and losses based on 10000 Monte Carlo simulations (in percentage of total assets).
'Flows' represent the total redeemed and withdrawn amount of liquidity in the endogenous loops. 'Losses' cover final losses from endogenous defaults and fire sales at the system level. Each dot represents the outcome of an individual Monte Carlo simulation.
Source: Authors' calculations.

6.3 The amplification role of funds

In this section we study how much the presence of funds adds to system-wide losses. This leads to the question how large is the feedback effect of funds on banks, with the former being traditionally not the subject of stress testing exercises. In the previous sections, we have already seen that fund losses are in general similar to bank losses but they have also an effect on bank losses via interconnectedness: overlapping tradable portfolios, banks' holdings of fund shares and bank loans to investment funds (the performance of the latter can also be at risk).

To understand better the amplification role of funds, we perform an experiment where we exclude the fund sector from our simulations. As a consequence, the banks' holdings of redeemable assets and loans to funds do not play any role anymore. Moreover, funds are also not active in the market, which narrows the amounts sold and, thus, limits the possible price depreciation. We run the simulations for exactly the same set of shocks as before and calculate the difference in losses for banks compared to the original simulations.

Figure 18 shows how much additional losses banks suffer in the simulation that includes funds compared to the experiment without funds. We observe additional losses across all dimensions but on different scales, except for endogenous defaults. The impact of funds on the banks' exogenous losses is, as expected, the smallest. The difference is due to the fact that NFC defaults also have a price impact, which reduces funds' net asset values immediately as described in section 5.2. The same consideration applies to exogenous market losses. These differences are only marginal compared to the original losses in Figure 13.

On the other hand, additional endogenous market losses are very large, which confirms that funds have a very large impact in this segment. Combined with the evidence shown in Figure 32 in Appendix G, this suggests that endogenous market losses without funds are almost zero, highlighting the important role of investment funds and the overall high amount of liquidity in the banking system.[28] However, the most surprising result comes from the amount of additional losses from endogenous defaults. One can observe a slight decrease in the presence of funds for a number of simulations. Though the amount of additional market losses is much higher, it is still possible that the presence of some funds has a mitigating role for some banks when they are also able to redeem their investment fund shares besides selling tradable assets. Nevertheless, as we have shown in Figure 15, the presence of funds increases bank capital depletion by 1 percentage point on average. Appendix G reports more details on the outcome from the no-fund experiment.

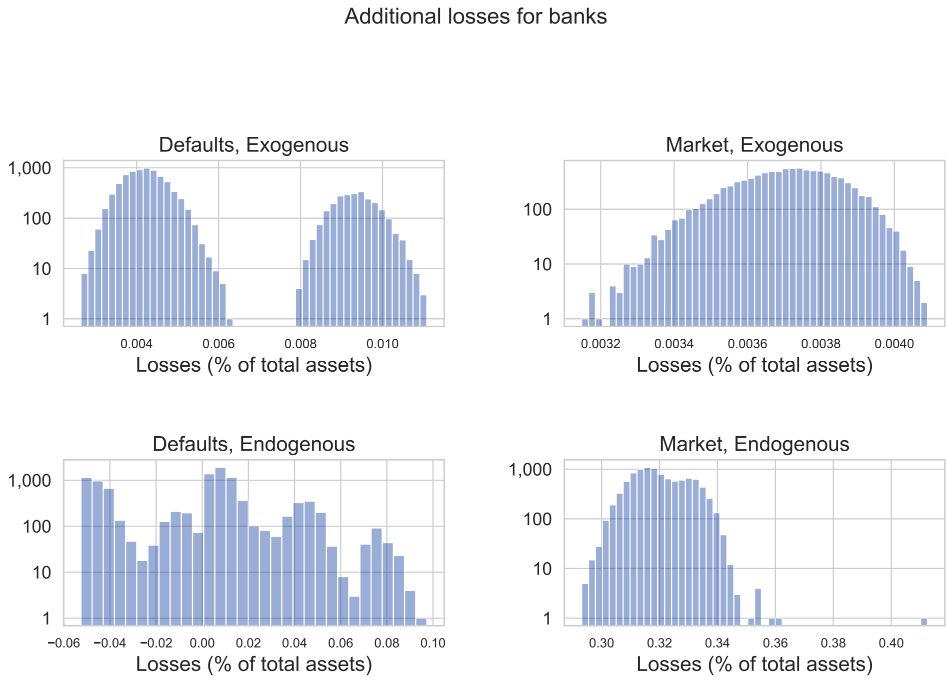

Figure 18: Additional losses for banks compared to the no funds case based on 10000 Monte Carlo simulations (in percentage of total banking sector assets).
'Defaults, Exogenous' refer to NFC defaults. 'Market, Exogenous' refers to exogenous market losses both from the market scenario and from the price drop of exogenously defaulting NFCs issuing securities. 'Endogenous' losses are model-driven.
Source: Authors' calculations.

[28]The same is likely true in case other financial sectors were added to the model. For example, if hedge fund behaviour was modelled, market losses for banks would probably be much higher as well.

7 Conclusion

In this paper, we present a stress testing approach to modelling short-term effects of financial stress in a system of banks and investment funds. Introducing dynamic responses of individual institutions to a macro-financial shock caused by the Covid-19 outbreak, we show that inter-sectoral contagion significantly increases initial losses in the system.

As to the channels of risk amplification, the joint asset fire sales of banks and funds have the largest systemic effect in our model, but the redemption of fund shares also turns out to be important. Thus, our findings highlight the significance of expanding the modelling capacity of macroprudential stress tests such as to include not only banks but also other financial institutions.

Despite the detailed complex mechanisms and the highly granular data used, one has to keep in mind the limitations of our framework. For instance, we are not yet modelling derivatives and margin calls.[29] However, when adding new model features for a given sector or additional inter-sectoral mechanisms, we need to strike a balance between complexity and understandability, not mentioning computational constraints.

Another dimension that is not covered by our model is policy response, i.e. central bank or other policy interventions. Such policy response – as seen in the Covid-19 crisis – can be in the form of a relaxation of banks' capital requirements or different types of loan moratoria, as initiated by various euro area governments. Moreover, in case of extreme liquidity stress, banks that are considered as viable could also turn to their lender of last resort by requesting Emergency Liquidity Assistance (ELA).[30]

In addition, a framework as the one described in this paper is not only extremely data-intensive but also carries a high level of uncertainty. Thus, with the different behavioural assumptions embedded in our model, we think that the results should rather be used for counterfactual analysis, conditional on different stress scenarios, and not be interpreted as point forecasts.

Furthermore, besides the advantages of granular information, the data quality usually decreases with the level of granularity - mainly due to data mining and matching biases. For example, private data is reported by funds only voluntarily.

Along these lines, our future work will focus on the model extension with regard to other financial sectors such as insurance corporations, hedge funds, money market funds, pension funds and CCPs. This will call for an extension of the model's time horizon. Moreover, we intend to refine the institutions behaviour, e.g. by assuming a simple portfolio optimization at a quarterly frequency, in order to discover the longer-term effects for the system.

Given the complex mechanics of our two-sector stress testing model, it will be important to conduct further robustness checks to understand better how, e.g., other stress scenarios or the individual exclusion of certain model dynamics change the results.

[29] Extending our current dataset further with granular data to cover, e.g., credit register information from Anacredit, individual derivative exposures from EMIR, secured and unsecured money market data from MMSR and data covering securities financing transactions from SFTDS will improve the results and increase the possibilities to model some of our risk channels in more detail.

[30] Regulatory and central bank responses could also be carried out following a theoretical, model-based optimization as done by Fukker and Kok (2021), who operate a default contagion model with different fire sale mechanisms and endogenous recovery rates.

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A Mathematical and balance sheet notations

Mathematical notations. Throughout the paper we have to deal with a large number of variables in different dimensions. To ease the understanding, we apply the following common standards: a small letter a is a real number, $\mathbf{a} = \{a_i\}_{i\in\mathcal{I}}$ is a vector of real numbers, $\mathbf{A} = \left\{a_{i,j}\right\}_{i\in\mathcal{I},j\in\mathcal{J}}$ is a matrix of real numbers where $\mathcal{I}$ and $\mathcal{J}$ are index sets of integers. The fraction of two vectors is to be interpreted as element-wise division: $\frac{\mathbf{a}}{\mathbf{b}} = \left\{\frac{a_i}{b_i}\right\}_{i\in\mathcal{I}}$; similarly, $\frac{1}{\mathbf{b}} = \left\{\frac{1}{b_i}\right\}_{i\in\mathcal{I}}$. Superscript $\cdot^{\mathrm{T}}$ means transpose of a matrix or vector. The product of a matrix $\mathbf{A}$ and a column vector $\mathbf{b}$ results in a vector $\mathbf{c}$ and is denoted following the mathematical standards: $\mathbf{c} = \mathbf{A}\cdot\mathbf{b} = \left\{\sum_{j\in\mathcal{J}} a_{i,j} b_j\right\}_{i\in\mathcal{I}} = \{c_i\}_{i\in\mathcal{I}}$. The inverse of a matrix $\mathbf{A}$ is denoted as $\mathbf{A}^{-1}$, and given $\mathbf{A}$ and $\mathbf{b}$, it solves $\mathbf{A}\mathbf{x} = \mathbf{b}$ for $\mathbf{x}$ as $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$. We also use matrix-vector multiplication where the vector is a row vector $\mathbf{b}^{\mathrm{T}}$. In this case, $\mathbf{A}\mathbf{b}^{\mathrm{T}} = \left\{a_{i,j} b_j\right\}$ or, in other words, column j of the matrix is multiplied by b_j. The column vector of ones will be denoted by $\mathbf{1}$ and, using the matrix multiplication $\mathbf{A}\cdot\mathbf{1} = \sum_j a_{i,j}$, we obtain the sum of the rows of the matrix. Hence, $\mathbf{A}^{\mathrm{T}}\cdot\mathbf{1}$ gives the sums of the columns.

