EUROPEAN CENTRAL BANK

# **Working Paper Series**

Jean-Edouard Colliard, Thierry Foucault, Peter Hoffmann Inventory management, dealers' connections, and prices in OTC markets



**Disclaimer:** This paper should not be reported as representing the views of the European Central Bank (ECB). The views expressed are those of the authors and do not necessarily reflect those of the ECB.

Abstract: We propose a new model of trading in OTC markets. Dealers accumulate inventories by trading with end-investors and trade among each other to reduce their inventory holding costs. Core dealers use a more efficient trading technology than peripheral dealers, who are heterogeneously connected to core dealers and trade with each other bilaterally. Connectedness affects prices and allocations if and only if the peripheral dealers' aggregate inventory position differs from zero. Price dispersion increases in the size of this position. The model generates new predictions about the effects of dealers' connectedness and dealers' aggregate inventories on prices.

*Keywords*: OTC markets, Interdealer trading, Inventory management. *JEL Codes*: G10, G12, G19.

#### **Non-technical Summary**

Many important financial assets (e.g. bonds, derivatives, currencies) trade in decentralized markets, often referred to as "over-the-counter" (OTC) markets. In these markets, dealers play a key role because they intermediate trades between end-investors. To do so, they accumulate substantial inventory positions. These positions are costly to hold, and are well-known to have a significant bearing on market liquidity.

In order to minimize their inventory costs, dealers trade among each other in the inter-dealer market. However, dealers in OTC markets are typically heterogeneous. While some are wellconnected, others have only few trading connections, and thus may find it more difficult to adjust their portfolio in the desired direction.

This paper proposes a model of trading that studies the *joint* effects of dealers' connectedness and inventory costs on prices and allocations in a decentralized OTC market. Consistent with stylized facts, we assume a two-tiered structure of a densely connected "core", and a more loosely connected "periphery". While trade is frictionless in the core, peripheral dealers bargain over the price with a limited set of other peripheral dealers. Importantly, only some of them are "connected" and are able to trade with core dealers.

The price in the core acts as a reference point for peripheral dealers because it is the price at which connected dealers can trade when their bilateral negotiations fail. Accordingly, changes to this price trickle down to the periphery, and affect the relative bargaining position of buyers and sellers.

Given the structure of the model, there are two sources of market power among peripheral dealers. First, connected dealers have the option to resort to trading in the core, which improves their bargaining position relative to unconnected dealers. Second, a dealer's inventory position relative to that of his/her competitors matters. In a market that is mainly populated by buyers, a seller will find it easy to get his/her trade done. In contrast, a buyer will have a hard time finding a seller, and may thus not be able to trade. Importantly, we show that there is price dispersion in the periphery (a sign of some traders exerting market power over others) if and only if both frictions are present, i.e. if some dealers are unconnected and, at the same time, there is an aggregate imbalance between buyers and sellers.

Our model makes precise predictions concerning how empirical researchers can measure the value of connectedness in OTC markets, a topic that has deserved considerable attention in the literature.

We also illustrate these insights using simulated data, and show that failing to control an interaction effect of a dealers connectedness and peripheral dealers' aggregate inventory position can lead to incorrect inference.

# 1 Introduction

Many assets (fixed income securities, securitizations, currencies etc.) trade in over-thecounter (OTC) markets. Given their size and economic function, understanding the process of price formation and liquidity provision in these markets is important. Dealers play the lead role in this process: they set prices at which investors can immediately execute their trades. As a result, they build up substantial inventory positions. Recent evidence shows that inventory holding costs have significant liquidity and price effects in OTC markets. For instance, Friewald and Nagler (2019) find that these costs explain a larger fraction of yield spreads changes in corporate bonds than search frictions.<sup>1</sup>

Dealers manage their inventory positions by trading with each other.<sup>2</sup> Thus, the effects of inventory holding costs in OTC markets are likely to depend on the structure of the interdealer market. Recent empirical findings show that dealers do not trade with each other randomly. Instead, they establish persistent trading relationships and the size of their trading network affects the prices at which they trade among themselves and with their clients (Di Maggio, Kermani, and Song (2017), Hollifield, Neklyudov, and Spatt (2017), Li and Schürhoff (2019)). Moreover, some dealers ("core dealers") are highly interconnected while others ("peripheral dealers") are more sparsely connected to other dealers (e.g., Li and Schürhoff (2019)).

Given these observations, we propose a new model of trading in OTC markets with a focus on the *joint* effects of inventory *and* network frictions on prices and allocations. Our main novel message is that these effects are interdependent. In our model, a better connected peripheral dealer derives market power from his network position and he therefore trades at more favorable prices. However, the magnitude of this effect depends on whether the dealer is on the crowded side of the market (e.g., a seller when the majority of dealers are sellers), and on the size of peripheral dealers' aggregate inventory. As a

<sup>&</sup>lt;sup>1</sup>See also Anderson and Liu (2019) and Randall (2015a). The importance of inventory holding costs for understanding prices in equity markets is also well-documented (see Hendershott and Menkveld (2014) for recent evidence).

<sup>&</sup>lt;sup>2</sup>For instance, Schultz (2017) reports that about half of the corporate bonds trades in his sample are due to interdealer trading. He also shows that these trades mainly serve to reduce inventories for both parties. See also, e.g., Li and Schürhoff (2019), Collin-Dufresne, Junge, and Trolle (2020), Hollifield, Neklyudov, and Spatt (2017), or Anderson and Liu (2019).

result, the size of bid-ask spreads charged by dealers to their clients depends on dealers' aggregate inventory, in contrast to standard inventory models (e.g., Ho and Stoll (1983), Biais (1993)). We explain below the underlying economic intuition and the implications of our theory for empirical work.

There are three groups of dealers in our model: core dealers, connected peripheral dealers, and unconnected peripheral dealers. Dealers trade with their clients and then trade with each other. They make their trading decisions to maximize their expected trading revenues net of their expected inventory holding costs, which increase with the position they hold after interdealer trading. Differences in dealers' positions generate gains from trade: when a dealer with a short position buys the asset from a dealer with a long position, both dealers reduce their inventory holding cost.

Trading among core dealers is frictionless and competitive.<sup>3</sup> In contrast, peripheral dealers bargain bilaterally with a limited set of other peripheral dealers, reflecting the sparsity of connections among these dealers. In addition, peripheral dealers are heterogeneously linked to core dealers, as is the case in reality (see Section 3.3).<sup>4</sup> Moreover, the net aggregate inventory of peripheral dealers is not necessarily zero. For instance, there can be more peripheral dealers with a long position (who want to sell) than peripheral dealers with a short position (who want to buy). The peripheral market is then crowded on the sell side and peripheral sellers are less likely to find a counterparty than peripheral buyers.

We solve for equilibrium allocations and prices both in the dealer-to-customer (D2C) and the interdealer (D2D) markets. In the D2D market, we characterize (i) the offer made by each type of peripheral dealer (connected or unconnected) given his inventory position, (ii) the likelihood that this offer is accepted by another peripheral dealer, and (iii) the price in the core market. This characterization enables us to compute the expected unwinding price for long and short positions, and to tie the trading costs of clients to the conditions

<sup>&</sup>lt;sup>3</sup>This assumption captures the idea that the high level of interconnectedness among core dealers should lead to a frictionless outcome. In practice, there exist trading venues that are only accessible to core dealers. Centralization is achieved either via inter-dealer brokers (e.g., ICAP, BGC Partners, or GFI) who match buy and sell orders, or via limit order book markets (to mention only two examples, de Roure, Moench, Pelizzon, and Schneider (2018) document this structure for the European sovereign bond market and Amstad and He (2019) for the Chinese corporate bond market).

<sup>&</sup>lt;sup>4</sup>For simplicity, we assume that some dealers have no connection to core dealers. As explained in Section 3.3, this assumption can be relaxed. The important point is that some peripheral dealers obtain a better price from core dealers because they have more trading relationships with these dealers.

in the D2D market.

The price in the core market is an important reference point for negotiations among peripheral dealers because connected peripheral dealers can trade at this price if their negotiations with other dealers fail. Consider a positive shock to core dealers' aggregate inventory (e.g., due to a new bond issuance). As is standard in models with inventory holding costs (e.g., Grossman and Miller (1988)), core dealers adjust the price at which they trade downward after such a shock. This adjustment trickles down to the periphery: it improves the connected buyers' outside option and weakens the connected sellers' outside option. In equilibrium, this effect leads all peripheral dealers to offer lower prices. Prices in the periphery also depend on the peripheral dealers' aggregate inventory position. When peripheral dealers have an aggregate long (short) position, trades among peripheral dealers take place at lower (higher) prices, holding the core market price constant. Thus, a first source of market power for peripheral dealers stems from being on the uncrowded side of the market.

The second source of market power comes from a dealer's connectedness. Interestingly, we find that it plays a role if and only if the net aggregate inventory in the periphery is different from zero. Indeed, when only one source of market power is present, all trades in the periphery occur at the core price. When both are present, their combined effects generate price dispersion: a connected buyer trades the asset at a lower average price than an unconnected buyer, and a connected seller trades the asset at a higher average price than an unconnected seller.

The difference in the average price at which connected and unconnected dealers trade is a measure of the effect of connectedness. The model predicts that, for dealers on the crowded side, this effect increases with the size of peripheral dealers' aggregate inventory. To grasp the intuition, consider the case in which there are more sellers than buyers in the periphery. As the number of sellers increases, it is more likely that a seller will not find a buyer, which reduces his market power. However, this effect is weaker for a connected seller, who has the option to trade with core dealers. Thus, the wedge between prices at which connected and unconnected sellers trade becomes larger. This finding has implications for empirical work. For instance consider a regression explaining the prices in the interdealer market. Our model implies that the regression should not only control for a dealer's connectedness, but also include an interaction term of the dealer's connectedness with his inventory position relative to peripheral dealers' aggregate inventory. We illustrate this point using data generated by simulations of our model in Section 5.2.

Another implication is that a positive shock to core dealers' aggregate inventory increases the dispersion of prices in the interdealer market if peripheral dealers have an aggregate long position in the asset, and decreases it otherwise. The reason for this asymmetry is that such a shock differentially affects connected dealers on the crowded and uncrowded sides. In equilibrium, the highest (resp. lowest) prices observed are offered by connected sellers (resp. buyers). When there is a positive shock to core dealers' aggregate inventory, all connected dealers reduce their offered prices. However, if connected sellers are on the crowded side, they are more likely to trade with core dealers. Hence, their offers are more elastic to the shock than the offers made by connected buyers. This implies that, following such a shock, the highest equilibrium price decreases more than the lowest price, and dispersion decreases. The opposite happens if the market is crowded on the buy side.

The model also generates several implications for D2C prices. First, connected peripheral dealers charge smaller bid-ask spreads than both core dealers and unconnected dealers. Indeed, due to their market power in the periphery, they can unwind their positions at a price more favorable than the core price. Thus they earn a larger trading surplus than other dealers on average, and pass through a fraction of this surplus to their clients. The flip side is that unconnected dealers charge larger bid-ask spreads than core dealers because connected dealers' trading surplus is obtained at their expense. On average (across all peripheral dealers), peripheral dealers charge larger bid-ask spreads than core dealers, reflecting the fact that the latter have access to a more efficient trading technology. Thus, the model implies a centrality discount, as found in Hollifield, Neklyudov, and Spatt (2017).

Second, an increase in the size of peripheral dealers' aggregate inventory reduces connected dealers' bid-ask spreads. Indeed, as the imbalance between peripheral buyers and sellers becomes larger, connected dealers gain market power and obtain increasingly better prices when trading with unconnected dealers. Thus, gains from trade with their clients are larger and therefore the latter obtain better prices, holding their bargaining power constant. Third, an increase in the size of core dealers' aggregate inventory has a positive effect on connected dealers' bid-ask spreads when peripheral dealers have a long position in aggregate, and a negative effect otherwise. This prediction follows from the aforementioned asymmetry of the effect of core dealers' aggregate inventory on price dispersion. These last two implications contrast with those of standard inventory models in which the size of dealers' inventory affects the bid-ask midpoint but not the bid-ask spread. Therefore, they offer a sharp way to test whether the mechanisms in our model have explanatory power for dealers' prices.

Finally, we show that the total gains from trade (for dealers and their clients) are inversely related to the inefficiency of inventory allocations in the interdealer market. Thus, any inefficiency in interdealer trading directly translates into higher trading costs for clients. Moreover, we show that the equilibrium outcome is different from the first-best when the two sources of market power for dealers are present. In this case, the trading costs of clients are inefficiently high. Otherwise (when all dealers are connected or when dealers' aggregate inventory is nil), the trading outcome is efficient (and clients' trading costs are minimal), as obtained when interdealer trading is centralized.

# 2 Contribution to the Literature

Our paper is related to models of decentralized trading in asset markets in exogenously specified trading networks. We briefly review the most related papers in this literature.<sup>5</sup>

Malamud and Rostek (2017) analyze a model in which strategic CARA agents trade in multiple, but not necessarily all, markets. Their main result is that market fragmentation (the fact that some investors are not connected to all markets) can improve welfare. In their model all agents trade at the same price in a given market (set in a uniform price double auction). In contrast, in our model, prices in the peripheral market are dispersed

<sup>&</sup>lt;sup>5</sup>Some papers endogenize the existing structure of OTC markets, and in particular three features: (i) The presence of intermediaries (dealers) (Atkeson, Eisfeldt, and Weill (2015), Farboodi, Jarosh, and Shimer (2017), Farboodi, Jarosch, Menzio, and Wiriadinata (2019), or Chang and Zhang (2016)); (ii) the core-periphery structure of the dealer market (e.g., Neklyudov (2019), Sambalaibat (2018), Wang (2017)); and (iii) the fragmentation of dealers' clientele (Babus and Parlatore (2018)).

because they are set by bilateral bargaining between dealers with different outside options. This approach enables us to relate the average price obtained by a peripheral dealer in a given trade to (i) his connectedness, (ii) his counterparties' connectedness and (iii) his inventory position relative to peripheral dealers' aggregate inventory position. In contrast, in Malamud and Rostek (2017), a dealer's market power in a given market is not determined by his inventory position.<sup>67</sup>

In Eisfeldt, Herskovic, Rajan, and Siriwardane (2018), investors trade bilaterally with the set of investors they are connected to. Investors are averse to counterparty risk, which results in imperfect risk sharing and price dispersion even though all investors behave competitively. In contrast, there is no counterparty risk in our model, but dealers behave strategically. This feature, combined with heterogeneity in connectedness and inventory positions, generates price dispersion.

Other related models analyze other frictions than inventory holding costs. Thus, our model and these are complementary in the sense that they build on different economic forces that could all jointly affect trading outcomes in reality. For instance, in Babus and Kondor (2018), dealers receive private signals about the payoff of a risky asset, but bear no inventory holding costs. A dealer's connectedness and his counterparties' connectedness matter for the terms of trade because they affect the resulting information asymmetry between the two.<sup>8</sup> Differently, in our model, they matter because they affect dealers' outside options, and this effect interacts with dealers' inventory position relative to peripheral dealers' aggregate inventory position.

Gofman (2014) considers a model of bilateral trading for one unit of an indivisible asset and studies whether an efficient outcome is reached, depending on the network structure.

<sup>&</sup>lt;sup>6</sup>In Malamud and Rostek (2017), a trader has more market power in a given market if he affects the clearing price more by shading his demand for the asset in this market. This impact is endogenous and independent of traders' inventory positions (e.g., when trading is centralized, the price impact of trader i depends on traders' risk aversions and the risk of the asset, but not traders' endowments (inventories); See Proposition 1 in their paper).

<sup>&</sup>lt;sup>7</sup>These differences in economic mechanisms between our paper and Malamud and Rostek (2017) also apply to Gallien, Kassibrakis, Malamud, Klimenko, and Teguia (2018), who consider a special case of Malamud and Rostek (2017) in which the interdealer market is centralized, and analyze implications of dealers' market power in the interdealer market for clients' trading costs.

<sup>&</sup>lt;sup>8</sup>For instance, in Babus and Kondor (2018) a dealer bears smaller trading costs when he trades with better connected dealers because more central dealers are less concerned by adverse selection. See their Figure 5, Panel F.

As the asset is indivisible, the distribution of aggregate inventories across different segments of the network plays no role, in contrast to our model.

Some papers (e.g., Afonso and Lagos (2015), Hugonnier, Lester, and Weill (2019), Neklyudov (2019), or Yang and Zeng (2019)) are also related because they allow for decentralized (and non frictionless) trading between dealers in the Duffie, Garleanu, and Pedersen (2005) framework.<sup>9</sup> Our modelling approach differs in many respects, reflecting the fact that we do not address the same questions. The most important difference is that in our model dealers are heterogeneous in their connections to core dealers. Thus, we can generate predictions about the effects of dealers' connectedness on trading outcomes and how these effects interact with their inventory position.

Our paper is also connected to models in which some market participants trade bilaterally (similar to peripheral dealers in our model) and only a subset of them can trade in a centralized trading platform (similar to core dealers). In particular, Dugast, Weill, and Uslu (2018) focus on the decision whether to connect to the centralized platform and show conditions under which mandating centralized trading improves welfare. In Dunne, Hau, and Moore (2015), clients trade bilaterally with dealers, who have access to a centralized market. There is no heterogeneity among dealers in their framework, while it plays an important role in ours.

# 3 Model

We first describe the agents in our model (Section 3.1) and how they trade together (Section 3.2). Then, in Section 3.3, we discuss our key modeling assumptions.

# 3.1 Market Participants and Timing

We consider the market for an asset populated by clients ("end-investors") and dealers. Dealers are split into: (i) core dealers, who can be viewed as large and tightly interconnected broker-dealers; (ii) peripheral dealers, representing smaller intermediaries with fewer

<sup>&</sup>lt;sup>9</sup>Most of the literature on interdealer trading with inventory holding costs (e.g., Ho and Stoll (1983), Vogler (1997), Randall (2015b), Babus and Parlatore (2018) or Anderson and Liu (2019)) assumes that trading among dealers is centralized.

trading relationships. Importantly, some peripheral dealers are more poorly connected to core dealers. We model this in a stylized way by assuming that a fraction  $\lambda \leq 1/2$  of peripheral dealers are "unconnected" and cannot trade with core dealers.<sup>10</sup>

# [INSERT FIGURE 1]

The model has three periods  $t \in \{1, 2, 3\}$ , see Figure 1. The asset payoff, v, is realized in period 3. Its distribution is common knowledge to all agents and we normalize  $\mathbb{E}(v) = 0$ . In period 1, each client is randomly matched with one and only one dealer (there are no trades directly between clients). A client's final payoff if she trades  $q \in \{-1, 0, 1\}$  shares of the asset with a dealer at price p is  $U(q, \ell, p) = q(v + \ell - p)$ , where  $\ell \in \{-L, L\}$  is the client's private valuation for the asset.<sup>11</sup>

Dealers start with no position in the asset. After trading with a client, dealer *i* has a position  $z_{i0} = -1$  if he sold the asset to the client, or  $z_{i0} = +1$  if he bought the asset from him. Dealers incur a per unit cost  $C^s$  (resp.,  $C^b$ ) if they hold a long (resp., short) position in the asset in period 3. This cost captures, in reduced form, dealers' risk aversion (as in Stoll (1978)) or the cost of funding a long (short) position in the asset overnight (as in Huh and Infante (2018)). Let  $C(x) = C^s \cdot x$  if x > 0 and  $C(x) = C^b \cdot |x|$  if x < 0. None of our results requires  $C^s \neq C^b$  but distinguishing the cost of short/long positions helps to understand the results. The final payoff of dealer *i* is:

$$\Pi_{i3}(z_{i3}, m_{i3}) = v \cdot z_{i3} - C(z_{i3}) + m_{i3}, \tag{1}$$

where  $z_{i3}$  and  $m_{i3}$  are, respectively, the asset and cash holdings of dealer *i* in period 3. As is often assumed in models of OTC markets, a peripheral dealer's position must be equal to minus one, zero, or one at any time  $(z_{it} \in \{-1, 0, +1\})$ .<sup>12</sup> Moreover, we assume that dealers can perfectly net out long and short positions. The heterogeneity in dealers'

<sup>&</sup>lt;sup>10</sup>The assumption  $\lambda \leq \frac{1}{2}$  limits the number of cases to discuss and is sufficient to highlight the most important testable implications of the model. Section III. E in the Internet Appendix studies the case  $\lambda > 1/2$ .

 $<sup>^{11}\</sup>mathrm{We}$  refer to dealers as "he", and clients as "she".

<sup>&</sup>lt;sup>12</sup>After trading with their clients, all dealers have either a long or a short position. One can easily assume that some core dealers do not (i.e., do not trade with clients), since only their aggregate inventory position will affect equilibrium outcomes. Doing so for peripheral dealers would make the analysis more complex by adding another type of peripheral dealers. The cap on peripheral dealers' positions is required to exclude the possibility of intermediation trades between peripheral dealers (i.e., trades in which two

positions after trading with their clients generates gains from trade among dealers. For instance, suppose  $z_{i0} = 1$  and  $z_{j0} = -1$ . If dealer *i* sells the asset to dealer *j*, they both end up with a zero position and save  $C^b + C^s$ . Thus, we refer to dealers with a long position as sellers and to dealers with a short position as buyers (indexed by *s* and *b*, respectively).

Trading among dealers takes place during periods 1 and 2 as described in the next section. Let  $q_{it}$  be dealer *i*'s signed trade in period  $t \in \{1, 2\}$  with other dealers  $(q_{it} > 0$  means that dealer *i* buys in period *t*). His final inventory position is:

$$z_{i3} = z_{i0} + q_{i1} + q_{i2} + \epsilon_{i3}, \tag{2}$$

where  $\epsilon_{i3}$  represents a shock to dealer *i*'s inventory position after trading with other dealers (e.g., due to "end-of-day" transactions with his clients). For peripheral dealers, we assume that  $\epsilon_{i3} = 0$  with certainty. For core dealers, the  $\epsilon_{i3}$  are i.i.d. random variables, with  $\Pr(\epsilon_{i3} \leq x) = \Phi(x/\kappa)$ , with  $\kappa$  a constant defined below.  $\Phi(.)$  is a continuous cumulative probability distribution with support over  $\mathbb{R}$  and symmetric around 0. We explain the role of this assumption in Section 4.1. Dealer *i*'s final cash holding is:

$$m_{i3} = m_{i0} - p_{i1}q_{i1} - p_{i2}q_{i2}, (3)$$

where  $p_{it}$  is the price at which dealer *i* trades in period *t* (if he does) and  $m_{i0}$  is dealer *i*'s cash holding *after* trading with his client in period 0.