The time dimension has no clear measurable interpretation in our model and only indicates that $t+1$ is taking place after t. Thus, b_t indicates the state of b in time t. When it is not necessary, we do not use the index t.

We will denote random variables by capital letters with time indices like W_t, a vector of random variables is denoted by $\mathbf{W}_t$.

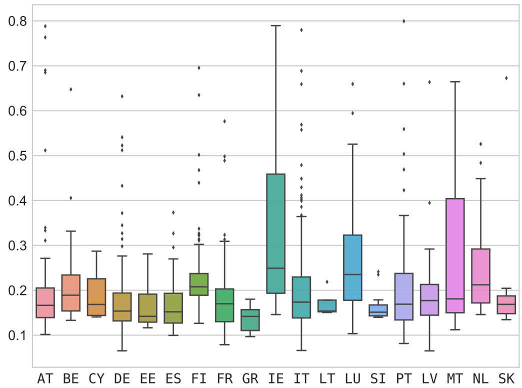
Common balance sheet notations. Both banks and funds hold securities on their asset side. The matrix of all securities holdings is denoted then as $\mathbf{H} = \{h_{i,j}\}$ where $h_{i,j}$ represents the securities issued by j held on i's balance sheet, hence $\sum_j h_{i,j}$ is the amount of securities held by i. Similarly, $\sum_j h_{j,i}$ is the amount of securities issued by i. Securities are aggregated to the issuer and asset type (equity or bond) level. They can be either redeemable assets (open-end investment funds) or tradable assets (like bonds and equities), $\mathbf{H}^{\mathrm{red}}$ and $\mathbf{H}^{\mathrm{trd}}$ respectively. To ease the understanding, we generally only index the holdings by issuers in the paper, but keep track of their evolution at asset type level: equity or bond. Only if necessary, we will denote a security holding of entity i of a specific security ϕ as $h_{i,\phi}$ but we still refer to the matrix of holdings as $\mathbf{H}^{\mathrm{red}}$ and $\mathbf{H}^{\mathrm{trd}}$. For banks, we assume that tradable assets only consist of assets that are not eligible for central bank operations. On the other hand, funds can start selling all kinds of assets as they don't have access to a central bank facility.

Another considered balance sheet item is the amount of cash or liquid buffer, denoted either by c_i for entity i or denoted as $\mathbf{c}$ when stacked into a vector for all entities. For banks, we assume that for banks, the amount of high-quality liquid assets is cash-equivalent. Similarly, we denote capital-like items by k_i or $\mathbf{k}$ in vector notation. As loans are only provided by banks, we introduce them in subsection 2.1.

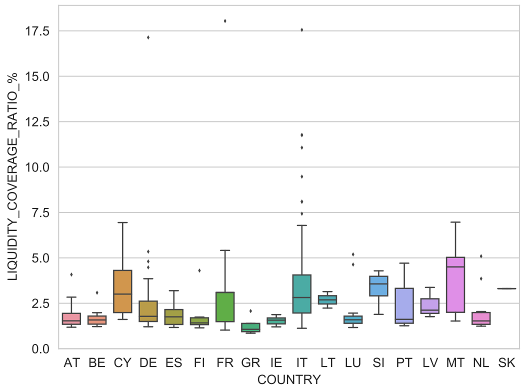
All institutions can get into solvency default or distress as well as liquidity default or distress. τ_i^{def} and τ_i^{dis} are the default and distress thresholds with respect to capital, $\tau_i^{\mathrm{c,dis}}$ and $\tau_i^{\mathrm{c,def}}$ the distress and default thresholds with respect to liquidity buffers or cash for institution i, also denoted as vectors $\boldsymbol{\tau}^{\mathrm{def}}, \boldsymbol{\tau}^{\mathrm{dis}}, \boldsymbol{\tau}^{\mathrm{c,def}}, \boldsymbol{\tau}^{\mathrm{c,dis}}$. Using these, an institution i is in solvency default, if $k_i < \tau_i^{\mathrm{def}}$; in solvency distress, if $k_i < \tau_i^{\mathrm{dis}}$; in liquidity distress, if $c_i < \tau_i^{\mathrm{c,dis}}$; and in liquidity default, if $c_i < \tau_i^{\mathrm{c,def}}$.

B Banking data

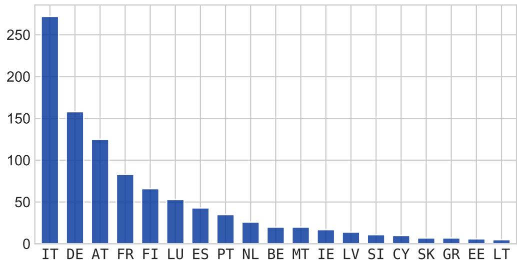
We reconstruct the banks' balance sheet from two different data sources available at the ECB: namely the COREP large exposures and the SHS-G dataset. They map connections from banks to funds and the real economy. Moreover, this dataset is combined with probability of defaults (PDs) data from Moody's. Each bank is reported at the highest level of consolidation within the euro area combined with a unique identifier. Figure 19 provides a high-level description of key banking indicators across euro area countries.



(a) CET1 ratios



(b) Liquidity Coverage Ratios



(c) Banks' locations (by number of banks)

Figure 19: Descriptive statistics of the network of banks used as input to the model, with data for Q4 2019.
Source: Authors' calculations.

B.1 Large Exposures dataset (LE)

The large exposure (LE) reporting framework, introduced in 2014, requires all EU institutions to report exposures exceeding either EUR 300 million or 10% of their eligible capital, towards either a single counterparty or group of connected entities. From these data we obtain exposures of each individual financial institution towards the rest of the system. The long-term and short-term exposure matrices $\mathbf{L}^L$ and $\mathbf{L}^S$ are then constructed from this data. In particular, loan exposures from banks to investment funds captured in the LE dataset amount to EUR 250 billion, of which EUR 65 billion have a maturity lower than one month.

The LE reporting framework includes maturity splits, which allows us to differentiate long-term from short-term exposures. For each exposure in the LE dataset we know the original exposure amount and the net exposure amount after credit risk mitigation. The latter can approximate the unsecured part of the exposure, which we use also as a proxy for the Loss Given Default (LGD) of the exposures (see Figure 20).

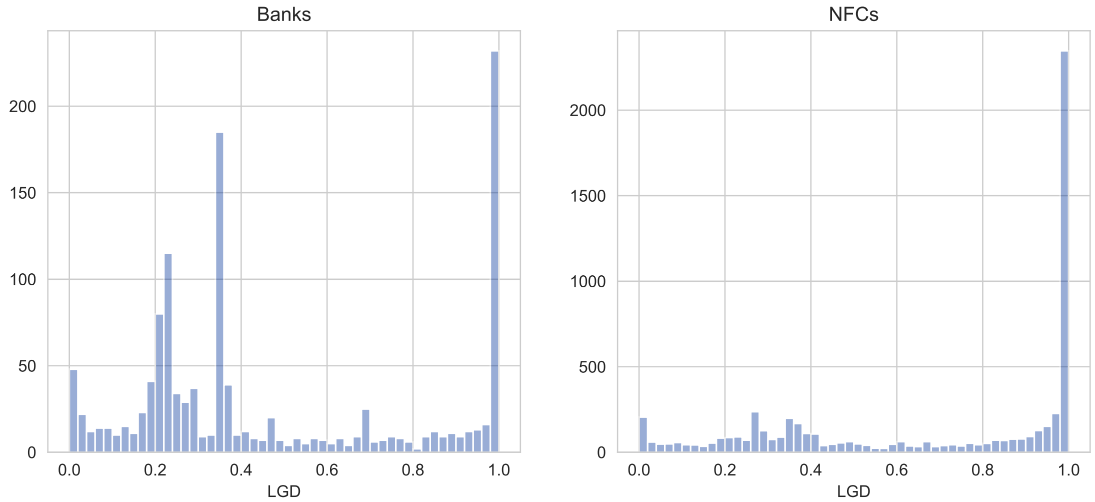


Figure 20: Histogram of loan-level LGDs for banks and NFCs as of Q4 2019.
For loans to funds, no LGDs are available and we assume that they are equal to 1.
Source: Authors' calculations.

Thus, for each edge[31] from i to a we have a breakdown along two dimensions such that

$$\text{Edge}_{i,a,t} = \left\{ l_{i,a,t}, \left(l^S_{i,a,t}, l^L_{i,a,t} \right), \left(l^{SU}_{i,a,t}, l^{SS}_{i,a,t}, l^{LU}_{i,a,t}, l^{LS}_{i,a,t} \right) \right\} ,$$

where in the superscripts S and L mean short-term and long-term, SU and SS denote short-term unsecured and secured exposures, LU and LS denote long-term unsecured and secured exposures. The items in brackets add up to the total amount of gross exposures $l_{i,a,t}$.

Nevertheless, households and most of the NFCs are not captured within the LE framework. Thus, we complement the dataset with aggregate exposures coming from FINREP, reported at the country-level. In a next step, we check for each bank, reported period as well as counterparty-country and -sector the average LGD and compare it with the one provided by the EBA stress test methodology using ECB satellite models. If the average that we obtain at an aggregate level is larger than the average LGD, we rescale observed LGD for all granular exposures by a multiplier in order to get the average LGD for each bank vis-à-vis each counterparty-country and -sector equal to the one provided by the ECB. In case the average LGD computed in our dataset is smaller than the short-term one, it is kept unchanged.

The ECB satellite model stress test LGD used to rescale the granular LGD are those retrieved for the baseline scenario. In case of country-sector specific stress testing, we use the stress test LGDs from the adverse scenario derived by ECB satellite models. As an

[31] The entities in our joint model of banks and funds are referred to as nodes, while the connections are edges.

example, let's suppose bank A has 3 exposures of 100 in gross terms each vis-à-vis three Turkish NFCs. The unsecured long-term parts (our initial LGD) of those exposures are respectively 80%, 60%, 40%, that is an average of 60% of the gross exposure. In the EBA scenario, the LGD vis-à-vis Turkish NFC is 30%. This means that we are overestimating the exposure-specific LGD by a factor of 2 on average. Hence, we rescale each LGD by 2, obtaining finally for the three exposures an LGD of 40, 30, 20, whose average resembles the one of the ECB satellite models developed for the EBA stress test. Alternatively, if a bank already shows an LGD below the one derived from ECB satellite models, those exposures remain untouched, that is they are not scaled up.

B.2 List of COREP data used in our framework

- Large Exposure (LE) (COREP C.27 to C.30)
  - All EA banks (significant institutions and less significant institutions)
  - Quarterly bilateral exposures with a value larger than 10% of a bank's eligible capital or 300ml Euro
  - Information on counterparties (Name, LEI, Country, Sector, NACE)
  - Exposure-Specific information (asset class, secured/unsecured, maturity)
- Large Liabilities data (COREP C.67)
  - All EA banks (significant institutions and less significant insitutions)
  - Top 10 quarterly granular bilateral liabilities with a value $>$ 1% total assets
  - Information on counterparties (Name, LEI)
  - Exposure-Specific information (asset class, secured/unsecured, maturity)

B.3 List of FINREP data used in our framework

- Primary statements (balance sheet and income statement as well as comprehensive income and equity)
- Disclosure of financial assets and liabilities
- Financial assets disclosures and off-balance sheet activities
- Non-financial instrument disclosure

C Investment funds data

We reconstruct the funds' balance sheets using data from Lipper IM, provided by Refinitiv. A first part is referred to as the static data and contains fund specific characteristics, independent of time. The second part contains time-varying information on funds' holdings $\mathbf{H}_{i,\phi,t}$, with a security-level granularity, although aggregates are also computed by Lipper.

We provide in Table 2 the breakdown by country of our coverage in Lipper, compared to the data reported to the ECB and available through the IVF database. The most important feature is that, when aggregated at the EA level, the median coverage of funds' assets that we get from Lipper are 59% for equity funds, 47% for bond funds and 25% for mixed funds compared with the data from the IVF database. Some of the coverage ratios in the tables are above 100%. Most of the countries concerned have a small investment fund sector. So, these discrepancies in the reporting may be driven by a very small number of funds. To get a better understanding, we provide as well a comparison of Lipper IM data with data from European Fund and Asset Management Association (EFAMA), with corresponding coverage ratios shown in Table 3. At the EA level, the median coverage of funds' assets that we get from Lipper are 64% for equity funds, 67% for bond funds and 39% for mixed funds compared with data provided by EFAMA. Coverage ratios larger than 100% are in this case as well reported (albeit somewhat smaller). So, the explanation provided above would still hold.

domicile	Coverage ratio (%) min	median	max	Totals 2019Q4 Lipper	IVF
AT	41.8	50.0	54.7	13.8	33.0
BE	87.2	99.4	211.3	50.3	57.5
CY	0.0	0.0	8.0	0.2	2.4
DE	31.5	37.1	48.8	165.3	338.4
EE	25.9	55.5	72.7	0.1	0.4
ES	81.5	87.8	93.6	39.3	44.0
FI	74.1	81.4	85.6	37.3	48.7
FR	46.4	54.3	60.7	201.7	359.2
GR	58.2	77.2	85.3	1.2	1.4
IE	42.4	52.2	58.7	415.7	932.2
IT	70.9	78.6	85.2	20.8	25.9
LT	24.6	111.9	207.5	0.0	0.1
LU	75.1	78.9	81.4	1221.5	1569.4
LV	33.3	64.8	217.4	0.0	0.0
MT	4.4	20.7	345.1	0.6	4.0
NL	12.8	16.0	50.7	59.5	394.1
PT	82.9	89.1	96.8	2.0	2.4
SI	35.7	43.2	47.2	0.8	1.9
SK	24.3	63.3	70.3	0.3	0.7
Total EA	55.8	58.8	66.1	2230.3	3815.7

(a) Equity funds assets

domicile	Coverage ratio (%) min	median	max	Totals 2019Q4 Lipper	IVF
AT	37.5	46.3	49.5	25.7	68.3
BE	27.0	59.1	131.7	8.9	22.0
CY	0.0	1.6	59.0	0.1	0.2
DE	7.9	9.1	13.1	57.6	502.4
EE	0.0	81.3	120.8	0.0	0.0
ES	32.3	95.4	101.3	89.0	93.3
FI	47.0	51.1	86.5	33.8	58.6
FR	36.4	40.2	49.0	148.1	354.5
GR	58.8	100.3	138.6	1.9	1.9
IE	30.1	38.1	54.2	453.2	937.7
IT	46.8	58.5	102.3	59.5	116.3
LT	32.6	72.1	104.7	0.0	0.1
LU	57.9	67.8	80.5	1246.9	1549.4
LV	57.7	81.9	118.1	0.2	0.2
MT	67.5	133.8	182.2	1.4	2.1
NL	7.9	9.8	24.5	36.9	204.5
PT	25.4	44.4	56.1	3.5	8.0
SI	25.1	31.7	48.7	0.1	0.2
SK	25.9	39.0	80.2	0.6	1.6
Total EA	42.1	46.9	55.3	2167.4	3921.2

(b) Bond funds assets

domicile	Coverage ratio (%) min	median	max	Totals 2019Q4 Lipper	IVF
AT	11.4	13.3	16.2	12.1	83.7
BE	29.6	35.4	153.3	50.4	98.8
CY	0.0	0.0	27.9	0.1	0.5
DE	5.1	6.1	9.2	103.8	1130.7
EE	63.0	76.7	101.2	0.0	0.0
ES	47.4	71.6	137.5	43.5	69.4
FI	23.0	31.4	54.7	2.9	12.5
FR	12.7	19.5	20.7	79.0	420.9
GR	50.9	74.9	81.7	0.7	0.9
IE	6.6	17.3	22.3	75.3	348.9
IT	82.0	98.8	127.6	145.4	140.3
LT	47.0	201.1	inf	0.0	0.0
LU	30.0	36.9	52.8	592.0	1122.3
LV	6.6	58.4	94.2	0.0	0.0
MT	23.6	59.0	451.3	0.4	1.6
NL	29.6	39.2	45.8	7.5	25.5
PT	150.8	188.2	233.3	4.7	2.2
SI	23.8	31.3	55.5	0.2	0.9
SK	22.3	34.7	50.6	0.8	3.6
Total EA	20.0	24.5	32.5	1118.8	3462.6

(c) Mixed funds assets

	Total Lipper	Total EFAMA	Ratio (%)
AT	695	2019	34.4
BE	708	932	76.0
CY	32	296	10.8
DE	1165	6536	17.8
EE	5	0	inf
ES	956	2669	35.8
FI	323	494	65.4
FR	2277	10856	21.0
GR	89	213	41.8
IE	2184	7285	30.0
IT	1085	1814	59.8
LT	8	0	inf
LU	8831	14898	59.3
LV	5	0	inf
MT	36	687	5.2
NL	258	1712	15.1
PT	106	371	28.6
SI	34	115	29.6
SK	0	86	0.0
Total EA	18797	50983	36.9

(d) Number of firms

Table 2: Lipper coverage of funds: comparison to IVF by assets for each fund family.
The ratios of coverage by assets are computed for each quarter where IVF data is available (that of Lipper going back further in time), and the minimum, median and maximum presented are taken over time. Monetary values in the totals give are in EUR billions.
Source: Lipper IM, IVF, and authors' calculations.