# 3.2 Trading Mechanism

Period 1 is divided into a random number  $\tilde{T}$  of subperiods, indexed by  $\tau = 1, 2...\tilde{T}$ . Each subperiod is the last with probability  $(1 - \psi)$ . In each subperiod  $\tau \leq \tilde{T}$ ,  $(\kappa + 1)$  new clients arrive. One is matched with a new peripheral dealer while the remaining  $\kappa$  clients are matched with a new core dealer. Thus,  $\kappa$  captures the core dealers' share of the D2C market. Each dealer (either core or peripheral) trades with his client at a price set by

dealers with the *same* position trade together). Studying such "intermediation chains" is beyond the scope of this paper. Importantly, they would not suppress the effects of heterogeneity among peripheral dealers in our model as long as connected dealers face a position constraint that prevents them from fully absorbing unconnected peripheral dealers' aggregate inventory.

Nash bargaining in which the client's bargaining power is  $\beta \in (0, 1)$ . The bargaining process between dealers and their clients is deliberately simple so as to focus the analysis on how frictions in interdealer transactions affect trading costs for dealers' clients. We set  $L > max(C^s, C^b)$  so that gains from trade always exist between clients and dealers. Thus, in equilibrium, clients with a private valuation  $\ell = +L$  buy the asset from dealers while those with a private valuation  $\ell = -L$  sell it to dealers (see below).

The rest of this section describes the arrival process for dealers' clients and how dealers are matched for inter-dealer trading.

#### 3.2.1 Peripheral market

The first client ( $\tau = 1$ ) matched with a peripheral dealer is a seller with probability  $\alpha^{pe}$ or a buyer with probability  $1 - \alpha^{pe}$ . Then, in each subperiod  $\tau > 1$ , a seller is followed by a buyer with probability  $\psi \pi_b(\alpha^{pe})$  and by a seller with probability  $\psi(1 - \pi_b(\alpha^{pe}))$ . Symmetrically, a buyer is followed by a seller with probability  $\psi \pi_s(\alpha^{pe})$  and by a buyer with probability  $\psi(1 - \pi_s(\alpha^{pe}))$ .  $\pi_b$  and  $\pi_s$  are functions of  $\alpha^{pe}$  that will be defined below. With probability  $(1 - \psi)$ , the flow of new clients stops, i.e.,  $\tau = \tilde{T}$ .

After buying (selling) the asset from (to) a client, a peripheral dealer has a long (short) position and seeks to sell (buy) the asset to avoid paying the inventory holding cost. Hence, we refer to dealers with a long (short) position as sellers (buyers). Peripheral dealers trade with each other bilaterally according to the sequence in which they are contacted by clients. Specifically, after trading with his client, peripheral dealer  $\tau$  makes a take-it-or-leave-it offer to peripheral dealer  $\tau + 1$ . Dealer  $\tau + 1$  receives the offer after trading with his client, and then decides whether to accept  $\tau$ 's offer. If he does, then dealers  $\tau$  and  $\tau + 1$  trade together and exit the market. Then, the bargaining game continues with dealer  $\tau + 2$  making an offer to dealer  $\tau + 3$ . If, instead, dealer  $\tau + 1$  rejects dealer  $\tau$ 's offer, the game continues with dealer  $\tau + 1$  making an offer to dealer  $\tau + 2$ . Dealer  $\tau$  then trades in the core market in period 2 if he is connected (see below), or does not trade otherwise. Note that if an unconnected dealer happens to have the same position as both the preceding and the succeeding dealers, then he is stuck with his initial position. This bargaining process, summarized in Figure 2, is repeated until subperiod  $\tilde{T}$ .

# [INSERT FIGURE 2]

Peripheral dealers do not know the total demand from clients,  $\tilde{T}$ . Thus, the bargaining game is stationary because each dealer has the same probability to be the last dealer in the sequence of peripheral dealers. Moreover, dealers do not know the types of other dealers (i.e., their connectedness to core dealers and their inventory position).<sup>13,14</sup>

We specify  $\pi_s(\alpha^{pe})$  and  $\pi_b(\alpha^{pe})$  as follows:

$$\pi_b(\alpha^{pe}) = \min\left(\frac{1-\alpha^{pe}}{\alpha^{pe}}, 1\right), \quad and \quad \pi_s(\alpha^{pe}) = \min\left(\frac{\alpha^{pe}}{1-\alpha^{pe}}, 1\right). \tag{4}$$

We choose this specification for two reasons. First, it is convenient to parameterize peripheral dealers' aggregate inventory in a simple way (see below). Second, and more importantly, it ensures that a buyer (among peripheral dealers) is more likely to find a seller when there are more sellers in the periphery  $(\pi_s(\alpha^{pe}) > \pi_b(\alpha^{pe}))$  if and only if  $\alpha^{pe} > \frac{1}{2}$ ). Accordingly, we say that the peripheral market is *crowded* on the sell side when  $\alpha^{pe} > \frac{1}{2}$ and on the buy side otherwise. Our specification also implies that dealers who are on the uncrowded side of the market are sure to find a counterparty when  $\psi \to 1$ . In this way, we make sure that there are no "built-in" inefficiencies due to the random arrival of customers: If a dealer on the uncrowded side of the peripheral market does not trade with another peripheral dealer, this is because of his strategic behavior, not because of matching frictions.<sup>15</sup>

<sup>&</sup>lt;sup>13</sup>It is natural to assume that dealers observe neither the total client demand, nor the inventory position of their counterparty. Moreover, in reality, it is likely that dealers are uncertain about the connectedness of their counterparties, i.e., their counterparties' counterparties (as assumed in Caballero and Simsek (2013) for instance). Indeed, the number of connections of a peripheral dealer with core dealers may vary over time and observable characteristics of a dealer might not be sufficient to resolve all uncertainty about his connectedness. For instance, we show in the Internet Appendix (Figures IB.7 and CB.7) that a trade size is only an imperfect signal of dealers' connectedness, using data discussed in Section 3.3.

<sup>&</sup>lt;sup>14</sup>As peripheral dealers do not know each others' types, the Nash bargaining approach cannot be used to model trades between dealers. As an alternative to take-it-or-leave-it offers, Section III. D of the Internet Appendix studies a version of the model with a bilateral double-auction protocol as in Duffie, Malamud, and Manso (2014). The equilibrium allocations with this protocol are identical to those obtained with take-it-or-leave-it offers.

<sup>&</sup>lt;sup>15</sup>In Section III. B of the Internet Appendix, we show that the functions  $\pi_b(\alpha^{pe})$  and  $\pi_s(\alpha^{pe})$  can be interpreted as matching technologies in a directed search model. Moreover, in Section III. F of the Internet Appendix, we extend the model by introducing search frictions so that both  $\pi_s$  and  $\pi_b$  are always less than one. While this extension complicates the analysis, the key economic mechanisms remain the same.

# 3.2.2 Core market

Each client of a core dealer is a seller with probability  $\alpha^{co}$  and a buyer otherwise. Core dealers and connected dealers trade with each other in Period 2, after the arrival of the last client, in a centralized Walrasian market. This assumption captures the idea that the tight connections among core dealers help them to trade multilaterally (rather than bilaterally) and achieve an efficient allocation of inventory holding costs as that obtained in a competitive centralized market (see Footnote 3). Moreover, it reflects the fact that connected peripheral dealers are connected to many core dealers so that they can trade with them at a competitive price (see Section 3.3 for a discussion).

## 3.2.3 Aggregate dealer inventories

Henceforth, we focus on the case  $\psi \to 1$ . This focus has two advantages. First, it simplifies the exposition and the derivation of the equilibrium. Second, in this case, the proportion of clients who buy the asset converges almost surely to  $\alpha^{pe}$  for peripheral dealers and  $\alpha^{co}$ for core dealers. Thus, the per capita aggregate inventories of peripheral dealers  $(z_0^{pe})$  and core dealers  $(z_0^{co})$  are deterministic and given by:<sup>16</sup>

$$z_0^{pe} = (2\alpha^{pe} - 1) \quad and \quad z_0^{co} = \kappa(2\alpha^{co} - 1).$$
(5)

Finally, let  $\Delta$  denote the difference between the proportions of connected dealers with long and short positions who trade with core dealers in equilibrium. These proportions, and therefore  $\Delta$ , are endogenous. We refer to this variable as peripheral dealers' order flow in the core market.

# 3.3 Discussion

#### 3.3.1 Mapping of the model to real-world interdealer markets

In our model, peripheral dealers have differential access to core dealers and trade with each other. In the Internet Appendix (Part I), we show that this is the case in the US corporate bond market (using the TRACE academic dataset) and the European interbank market

<sup>&</sup>lt;sup>16</sup>We prove this claim formally in the Internet Appendix II. B.1.

by establishing three facts about these markets:<sup>17</sup> (i) the number of peripheral dealers is large and the volume of trading between peripheral dealers accounts for a significant fraction of the total trading activity; (ii) peripheral dealers differ greatly in their number of relationships with core dealers; and (iii) a significant fraction of the trading volume of less well connected dealers is due to trading with other less well connected dealers.

The first and second facts matter the most for our paper. The first implies that modelling trades between peripheral dealers is important to interpret and predict trading outcomes in OTC markets, while the second motivates our approach with heterogeneous peripheral dealers. The third fact is consistent with our assumption that unconnected dealers can trade together (even though actual trades might be rare in equilibrium).

Real-world OTC markets are often described as having a core-periphery structure. The dealer network we consider here is more general because it allows for limited trading among peripheral dealers and heterogeneity in peripheral dealers' connectedness to core dealers (the case  $\lambda = 0$  corresponds to a "strict" core-periphery network). Our theoretical contribution is to analyze the implications of these features for the distribution of interdealer prices. We believe this analysis can generate new insights about OTC markets given the stylized facts mentioned above and complement other analyses of OTC markets as core-periphery networks. Moreover, we do not claim that peripheral dealers trade together in *all* OTC markets. For instance, in the U.S. single-name CDS market, only a few agents act as core dealers (see Eisfeldt, Herskovic, Rajan, and Siriwardane (2018)). They are well connected among each other, while the remaining "peripheral" market participants are all connected to them and do not trade with each other bilaterally (as when  $\lambda = 0$  in our model).

#### 3.3.2 Unconnected vs. Connected dealers

For simplicity, we assume that (i) peripheral dealers are either connected to all core dealers or not connected at all, and that (ii) connected dealers obtain competitive prices from core dealers. These assumptions simplify the analysis, but they can be relaxed. Section III. A of the Internet Appendix develops an extension of our model in which all peripheral dealers

<sup>&</sup>lt;sup>17</sup>We chose these two markets because they are the focus of a significant fraction of the literature on OTC markets. See references in Sections I. A.2 and I. B.2 of the Internet Appendix.

are connected to core dealers, but they differ in the number of core dealers they can request quotes from. Each core dealer responds to a quote request with a fixed probability and is selected if he posts the best price. In equilibrium, peripheral dealers with more connections obtain better priced from core dealers on average. This extension nests the model presented here, which is the special case in which peripheral dealers have either at most one connection ("unconnected") or a very large number of connections ("connected "). Our main results still hold in this more general model.

#### 3.3.3 Interpretation of dealers' aggregate inventories

One possible interpretation of our model is that it represents one trading day in an OTC market. In this case, our implications refer to daily equilibrium outcomes (e.g., the distribution of daily transaction prices) and one should interpret dealers' aggregate inventories  $(z_0^{pe} \text{ and } z_0^{co})$  as reflecting not only their clients' order flow on a given day, but also their residual aggregate inventories from the previous day (both sources of inventories would have the same effect in our model). Correspondingly,  $C^s$  and  $C^b$  should then be interpreted as overnight inventory holding costs.

# 4 Equilibrium

In Section 4.1, we derive the equilibrium in the core market taking connected dealers' order flow,  $\Delta$ , as given. Then, in Section 4.2, we solve for the equilibrium of the peripheral market, taking the price in the core market as given. In Section 4.3, we solve for the full interdealer market equilibrium by imposing mutual consistency among of the equilibrium in the core and the peripheral markets. Finally, in Section 4.4, we solve for the prices charged by dealers to their clients.

# 4.1 Equilibrium in the Core Market

Trading in the core market involves core dealers and connected peripheral dealers. First, we derive the net demand for core dealer i at price  $p^{co}$ . Using (1), a core dealer's expected

payoff if he trades  $q_{i2}$  shares of the asset is:

$$\mathbb{E}(\Pi_{i3} \mid q_{i2}; z_{i0}, m_{i0}) = m_{i0} - p^{co} q_{i2} - \bar{C}(q_{i2}; z_{i0})$$
(6)

where  $\bar{C}(q_{i2}; z_{i0}) \equiv \mathbb{E}(C(z_{i3}))$  and  $z_{i3} = z_{i0} + q_{i2} + \epsilon_{i3}$ . Thus,  $\bar{C}(q_{i2}; z_{i0})$  is core dealer *i*'s *expected* inventory holding cost upon trading  $q_{i2}$  units in period 2, given his initial position  $z_{i0}$ . We obtain that:

$$\bar{C}(q_{i2};z_{i0}) = C^s \int_{-(z_{i0}+q_{i2})}^{+\infty} (z_{i0}+q_{i2}+\epsilon) d\Phi(\kappa^{-1}\epsilon) - C^b \int_{-\infty}^{-(z_{i0}+q_{i2})} (z_{i0}+q_{i2}+\epsilon) d\Phi(\kappa^{-1}\epsilon).$$
(7)

Core dealer *i*'s optimal net demand in period 2,  $q_{i2}^{co*}$ , maximizes (6) and therefore must solve the following first-order condition:

$$p^{co} + \underbrace{C^{s}[1 - \Phi(-\kappa^{-1}(q_{i2}^{co*} + z_{i0}))] - C^{b}\Phi(-\kappa^{-1}(q_{i2}^{co*} + z_{i0}))}_{=\bar{C}'(q_{i2}^{co*};z_{i0})} = 0.$$
(8)

As the expected inventory cost  $\bar{C}$  is strictly convex in  $q_{i2}$ ,<sup>18</sup> a core dealer's expected payoff is strictly concave in his demand in period 2. Hence, (8) is necessary and sufficient for  $q_{i2}^{co*}$  to be core dealer *i*'s optimal demand, which equates the expected benefit of buying an extra unit of the asset (the expected payoff of the asset, i.e., zero) to the expected cost (the sum of the price paid in the core market,  $p^{co}$ , and the marginal expected inventory holding cost,  $\bar{C}'(q_{i2}^{co*}; z_{i0})$ ). Solving (8) yields:

$$q_{i2}^{co*}(p^{co}, z_{i0}) = -z_{i0} - \kappa \Phi^{-1}\left(\frac{p^{co} + C^s}{C^s + C^b}\right), \text{ for } p^{co} \in (-C^s, C^b).$$
(9)

A core dealer's demand decreases with the price of the asset in the core market and his initial inventory. We then deduce from (9) that core dealers' average net demand (per capita) in period 2 is:

$$q_2^{co*}(p^{co}, z_0^{co}) = -z_0^{co} - \kappa \Phi^{-1}\left(\frac{p^{co} + C^s}{C^s + C^b}\right), \text{ for } p^{co} \in (-C^s, C^b).$$
(10)

<sup>18</sup>We have  $\bar{C}''(q_{i2}) = \kappa^{-1} (C^s + C^b) \frac{\partial \Phi(-\kappa^{-1}(z_{i0} + q_{i2}))}{\partial q_{i2}} > 0.$ 

We now consider the trading of peripheral dealers in the core market. If a connected seller  $(z_{i1} = +1)$  liquidates his position in the core market, he reduces his expected inventory holding cost by  $C^s$  and increases his cash position by  $p^{co}$ . Thus, it is optimal for the dealer to fully unload his position in the core market if  $p^{co} > -C^s$ . Symmetrically, it is optimal for a connected buyer  $(z_{i1} = -1)$  to fully cover his position in the core market if  $p^{co} < C^b$ . Finally, a connected dealer with no position optimally avoids trade at any price in  $[-C^s, C^b]$ . In sum, a connected dealer's optimal demand in the core market  $q_{i2}^{pe*}(p^{co}, z_{i1})$  is  $-z_{i1}$ , for any  $p^{co} \in (-C^s, C^b)$ . Thus, the average net demand (per capita) of peripheral dealers is:

$$q_2^{pe*} = -\Delta, \text{ for } p^{co} \in (-C^s, C^b),$$

$$(11)$$

where  $\Delta$  is connected peripheral dealers' order flow.

The equilibrium price in the core market,  $p^{co*}$ , is such that the average net demand (per capita),  $q_2^{pe*} + q_2^{co*}$ , in this market is zero. Hence:

$$q_2^{co*}(p^{co*}, z_0^{co}) = \Delta.$$
(12)

Thus, in equilibrium, connected peripheral dealers who did not trade with other peripheral dealers transfer their aggregate inventory to core dealers at price  $p^{co*}$ . In this sense, core dealers provide liquidity to peripheral dealers. Combining (10) and (12), we obtain:

**Lemma 1.** Let  $z^* = \Delta + z_0^{co}$ . The equilibrium price in the core market belongs to  $(-C^s, C^b)$ and is given by:

$$p^{co*}(z_0^{co}, \Delta) = \Phi(-\kappa^{-1}z^*)C^b - (1 - \Phi(-\kappa^{-1}z^*))C^s.$$
(13)

It decreases with core dealers' initial per capita inventory,  $z_0^{co}$ , and the order flow from peripheral dealers,  $\Delta$ .

After trading, core dealers' per capita inventory position is  $z^*$ . The equilibrium price in the core market decreases with this position, as usual in models with inventory holding costs (e.g., Ho and Stoll (1983)). Note that when the number of clients per core dealer  $\kappa$  goes to infinity, the impact of peripheral dealers' order flow on core dealers' aggregate inventory becomes negligible and the equilibrium price in the core market,  $p^{co*}$ , is only determined by core dealers' aggregate inventory "per client",  $\kappa^{-1}z_0^{co} = (2\alpha^{co} - 1)$ . We refer to this case as the "thick core market" case.<sup>19</sup> Our assumption that  $\Phi(.)$  is scaled by  $\kappa$ guarantees that the final inventory shock remains sizable relative to core dealers' aggregate inventory after trading in period 2, even in the thick core market case.

Remark that a core dealer's expected inventory holding cost,  $\bar{C}(q_{i2}; z_{i0})$ , is a strictly convex function of his trade in period 2, even though his realized inventory cost is piecewise linear in his realized position at date 3. This is the reason why (i) core dealers' aggregate demand for the asset decreases continuously with the core market price and (ii) the core market price is a continuously decreasing function of core dealers' aggregate inventory (see Lemma 1). It can therefore take any values in the interval  $(-C^s, C^b)$  rather than just the extremes, as it would if core dealers faced no uncertainty on their final inventory.<sup>20</sup> This feature considerably enriches the model. Alternatively one can directly assume that core dealers' realized inventory holding costs are strictly convex instead of assuming that they receive a random inventory shock (see Section III. C of the Internet Appendix).

# 4.2 Equilibrium in the Peripheral Market

We now analyze the equilibrium in the peripheral market, taking the core market price  $p^{co} \in (-C^s, C^b)$  as given.

## 4.2.1 Equilibrium strategies

We use subscripts b and s to index buyers' and sellers' actions, and superscripts U and C to index unconnected and connected peripheral dealers' actions, respectively. Thus, there are four possible types of peripheral dealers:  $(k,i) \in \{b,s\} \times \{U,C\}$ . The offer received by a dealer can be summarized by a quantity  $q \in \{-1,0,1\}$  (q = 0 meaning no offer; q = 1 an offer to buy; and q = -1 an offer to sell), and a price p. A dealer's strategy specifies whether he accepts the offer (q, p), and, if not, which offer he makes. We focus on

<sup>&</sup>lt;sup>19</sup>Results are unchanged if core dealers can trade together before trading with connected peripheral dealers, rationally anticipating their trades with connected dealers.

<sup>&</sup>lt;sup>20</sup>In this case, core dealer's demand would be inelastic for any price in  $(-C^s, C^b)$ , exactly like connected peripheral dealers. The equilibrium price would then be  $-C^s$  if  $z^* > 0$  (excess supply) or  $C^b$  if  $z^* < 0$  (excess demand).

Markov perfect equilibria: a dealer's strategy can be contingent on his type and the offer he receives but not on the full history of the game. As dealers do not observe whether their counterparty is connected or not, their strategy cannot depend on this characteristic. We denote by  $\varphi_b(p_b)$  (resp.,  $\varphi_s(p_s)$ ) the equilibrium probability that an offer to buy (sell) at price  $p_b$  ( $p_s$ ) is accepted.

We must solve for the equilibrium strategy of each of the four types of dealers conditionally on each offer (q, p). The following remarks (i) to (iii) substantially reduce the dimensionality of the problem.

(i) Consider a dealer of type (k, i) and let  $V_k^i$  be his continuation value conditionally on *rejecting* (or not receiving) an offer. Using (1), for unconnected dealers we have:

$$V_b^U = \max_{p_b \in (-C^s, C^b)} -\varphi_b(p_b)p_b - [1 - \varphi_b(p_b)]C^b$$
(14)

$$V_s^U = \max_{p_s \in (-C^s, C^b)} \varphi_s(p_s) p_s - [1 - \varphi_s(p_s)] C^s.$$
(15)

When a connected dealer's offer is rejected, he optimally trades in the core market at price  $p^{co}$  (see Section 4.1). Consequently:

$$V_b^C = \max_{p_b \in (-C^s, C^b)} -\varphi_b(p_b)p_b - [1 - \varphi_b(p_b)]p^{co}$$
(16)

$$V_s^C = \max_{p_s \in (-C^s, C^b)} \varphi_s(p_s) p_s + [1 - \varphi_s(p_s)] p^{co}.$$
 (17)

We deduce that a buyer of type  $i \in \{U, C\}$  accepts a seller's offer at price  $p_s$  if and only if  $V_b^i \leq -p_s$ , and a seller accepts a buyer's offer at price  $p_b$  if and only if  $p_b \geq V_s^i$ .