	Bond			Equity			Mixed			Total		
	Lipper	EFAMA	% ratio	Lipper	EFAMA	% ratio	Lipper	EFAMA	% ratio	Lipper	EFAMA	% ratio
AT	250	483	51.8	187	328	57.0	219	1060	20.7	656	1871	35.1
BE	60	68	88.2	304	216	140.7	340	259	131.3	704	543	129.7
BG	1	8	12.5	4	45	8.9	2	63	3.2	7	116	6.0
HR	0	40	0.0	0	23	0.0	0	9	0.0	0	72	0.0
CY	10	11	90.9	3	29	10.3	20	77	26.0	33	117	28.2
CZ	27	43	62.8	23	34	67.6	27	73	37.0	77	150	51.3
DK	217	382	56.8	288	497	57.9	82	172	47.7	587	1051	55.9
FI	87	117	74.4	212	223	95.1	42	114	36.8	341	454	75.1
FR	695	1059	65.6	1144	1715	66.7	482	3028	15.9	2321	5802	40.0
DE	270	1051	25.7	425	1089	39.0	593	3475	17.1	1288	5615	22.9
GR	42	77	54.5	39	83	47.0	29	50	58.0	110	210	52.4
HU	75	76	98.7	73	105	69.5	118	110	107.3	266	291	91.4
IE	722	1472	49.0	1192	2660	44.8	332	1209	27.5	2246	5341	42.1
IT	262	204	128.4	91	105	86.7	747	553	135.1	1100	862	127.6
LI	0	320	0.0	0	405	0.0	0	230	0.0	0	955	0.0
LU	2759	3227	85.5	3550	4049	87.7	2236	3778	59.2	8545	11054	77.3
MT	27	55	49.1	28	99	28.3	24	52	46.2	79	206	38.3
NL	75	219	34.2	161	363	44.4	60	109	55.0	296	691	42.8
NO	86	187	46.0	167	477	35.0	29	84	34.5	282	748	37.7
PL	101	196	51.5	138	227	60.8	100	293	34.1	339	716	47.3
PT	25	28	89.3	44	41	107.3	50	71	70.4	119	140	85.0
RO	0	18	0.0	0	29	0.0	0	30	0.0	0	77	0.0
SK	6	24	25.0	5	14	35.7	5	47	10.6	16	85	18.8
SI	6	10	60.0	47	71	66.2	10	18	55.6	63	99	63.6
ES	578	701	82.5	313	1056	29.6	438	567	77.2	1329	2324	57.2
SE	93	118	78.8	325	345	94.2	96	157	61.1	514	620	82.9
CH	310	259	119.7	489	428	114.3	175	216	81.0	974	903	107.9
TR	63	71	88.7	59	57	103.5	78	69	113.0	200	197	101.5
GB	363	383	94.8	1230	1326	92.8	572	841	68.0	2165	2550	84.9
Total EA	5874	8806	66.7	7745	12141	63.8	5627	14467	38.9	19246	35414	54.3
Total EU	6388	9687	65.9	8596	13446	63.9	6052	15374	39.4	21036	38507	54.6

Table 3: Lipper coverage of funds: comparison to EFAMA by number of funds.
The ratios of coverage by assets are computed for each quarter where, and the minimum, median and maximum presented are taken over time. Monetary values in the totals give are in EUR billions.
The coverage has been computed for 2019Q4 and restricted to funds which are regulated under the UCITS or AIFMD frameworks. The data used is taken from *Trends in the European Investment Fund Industry in the Fourth Quarter of 2019 & Results for the Full Year of 2019* (2020).
Source: Lipper IM, EFAMA and authors' calculations.

D Visualization of exposure networks

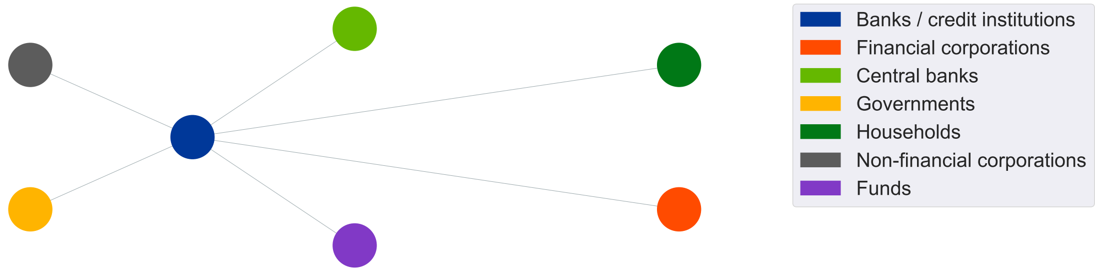


Figure 21: Aggregated loans from banks to all sectors in Q4 2019. Individual exposures are aggregated to sector level.
Source: Authors' calculations.

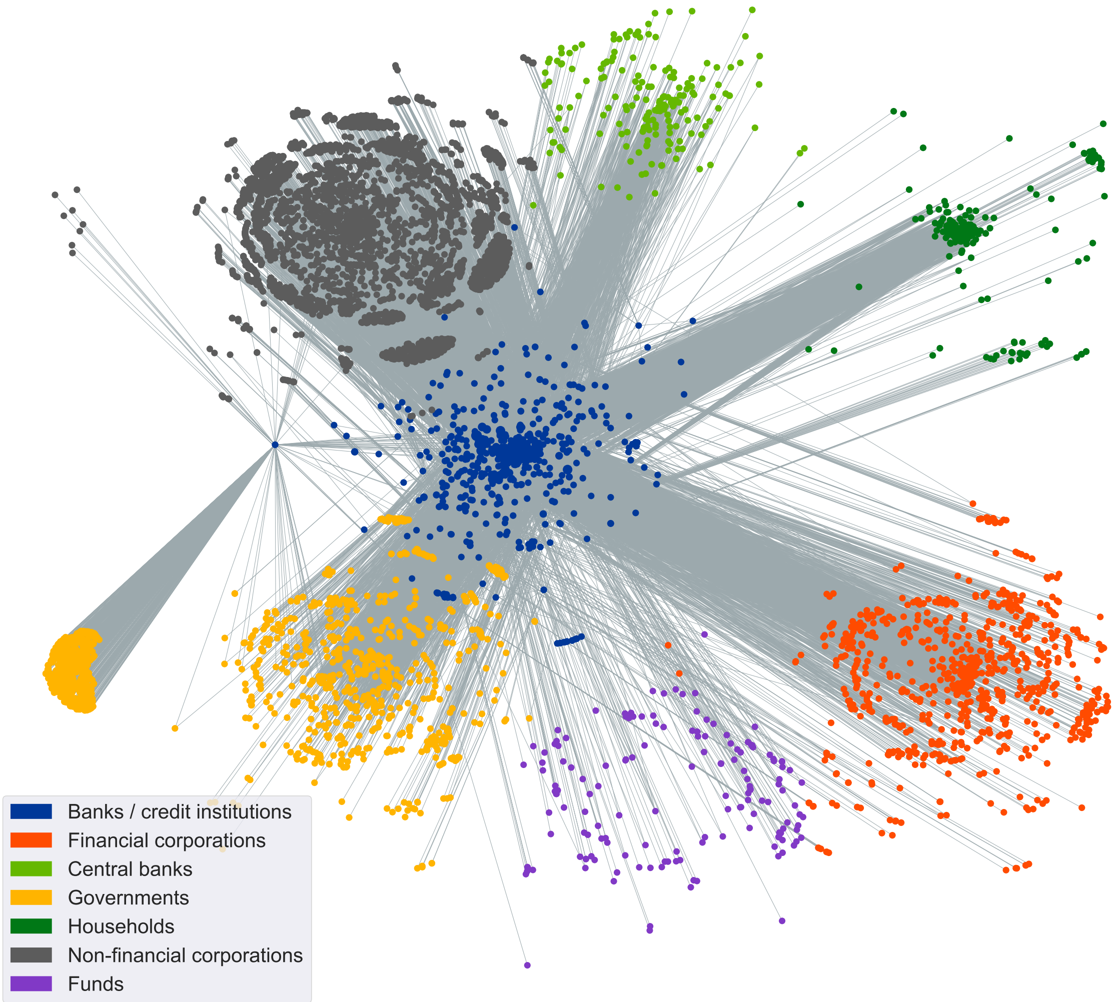


Figure 22: Exposures from banks to all sectors at entity level in Q4 2019.
Source: Authors' calculations.

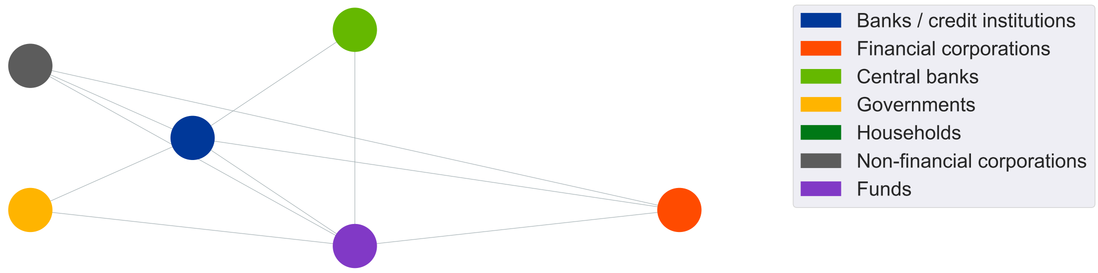


Figure 23: Portfolio holding connections at sector level in Q4 2019. An edge represents that a bank/fund holds assets issued by another sector.
Source: Authors' calculations.