(ii) It follows from (14)-(17) that  $V_k^C \ge V_k^U$ : Connected dealers have weakly higher continuation values than unconnected dealers with the same position because they can trade with core dealers. We deduce the acceptance probabilities  $\varphi_b$  and  $\varphi_s$ :

$$\varphi_b(p_b) = \begin{cases} 0 & \text{if } p_b < V_s^U \\ \lambda \pi_s & \text{if } p_b \in [V_s^U, V_s^C) , \varphi_s(p_s) = \\ \pi_s & \text{if } p_b \ge V_s^C \end{cases} \begin{cases} 0 & \text{if } p_s > -V_b^U \\ \lambda \pi_b & \text{if } p_s \in (-V_b^C, -V_b^U] \\ \pi_b & \text{if } p_s \le -V_b^C, \end{cases}$$
(18)

with  $\pi_s$  ( $\pi_b$ ) being the likelihood that the offer made by a peripheral buyer is received

by a peripheral seller (buyer), as defined in (4). Thus, an unconnected buyer optimally chooses an offer which is either  $p_b = V_s^U$  or  $p_b = V_s^C$ , since any other offer can be lowered without reducing its likelihood of acceptance. In choosing between these two offers, the buyer trades off his rent if the offer is accepted (higher when  $p_b = V_s^U$ ) with the risk of a rejection (smaller when  $p_b = V_s^C$ ). Symmetrically, an unconnected seller chooses an offer which is either  $p_s = -V_b^U$  or  $p_s = -V_b^C$ .

(iii) A connected dealer always has the possibility to trade at the core market price instead of accepting an offer, so that  $V_s^C \ge p^{co}$  for connected sellers, and  $V_b^C \ge -p^{co}$  for buyers. Thus, connected sellers (resp., buyers) only accept prices above (resp., below)  $p^{co}$ and therefore connected dealers never trade together. Hence, a connected buyer (seller) must choose either to make an offer at price  $p_b = V_s^U$  ( $p_s = -V_b^U$ ) or an offer with a zero chance of acceptance, which is equivalent to directly choosing to trade in the core market after rejecting an offer. The latter case is optimal when  $p_b = V_s^U > p^{co}$ .

### 4.2.2 Equilibrium characterization

The previous remarks imply:

**Lemma 2.** Any Markov-perfect equilibrium of the peripheral market is fully characterized by a strategy profile  $\Sigma^*(p^{co}, \alpha^{pe}, \lambda) = (\theta_s, \theta_b, \gamma_s, \gamma_b)$  such that:

- A seller (resp., buyer) of type  $i \in \{U, C\}$  accepts an offer  $(1, p_b)$  if  $p_b \ge V_s^i$  (resp., an offer  $(-1, p_s)$  if  $p_s \le -V_b^i$ ).

- A connected dealer of type  $k \in \{b, s\}$  who does not accept an offer trades in the core market with probability  $1 - \gamma_k$ . With probability  $\gamma_k$ , the connected dealer makes a new offer at price  $p_s^C = -V_b^U$  if k = s, or  $p_b^C = V_s^U$  if k = b.

- An unconnected dealer of type  $k \in \{b, s\}$  who does not accept an offer always makes a new offer. With probability  $\theta_k$ , the price is  $p_s^U = -V_b^C$  for k = s, or  $p_b^U = V_s^C$  for k = b. With probability  $1 - \theta_k$ , the price is  $p_s^U = -V_b^U$  for k = s, or  $p_b^U = V_s^U$  for k = b.

Thus, solving for the equilibrium of the peripheral market amounts to solving for the equilibrium strategy profile  $\Sigma^*$ . The equilibrium is in pure strategies if  $\theta_k$  and  $\gamma_k$  are all either zero or one. Otherwise, the equilibrium is in mixed strategies. Only 3 pure strategy profiles arise in equilibrium (see Lemma 3), namely:

- Active Connected Dealers (ACD):  $\Sigma^{ACD} = (1, 1, 1, 1)$ . Unconnected dealers make offers that are accepted by all dealers with an opposite trading need. Connected dealers' offers are only accepted by unconnected dealers with an opposite trading need. If a connected dealer does not accept an offer, he makes a new one.

- Inactive Connected Sellers (ICS):  $\Sigma^{ICS} = (1, 0, 0, 1)$ . Unconnected buyers make offers that are only accepted by unconnected sellers, while unconnected sellers make offers that are accepted by all buyers. If a connected buyer does not accept an offer, he makes a new one. Connected sellers trade in the core market only, and are thus inactive in the periphery.

- Inactive Connected Buyers (ICB):  $\Sigma^{ICB} = (0, 1, 1, 0)$ . Unconnected sellers make offers that are accepted by unconnected buyers only, while unconnected buyers make offers that are accepted by all sellers. If a connected seller does not accept an offer, he makes a new one. Connected buyers trade in the core market only, and are thus inactive in the periphery.

#### [INSERT FIGURE 3]

Figure 3 shows who makes offers to whom in each type of equilibrium. We refer to an equilibrium in which peripheral dealers' strategy profile is, say, of type ACD as an "ACD equilibrium."

**Lemma 3.** Let  $\omega_i = \frac{\lambda(1-\pi_k)}{1-\pi_k\lambda(2-\lambda)}$ , for  $k \in \{b, s\}$ . The unique equilibrium regime is given by:

$$\Sigma^*(p^{co}, \alpha^{pe}, \lambda) = \begin{cases} \Sigma^{ICS} & \text{if } p^{co} > (1 - \omega_b)C^b - \omega_b C^s \\ \Sigma^{ACD} & \text{if } \omega_s C^b - (1 - \omega_s)C^s < p^{co} < (1 - \omega_b)C^b - \omega_b C^s \\ \Sigma^{ICB} & \text{if } p^{co} < \omega_s C^b - (1 - \omega_s)C^s. \end{cases}$$
(19)

In the knife-edge case in which  $p^{co} = (1 - \omega_b)C^b - \omega_bC^s$  (resp.,  $p^{co} = \omega_sC^b - (1 - \omega_s)C^s$ ) then either an ICS (resp. ICB) or an ACD equilibrium obtains, or a mixed equilibrium.

Thus, the ICS equilibrium obtains when the proportion of peripheral sellers is high enough (since  $\omega_b$  increases with  $\alpha^{pe}$  and goes to 1 as  $\alpha^{pe}$  goes to one) or when the price in the core market is high enough. Two related forces explain this result. First, the wedge between the offers that connected and unconnected sellers are willing to accept becomes higher as the proportion of sellers increases, which makes it more attractive for unconnected buyers to make an offer that is only accepted by unconnected sellers. Second, as a result, connected sellers must make lower offers to attract unconnected buyers. If these offers are lower than the core market price, they are better off trading only with core dealers. For symmetric reasons, the ICB equilibrium obtains when the proportion of peripheral sellers or the core market price are low enough.

## 4.2.3 Equilibrium order flow to the core

To close this analysis, we derive the order flow from peripheral dealers in the core market,  $\Delta$ . This step is important because ultimately the price in the core market depends on this order flow (see Lemma 1).

Let  $\mu_s^{co*}(\alpha^{pe}, \lambda, \Sigma)$  and  $\mu_b^{co*}(\alpha^{pe}, \lambda, \Sigma)$  be, respectively, the proportions of connected sellers and buyers who trade in the *core market*. As explained in Section 4.1, connected sellers (resp., buyers) who trade in the core market are willing to sell (resp., buy) one unit at any price in  $(-C^s, C^b)$ . Thus, the order flow is:

$$\Delta^*(\alpha^{pe}, \lambda, \Sigma) = \mu_s^{co*}(\alpha^{pe}, \lambda, \Sigma) - \mu_b^{co*}(\alpha^{pe}, \lambda, \Sigma).$$
(20)

**Lemma 4.** For  $\alpha^{pe} \geq \frac{1}{2}$ , peripheral dealers' order flow in the core market is:

$$\Delta^*(\alpha^{pe}, \lambda, \Sigma) = \begin{cases} \frac{(1-\lambda)(1-\pi_b\lambda)}{1-\lambda\pi_b[2-\lambda-\lambda\pi_b(1-\lambda)^2]} \times (2\alpha^{pe}-1) & \text{if } \Sigma = \Sigma^{ACD}, \\ \frac{(1-\lambda)(1-\pi_b(1-\lambda(1-\lambda)))}{(1-\pi_b)(1-\pi_b\lambda)} \times (2\alpha^{pe}-1) & \text{if } \Sigma = \Sigma^{ICS}. \end{cases}$$

For  $\alpha^{pe} < 1/2$ , we have  $\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD}) = -\Delta^*(1 - \alpha^{pe}, \lambda, \Sigma^{ACD})$  and  $\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICB}) = -\Delta^*(1 - \alpha^{pe}, \lambda, \Sigma^{ICS})$ . In addition,  $\Delta^*(\alpha^{pe}, \lambda, \Sigma)$  has the following properties:

(i) It is positive if  $\alpha^{pe} > 1/2$ , negative if  $\alpha^{pe} < 1/2$ , and equal to zero if  $\alpha^{pe} = 1/2$ .

(ii) Holding  $\Sigma$  constant, it increases with  $\alpha^{pe}$ .

(iii) Holding  $\alpha^{pe}$  constant, it is higher in absolute value when connected dealers on one side only trade in the core market, i.e.,  $\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICS}) > \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD})$  for  $\alpha^{pe} > 1/2$ and  $\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICB}) < \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD})$  for  $\alpha^{pe} < 1/2$ . These properties are intuitive. For instance, when there are more sellers than buyers in the peripheral market, connected buyers are more likely to find a counterparty than connected sellers. Thus, more connected sellers than connected buyers trade in the core, which results in more sells than buys from peripheral dealers in the core ( $\Delta^* > 0$ ; point (i)). A larger number of sellers makes this order flow even more negative (point (ii)). Finally, in an ICS equilibrium, connected sellers only trade in the core market, which further increases the magnitude of the order flow from peripheral dealers (point (iii)).

# 4.3 Full Equilibrium of the Interdealer Market

The equilibrium strategy profile in the peripheral market depends on the core market price (Lemma 3). Conversely, the equilibrium core market price depends on the peripheral dealers' order flow (Lemma 1), which is determined by peripheral dealers' strategy profile (Lemma 4). The interdealer market is in a *full equilibrium* when the equilibrium of the peripheral market and the equilibrium price of the core market are mutually consistent.

**Definition 1.** A full equilibrium of the interdealer market is (i) a price in the core market  $p^{co*}$  and (ii) a strategy profile  $\Sigma^*$  for peripheral dealers, such that  $p^{co*} = p^{co*}(\alpha^{co}, \Delta^*(\alpha^{pe}, \lambda, \Sigma^*))$  (given by (13)) and  $\Sigma^* = \Sigma^*(p^{co*}, \alpha^{pe}, \lambda)$  (given by (19)).

The next proposition provides the full equilibrium  $(p^{co*}, \Sigma^*)$  in pure strategy of the interdealer market for all parameter values.<sup>21</sup>

**Proposition 1.** There exist four thresholds  $\alpha^+_{ACD}, \alpha^+_{ICB}, \alpha^-_{ACD}, \alpha^-_{ICS}$ , given in the Appendix, such that  $\alpha^+_{ICB} \ge \alpha^+_{ACD} > \frac{1}{2} > \alpha^-_{ACD} \ge \alpha^-_{ICS}$ .

1. For  $\alpha^{pe} > \frac{1}{2}$ , the unique full equilibrium strategy profile is  $\Sigma^* = \Sigma^{ICS}$  if  $\alpha^{co} < \alpha_{ICS}^-$  and  $\Sigma^* = \Sigma^{ACD}$  if  $\alpha^{co} > \alpha_{ACD}^-$ .

2. For  $\alpha^{pe} < \frac{1}{2}$ , the unique full equilibrium strategy profile is  $\Sigma^* = \Sigma^{ACD}$  if  $\alpha^{co} < \alpha^+_{ACD}$ and  $\Sigma^* = \Sigma^{ICB}$  if  $\alpha^{co} > \alpha^+_{ICB}$ .

3. For  $\alpha^{pe} = \frac{1}{2}$ , the unique full equilibrium strategy profile is  $\Sigma^* = \Sigma^{ACD}$ . Moreover, in the thick core market case  $(\kappa \to \infty)$ ,  $\alpha^+_{ACD} = \alpha^+_{ICB} = \frac{1}{2}(1 - \Phi^{-1}(\omega_s))$  and  $\alpha^-_{ACD} = \alpha^-_{ICS} = \frac{1}{2}(1 - \Phi^{-1}(1 - \omega_b))$ .

<sup>&</sup>lt;sup>21</sup>In Proposition 1, we do not report the equilibrium value of the core market price because it immediately follows from the condition  $p^{co*} = p^{co*}(\alpha^{co}, \Delta^*(\alpha^{pe}, \lambda, \Sigma^*))$  once  $\Sigma^*$  is known.

Consider first the thick core market case, for which Proposition 1 describes the full equilibrium of the interdealer market for all ( $\alpha^{co}, \alpha^{pe}, \lambda$ ). Figure 4 (Panel A) provides the equilibrium map in this case in the ( $\alpha^{pe}, \alpha^{co}$ ) space. The ICB or the ICS equilibria obtain when (i) core and peripheral dealers' aggregate inventories are of opposite signs and (ii) large enough in each segment. Otherwise, the ACD equilibrium obtains. For instance, the ICB equilibrium obtains only if (a) there is a sufficiently large excess of buyers in the peripheral market ( $\alpha^{pe} < \frac{1}{2}$ ) and (b) a sufficiently large excess of sellers in the core market ( $\alpha^{co} > \frac{1}{2}$ ). Intuitively, the price in the core market is low because sellers dominate in this market while prices in the peripheral market are relatively high because buyers dominate in this market. Hence, a connected buyer prefers to directly trade in the core market. Moreover, as connected buyers' outside option is attractive, unconnected sellers are better off making offers that are only accepted by unconnected buyers, even though this behavior raises the likelihood of not rebalancing their inventory.

### [INSERT FIGURE 4]

Figure 4 (Panel B) illustrates Proposition 1 when  $\kappa < \infty$ . While the effects of  $\alpha^{pe}$ and  $\alpha^{co}$  are identical to those in the thick core market case, the transition from the ACDstrategy profile to other profiles (ICS or ICB) is more complex and involves equilibria in mixed strategies when  $\alpha^{co} \in (\alpha^+_{ACD}, \alpha^+_{ICB})$  or  $\alpha^{co} \in (\alpha^-_{ICS}, \alpha^-_{ACD})$ .<sup>22</sup> We present a full analysis of these mixed strategy equilibria in Section II. A of the Internet Appendix.

The next proposition completes the characterization of the pure strategy equilibrium in the peripheral market with a closed-form solution for all prices.

**Proposition 2.** For 
$$k \in \{b, s\}$$
, denote  $\rho_k^C = \frac{1-\pi_k}{1-\pi_b\pi_s\lambda}$  and  $\rho_k^U = -\lambda \pi_k \rho_{-k}^C$ . In a full

<sup>&</sup>lt;sup>22</sup>The reason is as follows. For instance, suppose that  $\alpha^{co} = \alpha_{ACD}^-$  so that an ACD equilibrium obtains. Now consider a small shock  $-\xi$  to  $\alpha^{co}$ . If peripheral dealers' order flow to the core market were constant, this would increase the price in the core market and give rise to an ICS equilibrium. However, when switching from an ACD to an ICS equilibrium, the order flow does not remain constant: it experiences a discontinuous jump because suddenly all connected sellers only trade in the core market. If  $\alpha^{co} - \xi \ge \alpha_{ICS}^-$ , this countervailing effect pushes the core market price back to a level where an ACD equilibrium obtains. But then, the order flow reverts back to a lower level inconsistent with an ACD equilibrium. Thus, for  $\alpha^{co} \in (\alpha_{ICS}^-, \alpha_{ACD}^-)$ , there is no pure strategy equilibrium. The same reasoning applies to  $\alpha^{co} \in (\alpha_{ACD}^+, \alpha_{ICB}^+)$ .

equilibrium  $(\Sigma^*, p^{co*})$ , transactions among peripheral dealers occur at the following prices:

- If 
$$\Sigma^* = \Sigma^{ACD}$$
:  $p_s^U = p^{co*} + \rho_s^U(C^s + p^{co*}), \quad p_s^C = p^{co*} + \rho_s^C(C^b - p^{co*})$  (21)

$$p_b^U = p^{co*} - \rho_b^U(C^b - p^{co*}), \quad p_b^C = p^{co*} - \rho_b^C(C^s + p^{co*})$$
(22)

- If 
$$\Sigma^* = \Sigma^{ICS}$$
:  $p_s^U = p^{co*} + \rho_s^U(C^s + p^{co*}), \quad p_s^C \text{ is not observed}$  (23)

$$p_b^U = p^{co*} - \rho_b^C(C^s + p^{co*}), \quad p_b^C = p^{co*} - \rho_b^C(C^s + p^{co*})$$
(24)

- If 
$$\Sigma^* = \Sigma^{ICB}$$
:  $p_s^U = p^{co*} + \rho_s^C (C^b - p^{co*}), \quad p_s^C = p^{co*} + \rho_s^C (C^b - p^{co*})$  (25)

$$p_b^U = p^{co*} - \rho_b^U(C^b - p^{co*}), \quad p_b^C \text{ is not observed.}$$

$$(26)$$

# [INSERT FIGURE 5]

The prices posted by dealers are expressed as the sum of the core price plus  $\rho_k^i$  times the gains from trade between a connected and an unconnected dealer (either  $C^s + p^{co*}$  or  $C^b - p^{co*}$ ). The term  $\rho_k^C$  is (weakly) positive and reflects the fact that connected dealers always require an improvement over the core price. Conversely,  $\rho_k^U$  is negative because unconnected dealers need to make price concessions relative to the core price in order to attract connected dealers. Figure 5 illustrates the resulting price ordering.

Four remarks are in order. First, all prices increase with the core price. Intuitively, an increase in this price weakens buyers' continuation values when making offers, which allows sellers to raise their offers. Thus, sellers' continuation values after rejecting offers improves, so that buyers must raise their offers as well. Second,  $p_b^C$  and  $p_s^C$  are the lowest and highest prices, respectively (provided they are observed in equilibrium) because connected dealers are always able to extract the largest price concessions. Third, the relative ordering of  $p_b^U$ and  $p_s^U$  depends on the equilibrium type. For instance, when  $\alpha^{pe} > 1/2$ , sellers are on the crowded side of the market. Hence, unconnected sellers have to offer a price concession relative to the core price ( $p_s^U < p^{co}$ ). Unconnected buyers can either target connected sellers (ACD equilibrium) by offering  $p_b^U = p_s^C > p_s^U$ , or extract a price concession by offering the same price as connected buyers,  $p_b^C$ , which is below  $p_s^U$  (ICS equilibrium). Lastly, in the ACD equilibrium, connected dealers on the crowded side of the market make offers that are just equal to the core market price, i.e., they do not extract more surplus than what they can obtain by just trading with core dealers. This property, and more generally the ranking of prices in Figure 5, is best understood by analyzing the sources of dealers' market power in the model.

Define the market power of a peripheral dealer with type  $i \in \{U, C\}$  as:

$$M_b^i = \frac{C^b + V_b^i}{C^b - p^{co*}},$$
(27)

$$M_s^i = \frac{V_s^i + C^s}{p^{co*} + C^s}.$$
 (28)

To understand this definition, consider an unconnected seller. His continuation value  $V_s^U$  is equal to the price at which he expects to liquidate his position (see (15)). Thus,  $(V_s^U - (-C^s)) = (V_s^U + C^s)$  measures his trading surplus and  $M_s^U$ , his market power, is therefore his trading surplus normalized by the surplus that the unconnected seller would capture if he could trade with a core dealer (i.e.,  $(p^{co*} + C^s)$ ).

**Proposition 3.** 1. When  $\lambda = 0$  or  $\alpha^{pe} = 0$ , all peripheral dealers have equal market power  $(M_k^i = 1, \forall \{k, i\} \in \{b, s\} \times \{C, U\})$  and all transactions in the interdealer market take place at the core market price.

2. When  $\lambda > 0$  and  $\alpha^{pe} \neq \frac{1}{2}$ , dealers on the uncrowded side of the market have more market power than dealers on the crowded side, holding their connectedness constant (i.e.,  $M_b^i > M_s^i$  if and only if  $\alpha^{pe} > \frac{1}{2}$  for  $i \in \{C, U\}$ ). Moreover, connected dealers with a given position have more market power than unconnected dealers with the same position  $(M_k^C > M_k^U)$  for  $k \in \{b, s\}$ ).

3. Buyers' market power weakly increases with peripheral dealers' aggregate inventory  $\left(\frac{\partial M_b^i}{\partial z_0^{pe}} \ge 0\right)$  while sellers' market power weakly decreases with peripheral dealers' aggregate inventory  $\left(\frac{\partial M_s^i}{\partial z_0^{pe}} \le 0\right)$ .

Thus, there are two sources of market power for dealers, *connectedness:* and "uncrowdedness". Connected dealers have more market power than unconnected dealers, holding their position constant. Similarly, dealers on the uncrowded side of the market have more market power than dealers on the crowded side, holding connectedness constant (Part 2 of Proposition 3).

The outside option of unconnected dealers on the crowded side becomes increasingly worse as the market becomes more crowded on their side. Indeed, in this case, they become less likely to find a counterparty upon rejecting an offer. Thus, they are willing to accept increasingly worse prices, which in turn improves the outside option of dealers on the uncrowded side of the market. Accordingly, holding connectedness constant, the market power of dealers on the uncrowded (crowded) side gets larger (smaller) as crowdedness increases (i.e., peripheral dealers' aggregate inventory becomes larger in absolute value; Part 3 of Proposition 3).

The dispersion of transaction prices in the peripheral market stems from the interaction of these two sources of market power. Interestingly, price dispersion only arises if they are both present simultaneously (Part 1 of Proposition 3). When all dealers are connected, they are not willing to accept prices worse than the core price. Similarly, when peripheral dealers' aggregate inventory is nil, the likelihood of finding a counterparty is identical for buyers and sellers. This symmetry is sufficient to equalize peripheral dealers' market power across sides and all trades takes place at the core market price, even though some dealers cannot trade with core dealers.

The interaction of both sources of market power also explains why connected dealers on the crowded side trade at the core market price in the ACD equilibrium. Indeed, connectedness alone does not grant these dealers market power because their counterparties are always able to find someone else willing to trade at the core market price. In Section III. F of the Internet Appendix, we consider the more general case in which dealers on the uncrowded side of the market do not find a counterparty with probability 1. This feature is a new source of market power (absent from the baseline model) for the dealer making an offer because the recipient is himself not certain to find a counterparty upon rejecting the offer. When it is present, connected dealers trade at more favorable prices than  $p^{co}$  even when they are on the uncrowded side of the market (but the ranking of prices in Figure 5 and Proposition 3 are unchanged).

# 4.4 The Dealer-to-Client Market

We now use the characterization of the equilibrium in the interdealer market to derive the quotes at which dealers (both periphery and core dealers) trade with their clients at date 0. For brevity, we just sketch the main steps of the derivations that yield these quotes.

The full analysis is in Appendix C.8.