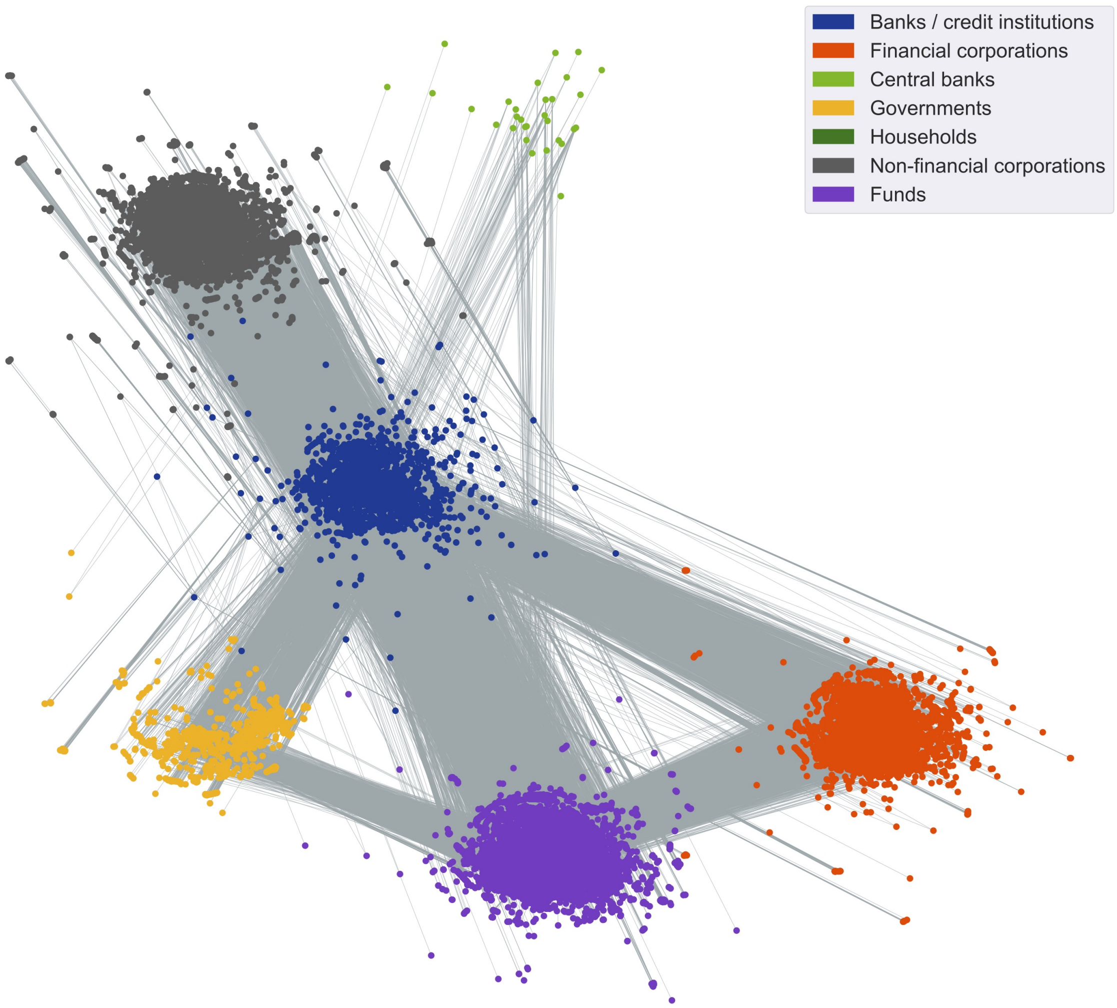


Figure 24: Portfolio holding connections at entity level in Q4 2019. An edge represents that a bank/fund holds assets issued by another entity of a sector.
Source: Authors' calculations.

E Methodological details

E.1 Establishing new unsecured loans

The heuristic algorithm that is used to create new unsecured lending relationships, as described in 5.4, is shown in Figure 25 below. For more sophisticated studies on endogenous network formation see also Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Cohen-Cole, Patacchini, and Zenou (2015) and Hałaj and Kok (2015).

Figure 25: Algorithm for the establishment of new inter-bank lending relationships

Data: State of the financial system
Establish a list L_b of borrowers
Compute from equation (26) the maxima $(u_k)_{k \in L_b}$ to be borrowed
Establish a list L_l of possible lenders
Compute the lending capacities $(v_k)_{k \in L_l}$ from (27)
while $L_b \neq \emptyset$ *and* $L_l \neq \emptyset$ **do**
    $i \leftarrow \arg\max_{k \in L_b} \{u_k\}$ // Select the borrower with the largest need
    $j \leftarrow \arg\max_{k \in L_l} \{v_k\}$ // Select the lender with the largest surplus
    // A new lending is established and we update our lists
    if $u_i < v_j$ **then**
        j lends an amount u_i to i
        We remove i from L_b
        $v_j \leftarrow v_j - u_i$
    else // $u_i < v_j$
        j lends an amount v_j to i
        We remove j from L_l
        $u_i \leftarrow u_i - v_j$

The proof of termination of the algorithm is simple: we have a finite number of entities in each of the two lists, and at every iteration either the borrower i or the lender j is taken out of its respective list. Therefore, the maximum number of iterations is bounded by the total number of entities in the two initial lists.

This algorithm is also meant to create only a small number of new lending relationships in the inter-bank network; thus, minimizing the impact on the network structure.

E.2 Proof of the price/capital resolution for funds

We prove in this part the result given by equation (15). First, we can rewrite the sum of redeemable assets for all funds in equation (14) as follows:

$$\mathbf{H}_t^{\text{red,f}} \cdot \mathbf{1}_{\text{red}} = \mathbf{H}_{t-1}^{\text{red,f}} \cdot \frac{\mathbf{k}_t}{\mathbf{k}_{t-1}} = \left(\mathbf{H}_{t-1}^{\text{red,f}} \frac{1}{\mathbf{k}_{t-1}^{\mathsf{T}}} \right) \cdot \mathbf{k}_t \tag{39}$$

where $\frac{1}{\mathbf{k}_{t-1}^{\mathsf{T}}}$ is a row vector $\left(\frac{1}{k_{i,t-1}}\right)_{i \in \text{InvF}}$ and, therefore, the matrix-row vector multiplication yields a matrix.

To prove equation (39), let us denote $(a_i)_{i \in \text{InvF}} = \mathbf{H}_t^{\text{red,f}} \cdot \mathbf{1}_{\text{red}}$, such that $\forall i, a_i = \sum_{j \in \text{InvF}} h_{i,j,t}$. We know that, $\forall i, j \in \text{InvF}, h_{i,j,t} = h_{i,j,t}^S \cdot p_{j,t}$, i.e. the value that i holds of j's shares is equal to the number of shares held multiplied by the price. Then, given that $h^S_{i,j,t} = h^S_{i,j,t-1}$ (because in the model no redemption happens at the exact same time as changes in market prices) we have

$$h_{i,j,t} = h^S_{i,j,t} p_{j,t} = h_{i,j,t-1} \frac{p_{j,t}}{p_{j,t-1}}. \tag{40}$$

Moreover, the price of j is proportional to its TNA (the factor between them being the number of shares issued). So we have $\frac{p_{j,t}}{p_{j,t-1}} = \frac{k_{j,t}}{k_{j,t-1}}$. Plugging this into equation 40 we get $h_{i,j,t} = h_{i,j,t-1} \frac{k_{j,t}}{k_{j,t-1}}$. Therefore,

$$a_i = \sum_{j \in \text{InvF}} h_{i,j,t-1} \frac{k_{j,t}}{k_{j,t-1}}$$

and it is immediate to verify that this is the expression of the i-th term of $\mathbf{H}^{\text{red,f}}_{t-1} \cdot \frac{\mathbf{k}_t}{\mathbf{k}_{t-1}}$.