Let  $P_k^i$  be the price at which a peripheral dealer of type  $(k, i) \in \{b, s\} \times \{C, U, co\}$ expects to unwind his position. For instance, consider an unconnected dealer who buys the asset from a client at date 0 and then seeks to resell it to other dealers. There are four possible trading outcomes in the interdealer market for this dealer: (i) he makes an offer at  $p_s^U$  and this offer is accepted; (ii) he accepts an offer at  $p_b^U$  from an unconnected buyer; (iii) he accepts an offer at  $p_b^C$  from a connected buyer; (iv) he does not trade and pays the inventory holding cost (which is equivalent to sell at  $-C^s$ ). The price  $P_s^U$  is an average of these four prices, weighted by the probabilities that the dealer trades at each price in equilibrium.

Now consider the trade of a type k dealer with a client who wants to sell the asset. If they trade, the total gains from trade are equal to the difference between the price  $P_s^k$  at which the dealer expects to resell the asset in the interdealer market, and the client's private valuation, -L. As the client's bargaining power is  $\beta$ , the dealer buys the asset at:

$$bid^{i} = P_{s}^{i} - (1 - \beta)(P_{s}^{i} + L).$$
<sup>(29)</sup>

Symmetrically, if a dealer's client is a buyer, the dealer sells the asset at:

$$ask^{i} = P_{b}^{i} + (1 - \beta)(L - P_{b}^{i}).$$
(30)

Thus, a dealer's connectedness affects the quotes he offers if and only if this position affects the price at which he expects to trade in the interdealer market. In this case, dealers' quotes differ even though clients are homogeneous, and even when they have full bargaining power  $(\beta = 1)$ . The link between a dealer's connectedness, position in the interdealer market, and quotes generates a rich set of predictions, analyzed in the next section.

# 5 Empirical Implications

In Section 5.1, we derive implications of our model for empirical studies of OTC markets, focusing on the effects of a peripheral dealer's connectedness on the terms of trade in the

D2D and D2C markets. Our main message is that the magnitude of these effects depends on whether a dealer is on the crowded or the uncrowded side of the market and thus peripheral dealers' aggregate inventories. In Section 5.2, to better convey the resulting implications for empirical research, we show how controlling for peripheral dealers' aggregate inventories affects inferences about the effect of connectedness on transaction prices, using simulations of our model.

# 5.1 Measuring the Effects of Connectedness

In this section we show how the heterogeneity in dealers' connections leaves different types of footprints in the data, which we interpret as measures of the effects of connectedness.

# 5.1.1 Measures using dealer-to-dealer transactions

A first natural way of measuring the effect of connectedness is to compare the price obtained by a connected seller (buyer) to the price obtained by an unconnected seller (buyer) in interdealer transactions. Specifically, let  $\bar{p}^{i,j}$  be the average price at which a seller of type  $i \in \{C, U\}$  trades with a buyer of type  $j \in \{C, U\}$ . This price is the weighted average of the prices at which the seller of type i makes an offer or receives an offer from a buyer of type j, with weights corresponding to the equilibrium probabilities of each event (derived in Section II.B of the Internet Appendix). We measure the effect of connectedness by  $(\bar{p}^{C,U} - \bar{p}^{U,U})$  for sellers and  $(\bar{p}^{U,U} - \bar{p}^{U,C})$  for buyers. This effect is positive when connected dealers trade at better prices on average than unconnected dealers with the *same* position.<sup>23</sup> We obtain the following:<sup>24</sup>

**Implication 1.** In the ACD Equilibrium, the effect of connectedness is positive:  $\bar{p}^{C,U} \geq \bar{p}^{U,U} \geq \bar{p}^{U,C}$ . Moreover, for dealers on the crowded side of the market, the effect of connect-

 $<sup>^{23}</sup>$ In theory, in a given trade, the price received by the seller (buyer) depends on whether he made the offer or accepted a buyer (seller)'s offer. However, standard datasets on OTC markets (e.g., TRACE) do not report which party made the offer. Thus, to be closer to what empiricists can effectively measure, we focus on the price at which a dealer with a given position trades in the D2C market, *averaged* across the prices at which the dealer trades when he makes an offer and when he receives an offer.

<sup>&</sup>lt;sup>24</sup>In an ICS (ICB) equilibrium, connected sellers (buyers) only trade with core dealers. Thus, there are no transactions involving connected sellers (buyers) in the peripheral market and therefore  $\bar{p}^{C,U}$  ( $\bar{p}^{U,C}$ ) cannot be measured. For this reason, we state Implication 1 only when an ACD equilibrium obtains. In the other equilibrium regimes, connected dealers still trade on average at better prices than unconnected dealers but this fact cannot be inferred from transactions among peripheral dealers alone.

edness increases in the size of peripheral dealers' aggregate inventory,  $|z_0^{pe}|$ :  $(\bar{p}^{C,U} - \bar{p}^{U,U})$ increases in  $|z_0^{pe}|$  for  $z_0^{pe} > 0$  and  $(\bar{p}^{U,U} - \bar{p}^{U,C})$  increases in  $|z_0^{pe}|$  for  $z_0^{pe} < 0$ .

The intuition for these results directly follows from Proposition 3: connected dealers have more market power than unconnected dealers, and this market power increases with the aggregate imbalance in the periphery. The point that more connected dealers trade at better prices on average is in line with the empirical findings of Di Maggio, Kermani, and Song (2017) (see their Table 4). To our knowledge, the prediction that this effect should increase with aggregate inventory imbalances has not been tested.

Testing Implication 1 requires data on dealer characteristics. If this information is not available, our model suggests that the effects of connectedness can alternatively be measured by the difference between the highest and the lowest prices observed in the interdealer market, which is a measure of price dispersion in the interdealer market. Indeed, in our model, this difference (denoted *Dispersion*) is equal to the difference in the continuation values of connected and unconnected dealers on the crowded side of the market (see the proof of Implication 2):

$$Dispersion = \begin{cases} V_s^C - V_s^U > 0 & \text{if } z_0^{pe} > 0 \\ 0 & \text{if } z_0^{pe} = 0 \\ V_b^C - V_b^U > 0 & \text{if } z_0^{pe} < 0 \end{cases}$$
(31)

This difference measures the extent to which connectedness to core dealers improves a peripheral dealer's continuation value (his outside option). Thus, it is another measure of the value of connectedness. Our model predicts that *Dispersion* varies with shocks to either peripheral or core dealers' aggregate inventories, as shown by our next testable implication.

**Implication 2.** Price dispersion in the interdealer market increases in the size of peripheral dealers' aggregate inventory,  $|z_0^{pe}|$ . Moreover, it increases (resp., decreases) in core dealers' aggregate inventory  $z_0^{co}$  for  $z_0^{pe} < 0$  (resp. for  $z_0^{pe} > 0$ ).

The intuition for the first part of Implication 2 is as follows. When the size of peripheral dealers' aggregate inventory,  $|z_0^{pe}|$ , increases, dealers on the crowded side of the market

have weaker outside options in case their negotiations fail because they are less likely to find a suitable trading partner. This effect is relatively stronger for unconnected dealers because they cannot resort to core dealers. Thus, the difference in continuation values for connected and unconnected dealers on the crowded side of the market increases with  $|z_0^{pe}|$ , which captures the same economic force as the second part of Implication 1.

The second part of Implication 2 also predicts that shocks to core dealers' aggregate inventories affect price dispersion in the interdealer market, and that the sign of this effect depends on the sign of peripheral dealers' aggregate inventory. Intuitively, a positive shock to core dealers' aggregate inventory triggers a drop in the core market price, which decreases the continuation values of sellers  $(V_s^C \text{ and } V_s^U)$  and increases the continuation values of buyers  $(V_b^C \text{ and } V_b^U)$ . For dealers on the crowded side of the market, the magnitude of this effect is larger for connected dealers. Their outside option is to trade in the core market, so they are directly exposed to changes in the core market price. In contrast, unconnected dealers are only affected indirectly: They cannot trade in the core themselves, but their counterparties may be able to do so. Thus, when the market is *crowded on* the sell side  $(z_0^{pe} > 0)$ , an increase in core dealers' aggregate inventory  $(z_0^{co})$  reduces the differential in continuation values between connected and unconnected sellers and therefore price dispersion (equation (31)). Conversely, when the market is crowded on the buy side  $(z_0^{pe} < 0)$ , an increase in core dealers' aggregate inventory enlarges the differential of continuation values between both types of buyers and therefore price dispersion. To our knowledge, these predictions are new and unique to our model. They could be tested by using shocks to dealers' aggregate inventory in a given asset due, for instance, to fire sales by institutional investors.<sup>25</sup>

#### 5.1.2 Measures using inventories

The difference in the frequency with which unconnected and connected dealers end the day with an inventory position is another way to measure the effects of connectedness empirically. In our model, this difference is the likelihood that an unconnected dealer does not unwind his inventory because connected dealers unwind their inventory with certainty

<sup>&</sup>lt;sup>25</sup>Li and Schürhoff (2019) find that core dealers in the U.S. municipal bond markets are more likely to buy bonds that experience large mutual fund outflows.

before date 3 (by trading either with another peripheral dealer or with core dealers).

**Implication 3.** The likelihood that an unconnected dealer on the crowded (resp., uncrowded) side ends with an inventory position weakly increases (resp., weakly decreases) in the size of peripheral dealers' aggregate inventory,  $|z_0^{pe}|$ , and the proportion of unconnected dealers,  $\lambda$ .

When the size of peripheral dealers' aggregate inventory increases, the crowded side of the market becomes even more crowded. Thus, unconnected dealers on the crowded side are less likely to find a trading partner before period 3. The effect of an increase in the proportion of unconnected dealers,  $\lambda$ , is more subtle. In equilibrium, dealers on the crowded side make offers that are accepted both by connected and unconnected dealers. Thus, an increase in  $\lambda$  has no effect on the likelihood that their equilibrium offers are accepted. However, it reduces the likelihood that they *receive* an offer and therefore unwind their inventory in equilibrium.

### 5.1.3 Measures using dealer-to-customer transactions

As shown in Section 4.4, dealers' quotes in the D2C market (given by (29) and (30)) are determined by the price  $P_k^i$  at which they expect to offload their inventory position after trading with their clients. This expected price depends on a dealer's connectedness because, as shown previously, it affects both the average price at which he trades (Implication 1) and his likelihood of ending with an inventory position (Implication 3). Thus, quotes in the D2C market reflect the combination of these two effects.

**Implication 4.** Suppose  $\lambda > 0$  and  $z_0^{pe} \neq 0$ . Connected dealers always post more competitive quotes than unconnected dealers. Moreover, for clients on the uncrowded side of the market, connected and unconnected dealers post strictly better prices than core dealers. Last, for clients on the crowded side of the market, connected dealers post the same price as core dealers, while unconnected dealers post inferior prices.

The intuition behind this result follows from Implications 1 and 3. Connected dealers can always unwind at the same price as core dealers, and hence can offer quotes that are at least as good. Moreover, when they are on the uncrowded side of the peripheral market, they can unwind their position at a strictly better than the core market price and pass through part of this improvement to clients on the uncrowded side. Unconnected dealers on the uncrowded side can also unwind their inventory with certainty at a better price than the core market price on average. Thus, they also make better offers than core dealers to clients on the uncrowded side of the market. In contrast, unconnected dealers on the crowded side incur the inventory holding cost with a positive probability and, if not, they unwind their inventory in the D2D market at a price worse than the core market price. Thus, they offer less favorable quotes than core dealers to clients on the crowded side.