Thus, from equations (14) and (39), and denoting by $\mathbf{I}_F$ the identity matrix of size F, we get[32]

$$\left(\mathbf{I}_F - \mathbf{H}^{\text{red,f}}_{t-1} \frac{1}{\mathbf{k}^{\mathsf{T}}_{t-1}} \right) \cdot \mathbf{k}_t \;=\; \mathbf{H}^{\text{trd,f}}_t \cdot \mathbf{1}_{\text{trd}} + \mathbf{c}_t - \mathbf{l}_t \;. \tag{41}$$

Since $\left(\mathbf{I}_F - \mathbf{H}^{\text{red,f}}_{t-1} \frac{1}{\mathbf{k}^{\mathsf{T}}_{t-1}} \right)$ is invertible as a straightforward consequence of the Neumann series expansion of its inverse, we obtain the solution:

$$\mathbf{k}_t \;=\; \left(\mathbf{I}_F - \mathbf{H}^{\text{red,f}}_{t-1} \frac{1}{\mathbf{k}^{\mathsf{T}}_{t-1}} \right)^{-1} \left(\mathbf{H}^{\text{trd,f}}_t \cdot \mathbf{1}_{\text{trd}} + \mathbf{c}_t - \mathbf{l}_t \right),$$

which is what we wanted to verify. □

E.3 Price impact calibration

The price impact method that we use to determine new prices relies on a sublinear relationship between the amount sold and the subsequent change in the price. The underlying foundation of the model is derived from the linear price impact specification described in Kyle's framework (Kyle, 1985):

$$\Psi = S\lambda \tag{42}$$

where S is the total amount sold of a single security and λ is the parameter known as Kyle's lambda, which represents the price impact parameter. For relatively small sized sales, this relation is known to provide a reliable description of the impact. However, when the size of a given sale increases, this model tends to overestimate the impact. To mitigate this problem the literature (see e.g. (Bouchaud, 2010)) suggests a square-root specification, which also satisfies a concave shape. Nevertheless, this shape implies an arbitrarily large impact as the size of the traded volume increases. Therefore, another stream of literature, e.g. (Schnabel and Shin, 2002), puts forward an exponential specification:

[32] This equation is actually the one used in our implementation.

$$\Psi = 1 - e^{-S\lambda} \tag{43}$$

The implementation of an exponential shape prevents the issue of extremely large impacts for increasing volumes. However, it does not prevent prices from dropping to zero. Moreover, it might be more realistic to assume that arbitrageurs step-in to buy securities whose price decline by a large fraction. Still, a buy-side of the financial market is notoriously difficult to model. Against this background, we approximate the price impact function using historical data:

$$\Psi_\phi(S_\phi) = B_\phi(1 - e^{-S_\phi \lambda_\phi / B_\phi}) \tag{44}$$

where λ_ϕ is the impact parameter, comparable to Kyle's lambda from before, for security ϕ, B_ϕ the corresponding impact boundary and S_ϕ is the total amount sold of security ϕ. A simple boundary might be the most negative return observed in the history of a security. Suppose we define the entire set of returns for security ϕ as R_ϕ. Then, a straightforward specification of the boundary reads as:

$$\text{B1:}\ \min(R_{\phi,t} | t = 0, .., T)$$

A drawback of this specification is that it might not work for extreme cases. Therefore, we ensure that the largest impact is included for all volumes that result in this return, establishing an additional boundary B2 that solves the following equation:

$$B_2(1 - e^{-(S_\phi | R = \min(R_\phi))\lambda_\phi / B_2}) - B_1 = 0. \tag{45}$$

E.4 Generation of correlated defaults

Traditionally, there are two approaches to modelling firm defaults: structural and reduced-form models. A structural model analyses the stochastic processes determining the unobserved market value of the asset (AVL) time-series of each firm. These variables represent the market-perceived values of the assets of each firm, given the known values of equity and its instantaneous volatility. If AVL of a firm falls below the value of its liabilities, the firm defaults. The correlation between AVLs of different firms also determines correlation between default event. By contrast, the reduced-form approach says nothing about the underlying processes; it rather directly models firms' defaults and correlations among them. An example of how to translate macro-economic shocks into correlated firm defaults in the stress testing context present (Tente, Westernhagen, and Slopek, 2019). They propose a copula model for tail-dependent country-specific and sector-specific factors in spirit of the reduced-from approach. In this paper, however, we follow the structural approach. As in (Merton, 1973), we model the data generating process of the unobserved AVLs as geometric Brownian motion with SDE:

$$\frac{dA_j}{A_j} = \mu_j dt + \sigma_j dW_j, \tag{46}$$

where $dW_j \sim N(0, dt)$. The components of the MLE estimator for the vector parameter $\theta = (\mu, \sigma)$ can be easily found to be

$$\hat{\sigma} = \sqrt{\frac{\sum_t (x_t - \sum_\tau x_\tau)^2}{(N-1)dt}}, \tag{47}$$

$$\hat{\mu} = \frac{\sum_t x_t}{N dt} + \frac{\hat{\sigma}}{2}. \tag{48}$$

As a key component, we additionally introduce the following correlation between the driving noises:

$$\mathrm{corr}(dW_i, dW_j) = \mathrm{corr}(d \log A_i, d \log A_j). \tag{49}$$

This is, thus, equivalent to considering

$$d \log \mathbf{A} = \mu dt + d\mathbf{Z}, \tag{50}$$

with $d\mathbf{Z} = \Sigma \cdot d\mathbf{W}$, and

$$\Sigma \cdot \Sigma^T = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \dots \\ \vdots & \ddots & \vdots \\ \rho_{N1}\sigma_N\sigma_1 & \dots & \sigma_N^2 \end{bmatrix}, \tag{51}$$

where $dW_i dW_j = \rho_{ij} dt$.

We estimate the parameters of the assumed data generating processes using a one-quarter rolling window for our quarterly time series and can, then, proceed with the generation of default events.

Each firm's probability of default is given by

$$P(A_j(t) \leq K_j) = \int_{[0,K_j]} p(x,t)dx, \tag{52}$$

where the threshold K_j is computed as current liabilities plus half of all future long-term liabilities. We can sample the points $\bar{x}_1(t), ..., \bar{x}_N(t)$ from $p(x,t)$ and, thus, estimate equation (52), using a *Monte Carlo* approach, as

$$I_N = \frac{1}{N} \sum_i \mathbb{1}_{[0,K_j]}(\bar{x}_i(t)), \tag{53}$$

so that:

$$\lim_{N \to \infty} I_N = \int_{[0,K_j]} p(x,t)dx. \tag{54}$$

The variance of the estimator (53) equals σ^2/N and the speed of convergence is given by $\sigma/\sqrt{N}$. Each iteration of the *Monte Carlo* simulation delivers a vector $\vec{\theta}$ of *correlated* default events

$$\theta_i = \mathbb{1}_{[0,K_j]}(\bar{x}_i(t)) = \begin{cases} 1, & \text{if firm } i \text{ defaults} \\ 0, & \text{otherwise} \end{cases}. \tag{55}$$

Notice that the cross-correlations of the underlying processes ρ_{ij} are not identical with the correlations of default events. The latter are given as:

$$\mathrm{corr}(\theta_1,\theta_2) = \frac{\mathbf{E}[\theta_1\theta_2] - \mathbf{E}[\theta_1]\mathbf{E}[\theta_2]}{\sqrt{(\mathbf{E}[\theta_1] - \mathbf{E}[\theta_1]^2)(\mathbf{E}[\theta_2] - \mathbf{E}[\theta_2]^2)}}, \tag{56}$$

$$\mathbf{E}[\theta_1\theta_2] = \int_{[0,T_1]}\int_{[0,T_2]} p(x_1,x_2,\rho,t)dx_1dx_2. \tag{57}$$

E.4.1 Macro-financial impacts on firms processes

Given the estimated time series of parameters obtained in the above passage, the next, optional, phase is to detect any dependency of the estimated parameters with respect to some exogenous macro-financial variables as done in standard scenario-conditional stress testing exercises. Following the ECB approach for the satellite model estimation of default probabilities, we consider a set of macro-financial variables commonly employed as regressors for ECB credit risk satellite models.

The plain vanilla example for it is an equation of the following form:

$$\hat{\theta}_{i,t} = \alpha + \beta(L)\hat{\theta}_{i,t-1} + \lambda(L)X + \epsilon_{i,t}, \tag{58}$$

where i refers to one of the estimated parameters of the data generating process, for which we can allow a lag structure, and λ represents the coefficients of the exogenous regressors. Estimation of equation (58) allows to capture the effects of an exogenous stress to the baseline path of our main macro-financial indicators on each firm's data generating process, by impacting directly the corresponding parameters. In the current exercise, we use a BMA regression to apply the macro-financial shock to the parameters of the data generating process for each entity.

E.4.2 Simulations

The estimated parameters (discretionarily stressed according to the procedure above) for the data generating process of each entity allow for *Monte Carlo* simulations of time-series paths with a one-year horizon. The random element of the simulations of each entity is determined through a multivariate Normal distribution, whose covariance matrix is based on the correlation matrix of the single firm noises and their volatility. At the end of a Monte Carlo iteration, we compare the last, end-of-path realisation of the data generation process for each entity with the entity-specific default threshold that we define using the one-year ahead Expected Default Frequency (EDF1) obtained from Moody's. If the end-of-path realisation is below this threshold, the firm defaults in the given iteration. This result, for each firm, forms the basis for the boolean vector that is fed into our contagion simulation.

E.5 Credit risk losses for aggregate exposures

We use residual exposures for banks to cover all exposures from FINREP as described in Section 3.1. Similarly to standard credit risk modelling, losses for these exposures are calculated as expectations by multiplying PDs, LGDs and exposures at default. For these exposures, shocked PDs and LGDs are derived from internal ECB models (Dees, Henry, and Reiner, 2017).[33]

Where possible, we take country-sector starting PDs ($\mathrm{PD}_{C,S}^{t_0,bank}$) from COREP reports at bank-level. For NFCs, we use country-sector average starting PDs as these are not available for this specific category in COREP. Then, given country-sector level PD multipliers ($\mathrm{PD}_{C,S}^{mult}$) for NFCs, FCs and households, the shocked PD is calculated by a starting point adjustment of top-down PDs ($\mathrm{PD}_{C,S}^{t_0,TD}$) as described in Dees, Henry, and Reiner (2017)):

$$\mathrm{PD}_{C,S}^{t,bank} = \Phi\left(\Phi^{-1}(\mathrm{PD}_{C,S}^{t_0,bank}) + \Phi^{-1}(\mathrm{PD}_{C,S}^{t_0,TD} \cdot \mathrm{PD}_{C,S}^{mult}) - \Phi^{-1}(\mathrm{PD}_{C,S}^{t_0,TD})\right).$$

For sovereigns, we use projections in levels.

Given the low variability of LGDs from stress testing experience, we use average shocked values for a country-sector breakdown conditional on the scenario:

$$\mathrm{LGD}_{C,S}^{t,bank} = \mathrm{LGD}_{C,S}^{t_0,TD} \cdot \mathrm{LGD}_{C,S}^{mult}$$

for all sectors, except for sovereigns, where we use constant values.

Finally, an expected loss for a residual exposure is given by

$$\hat{\ell}_{i,a} = l_{i,a} \cdot \mathrm{LGD}_{i,a} \cdot \mathrm{PD}_{i,a}.$$

Since only a fraction of the loans are defaulted in this case, we update the non-defaulted loan portfolio following

$$l_{i,a,t+1} = l_{i,a,t} \cdot (1 - \mathrm{PD}_{i,a,t}).$$

F Simulation details

This Appendix collects additional Figures that cover some of the main drivers of our simulation exercise. All of these Figures correspond to one simulation.

[33]Note that we do not use IFRS9 staging in the current model.

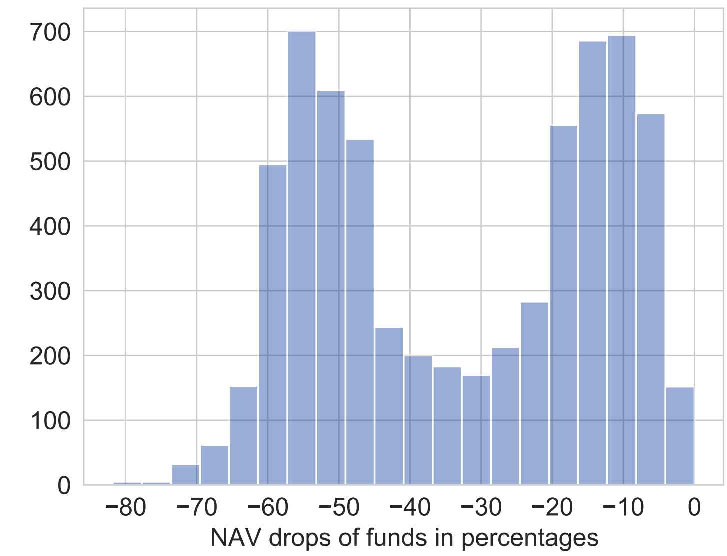


Figure 26: Distribution of declines in net asset values (NAV) for investment funds after convergence of the algorithm (percent).
Source: Authors' calculations.

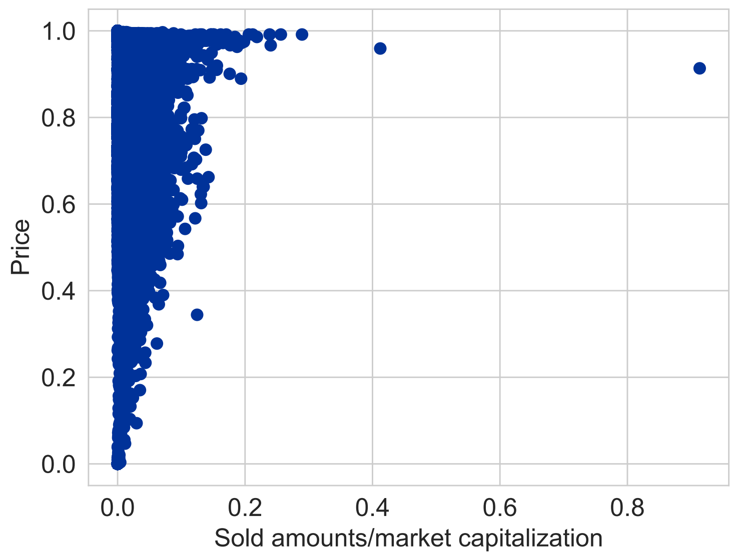


Figure 27: Sold volumes over market capitalization (decimal) and equilibrium prices (decimal; a value of 1 means no price change) after convergence of the algorithm.
Market capitalization is the total amount of holdings in the system for a given security, before the shock.
Source: Authors' calculations.

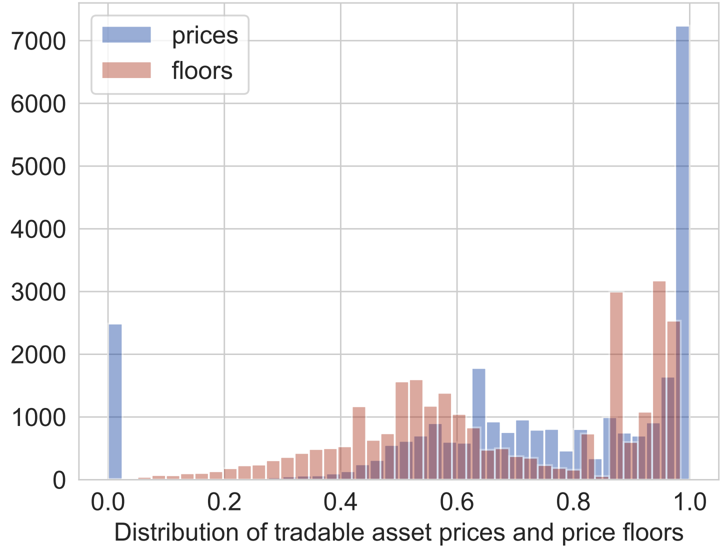


Figure 28: Distribution of tradable asset prices in equilibrium and distribution of estimated price floors after convergence of the algorithm (decimal; a value of 1 means no price change, a value of 0 the default of the issuer).
Source: Authors' calculations.

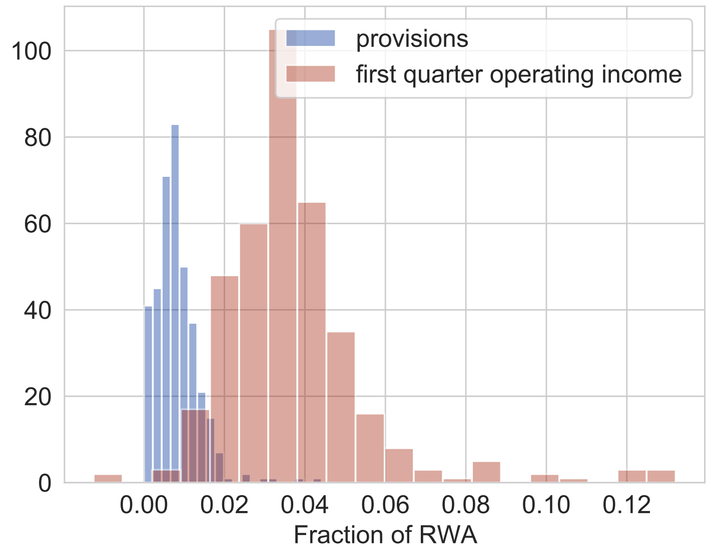


Figure 29: Distribution of the stock of provisions and first quarter net operating income as a fraction of total REA excluding provisions for banks before the start of the simulation (decimal).
Source: Authors' calculations.

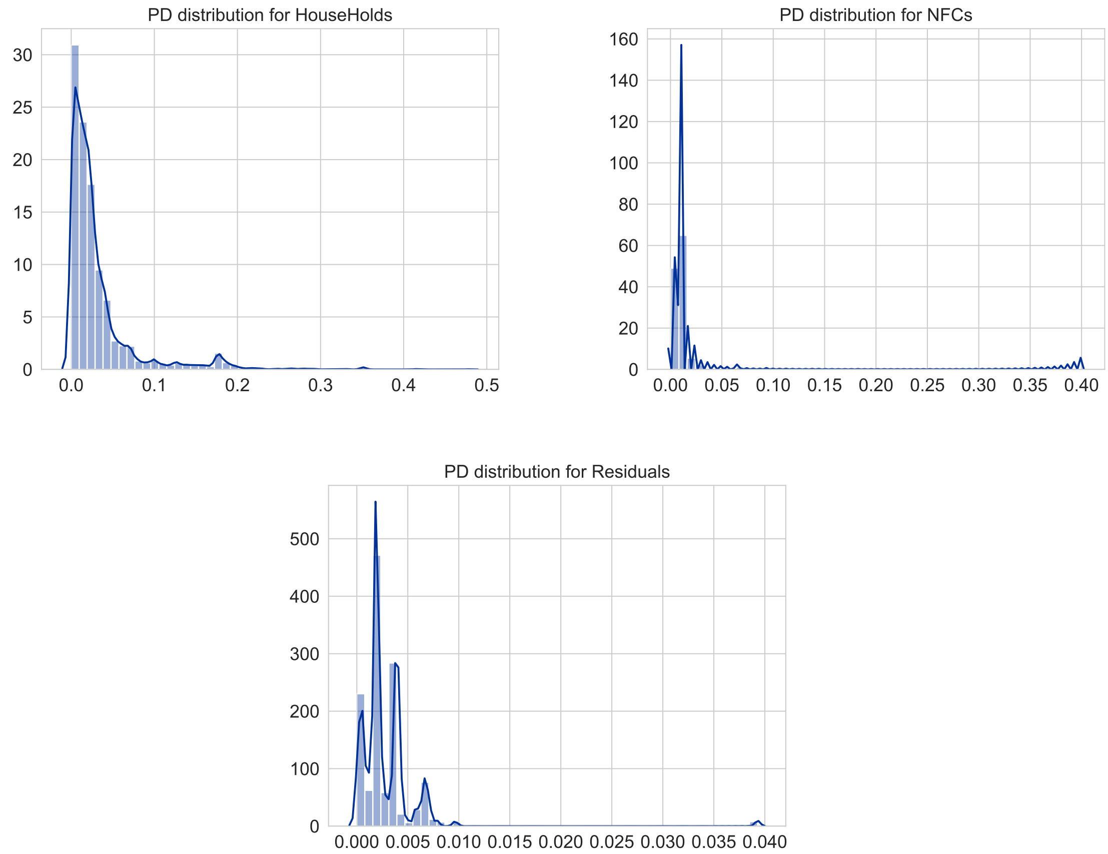


Figure 30: Distribution of probabilities of default (PDs) for different aggregate sectors (decimal).
'Residuals' cover all sectors that are not covered by the groups 'Households' and 'NFCs'.
Source: Authors' calculations.

G Results for the no-funds experiment

In this Appendix, we report additional figures for the experiment where investment funds are not included in the system, as discussed in subsection 6.3.

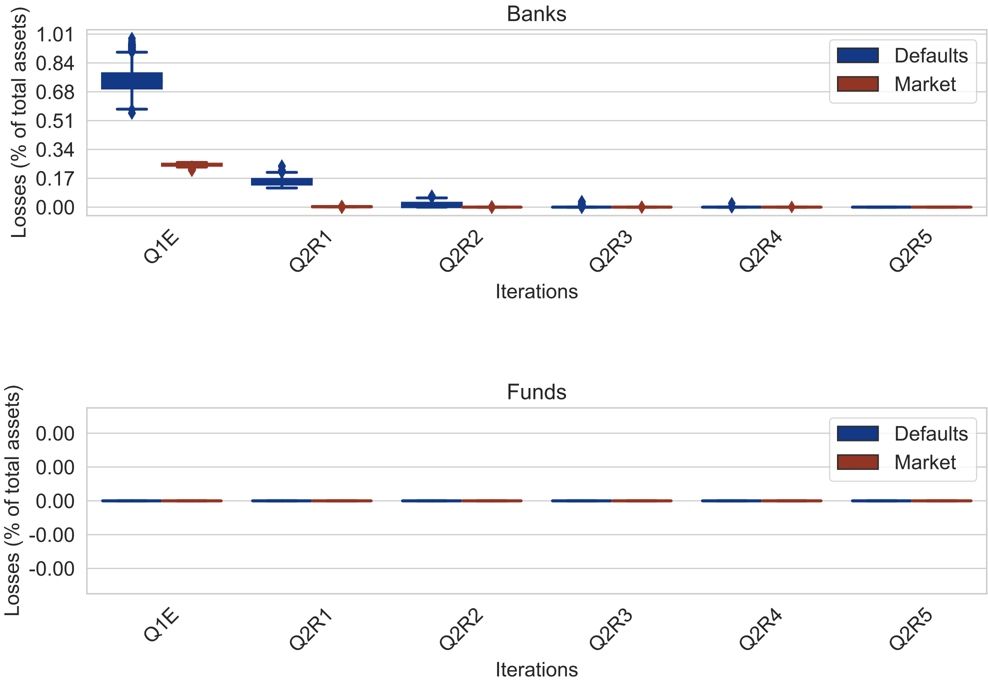


Figure 31: Market and default losses for 10000 Monte Carlo simulations, without funds (in percentage of total assets in the system of banks).
'Q1E' shows the reaction following the initial exogenous shocks in the first quarter. 'Q2R1' to 'Q2R5' represent the iterations in the second quarter until convergence of the algorithm. In 'Q1E, 'Defaults' refer to NFC defaults and 'Market' to exogenous market losses both from the market scenario and from the price drop of exogenously defaulting NFCs. From 'Q2R1' onward bank and fund defaults as well as market losses are model-driven. Candlesticks represent the 25th and 75th percentiles of the distribution of Monte Carlo simulations.
Source: Authors' calculations.

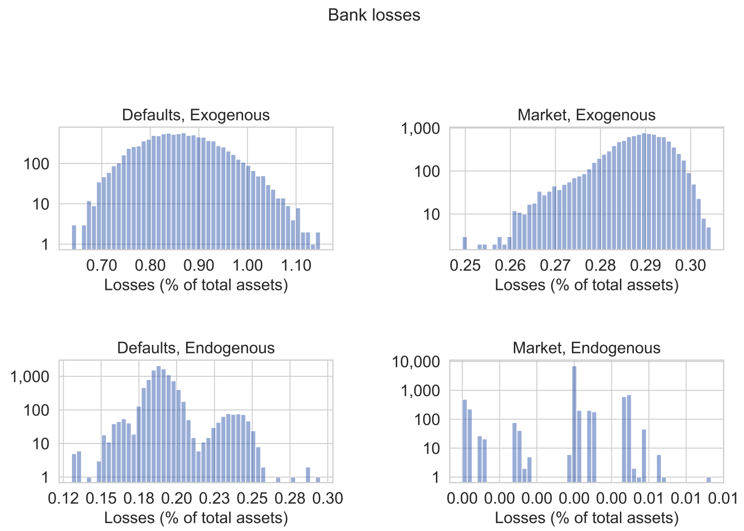


Figure 32: Losses for banks, without funds, based on 10000 Monte Carlo simulations (in percentage of total banking sector assets).
'Defaults, Exogenous' refer to NFC defaults. 'Market, Exogenous' refers to exogenous market losses both from the market scenario and from the price drop of exogenously defaulting NFCs issuing securities. 'Endogenous' losses are model-driven.
Source: Authors' calculations.

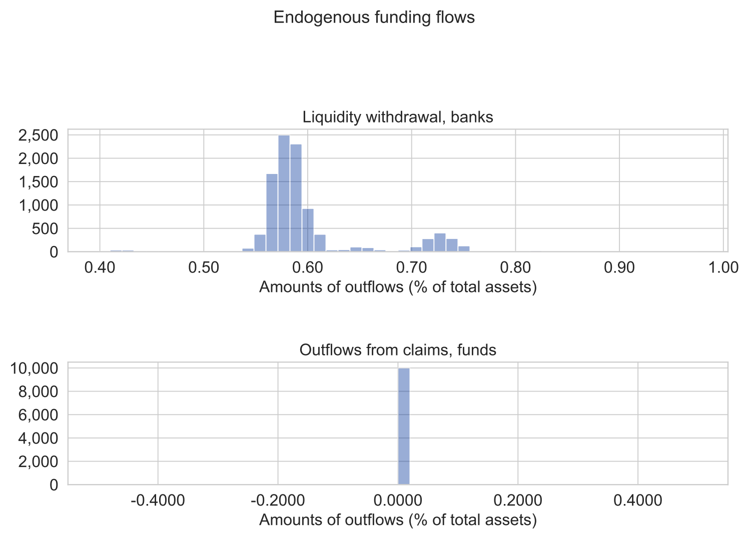


Figure 33: Funding flows for banks (without funds), following endogenous reactions, in form of liquidity withdrawal based on 10000 Monte Carlo simulations (in percentage of total assets of the respective sector).
Source: Authors' calculations.

Acknowledgements

The paper and the underlying model have been developed under the umbrella of the Financial Stability Committee's (FSC)Working Group on Stress Testing in a workstream focusing on system-wide stress testing, coordinated by Aurore Schilte and Matthias Sydow. We are grateful to Sergio Nicoletti-Altimari, Carmelo Salleo, Christoffer Kok as well as members of the FSC structures for helpful comments and discussions. We are indebted to Pierre-Alexandre Aranega and Michela Pellegrino for outstanding research assistance. The opinions presented in this paper are those of the authors alone, and do not represent the views of the European Central Bank or the Eurosystem.

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PDF ISBN 978-92-899-4804-3 ISSN 1725-2806 doi:10.2866/68326 QB-AR-21-072-EN-N