Implication 4 focuses on the case  $\lambda > 0$  and  $z_0^{pe} \neq 0$ . When either  $\lambda = 0$  or  $z_0^{pe} = 0$ , all trade in the interdealer market takes place at the core market price (see Part 1 of Proposition 3), and all dealers post the same quotes in the D2C market. Thus, our model explains the dispersion (across dealers) of quotes in the D2C market by heterogeneity in dealers' market power in the interdealer market, due to the combined effects of heterogeneity in dealers' connectedness to core dealers ( $\lambda > 0$ ) and an aggregate inventory imbalance among peripheral dealers ( $z_0^{pe} \neq 0$ ).

We define  $S^i = ask^i - bid^i$  as the (bid-ask) spread of a dealer with type  $i \in \{C, U, co\}$  in the D2C market. For instance, using (29) and (30),  $S^{co} = 2(1 - \beta)L$  because core dealers expect to trade at  $P_s^{co} = P_b^{co} = p^{co}$  in the interdealer market. We obtain the following implication:

Implication 5. When  $\lambda > 0$  and  $z_0^{pe} \neq 0$ , connected (unconnected) dealers charge smaller (greater) spreads than core dealers ( $S^C < S^{co} < S^U$ ) and the average spread of peripheral dealers is larger than the spread of core dealers ( $\lambda S^U + (1 - \lambda)S^C \geq S^{co}$ ). Moreover, core dealers's spreads are independent of peripheral and core dealers' aggregate inventories. In contrast, connected dealers' spreads decrease with  $|z_0^{pe}|$ , and increase (resp., decrease) in  $z_0^{co}$  when  $z_0^{pe} > 0$  (resp.,  $z_0^{pe} < 0$ ).

Several empirical papers compare bid-ask spreads of central and peripheral dealers in various OTC markets. While some studies (e.g., Hollifield, Neklyudov, and Spatt (2017)) document a "centrality discount" (central dealers charge smaller spreads on average), others (e.g., Li and Schürhoff (2019)) find a "centrality premium". Unconditionally, i.e. with-
out controlling for peripheral dealers' heterogeneity, our model predicts a centrality discount: the average spread across all peripheral dealers is larger than the spread charged by core dealers, reflecting the fact that the latter have access to a better trading technology.<sup>26</sup> However, as  $S^C < S^{co} < S^U$ , our model also predicts that connected dealers should charge a *smaller* spread than core dealers (a centrality premium) while unconnected dealers should charge a larger spread (a centrality discount). Thus, the effect of connectedness on the difference between peripheral and core dealers' spread in the D2C market should be negative. To our knowledge, this prediction is new and specific to our model. It follows from the fact that connected dealers share the rents obtained in the interdealer market with their clients.

This mechanism also explains why connected dealers' bid-ask spreads decrease in the size of peripheral dealers' aggregate inventory,  $|z_0^{pe}|$ . Indeed, an increase in  $|z_0^{pe}|$  means that the imbalance between buyers and sellers is stronger among peripheral dealers. Thus, connected dealers on the uncrowded side of the market have more market power, which allows them to extract larger rents from unconnected dealers and charge smaller bid-ask spread to their clients.

The same logic also explains why and how shocks to core dealers' aggregate inventory affect connected dealers' spreads (last part of Implication 5). Suppose the market is crowded on the sell side ( $z^{pe} > 0$ ). If a connected dealer buys the asset from a client, he expects to resell it with certainty at the core market price. Thus, the elasticity of his bid price to the core price is 1 (see (29)). If he sells the asset to a client, he expects to cover his short position at a price which on average is *smaller* than the core price because the market is crowded on the sell side. Specifically, the price at which the dealer covers a short position is a *weighted average* of the core price and the price at which he trades with other peripheral dealers ( $\bar{p}^{U,C} < p^{co*}$  when  $z^{pe} > 0$ ). Thus, when  $z^{pe} > 0$ , the elasticity of connected dealers' ask price to the core price is strictly less than 1 (see (30)). Consequently, an increase in core dealers' aggregate inventory (which triggers a drop in the core price) results in a larger drop in connected dealers' bid price than in their ask price and therefore an increase in their bid-ask spread. The symmetric argument applies to the case

 $<sup>^{26}</sup>$ See Uslu (2019) or Sambalaibat (2018) for other theoretical explanations of the centrality discount.

 $z^{pe} < 0$ , in which case an increase in core dealers' aggregate inventory triggers a decrease in connected dealers' bid-ask spread.

In standard inventory models (e.g., Ho and Stoll (1983) or Biais (1993)), the *size* of dealers' bid-ask spreads does *not* depend on dealers' aggregate inventories (only the midpoint of the bid-ask spread depends on dealers' aggregate inventory in these models). In contrast, it does in our model, at least for connected dealers, because aggregate inventories affect dealers' market power in the interdealer market and therefore the price at which they can unwind their inventories.<sup>27</sup>

## 5.2 Illustration using Simulated Data

In this section, we use numerical simulations to illustrate the main message of our model for empirical analyses of OTC markets, namely that controlling for peripheral dealers' aggregate inventory is important for understanding how the heterogeneity in dealers' connectedness affects price dispersion in these markets. Specifically, we simulate 250 "trading days". Each day is characterized by a new random draw of  $\alpha^{pe}$  and  $\alpha^{co}$ . Other parameters of the model remain constant across days, and we set  $\lambda = 0.4$ . Clients arrive sequentially and are matched with dealers as described in Section 3.2, and all dealers follow their equilibrium strategies. For each trade, we record (i) the price of the trade; (ii) the type of the seller (client, unconnected dealer, connected dealer, core dealer); and (iii) the type of the buyer. In addition, we record peripheral dealers' aggregate inventory positions on the day on which trades occur ( $z_0^{pe}$  and  $z_0^{co}$ ). The Internet Appendix II. F provides more details on the simulation procedure and gives a snapshot of the simulated data.<sup>28</sup>

We divide trading days into quartiles based on  $z_0^{pe}$  and estimate with OLS the following

<sup>&</sup>lt;sup>27</sup>Unconnected dealers' bid-ask spreads also depend on peripheral and core dealers' aggregate inventories. However, the effects of these inventories are in general ambiguous and depend on which equilibrium regime obtains. We thus do not to discuss them for brevity.

<sup>&</sup>lt;sup>28</sup>Our simulated data look like the data that can be directly obtained or inferred from some datasets (e.g., the TRACE academic dataset). See, for instance, Craig and von Peter (2014) or Eisfeldt, Herskovic, Rajan, and Siriwardane (2018) for methods for partitioning traders into different categories, and Friewald and Nagler (2019) for a construction of aggregate dealer inventories using data from various OTC markets.

equation separately on each subsample.

$$\hat{p}_{n,d} = \alpha + \sum_{(i,j) \in \{C,U,co\}, (i,j) \neq (co,co)} \beta^{i,j} D_{n,d}^{i,j} + \epsilon_{n,d},$$
(32)

where  $\hat{p}_{n,d} = p_{n,d} - p_d^{co}$  ( $p_{n,d}$  is the price of the  $n^{th}$  interdealer transaction on day d, and  $p_d^{co}$  is the core market price on day d) and  $D_{n,d}^{i,j}$  is a dummy variable equal to 1 when, in the  $n^{th}$  interdealer transaction on day d, the seller is of type i and the buyer of type j. Thus,  $\hat{p}_{n,d}$  measures the difference between the price of an interdealer transaction and the core market price. Given this specification,  $(\beta^{C,U} - \beta^{U,U})$  measures the average difference between the price received by connected sellers on the one hand and unconnected sellers on the other hand. Hence, it is the empirical analog of  $(\bar{p}^{C,U} - \bar{p}^{U,U})$  in our model and therefore measures the effect of connectedness for buyers. Table 1 reports estimates of the coefficients in (32) in columns (2) (lowest quartile of  $z^{pe}$ ) to (5) (highest quartile). In column (1), as a benchmark, we report estimates of the coefficients over all days.

### [INSERT TABLE 1]

Consider the results obtained for the highest quartile of  $z_0^{pe}$ , i.e., for days on which there are significantly more sellers than buyers among peripheral dealers (column (5) in Table 1). In this case, the effect of connectedness is much higher for sellers than for buyers  $(\beta^{C,U} - \beta^{U,U} \approx 0.34 \text{ while } \beta^{U,U} - \beta^{U,C} \approx 0.15)$ . Results for the lowest quartile (column (1)) are qualitatively symmetric: The effect of connectedness is much higher for buyers than for sellers (0.37 vs. 0.08). In contrast, for intermediate quartiles (columns (3) and (4)), the effect of connectedness is very similar for buyers and sellers.

In sum, in the interdealer market, connected dealers trade at better prices than unconnected dealers. However, this effect is stronger for connected dealers on the crowded side of the market and this asymmetry increases with the size of peripheral dealers' aggregate inventory ( $|z^{pe}|$ ) because dealers' market power depends both on (i) their connectedness and (ii) whether they are on the crowded side of the market.

[INSERT TABLE 2]

Second, we illustrate the effect of dealers' connectedness on transaction prices in the D2C market. On each day, we compute the average prices  $(a\bar{s}k_d^{co} \text{ and } b\bar{i}d_d^{co})$  at which clients buy or sell when they trade with core dealers. Then, for the  $n^{th}$  transaction in a given day d between a client and a peripheral dealer, we compute  $\hat{p}_{n,d}^{cu} = p_{n,d} - a\bar{s}k_d^{co}$  if the client buys and  $\hat{p}_{n,d}^{cu} = p_{n,d} - b\bar{i}d_d^{co}$  if the client sells. We then estimate the following regression:

$$\hat{p}_{n,d}^{cu} = \alpha + \sum_{i \in \{C,U\}} \beta^{cu,i} D_{n,d}^{cu,i} + \sum_{i \in \{C,U\}} \beta^{i,cu} D_{n,d}^{i,cu} + \epsilon_{n,d},$$
(33)

where  $D_{n,d}^{cu,i}$  ( $D_{n,d}^{i,cu}$ ) is a dummy variable equal to 1 if a customer sells (buys) the asset to (from) a dealer of type  $i \in \{C, U\}$ . Table 2 reports the results of this regression for all days (column (1)) and for each of our subsamples (as in Table 2). Column (1) shows that, on average, clients get better prices from connected dealers ( $\beta^{cu,C} > 0$  and  $\beta^{C,cu} < 0$ ) than from core dealers, and worse prices from unconnected dealers ( $\beta^{cu,U} < 0$  and  $\beta^{U,cu} > 0$ ). However, this average effect masks significant heterogeneity due to variations in peripheral dealers' aggregate inventory (as per our Implication 4). For instance, in the highest quartile of  $z_0^{pe}$  (column (5)), customers get good prices from *both* connected and unconnected dealers when they buy the asset and more so from connected dealers ( $\beta^{U,cu} < \beta^{C,cu} \leq$ 0). In contrast, when they sell the asset, they obtain no price improvement (relative to core dealers) from connected dealers ( $\beta^{cu,C} \approx 0$ ) and a worse price from unconnected dealers ( $\beta^{cu,U} < 0$ ). Symmetric results are obtained for the lowest quartile of  $z_0^{pe}$  (column (2)). Again, these findings highlight the importance of conditioning on peripheral dealers' aggregate inventory position to understand the dispersion of prices in D2C markets.

The breakdown of unconnected dealers' trading volume with other unconnected dealers and connected dealers is governed by  $\lambda$  in the model. In our simulations, we assume that  $\lambda = 0.4$ . In this case, about 19% of trades between peripheral dealers are between two unconnected dealers. For lower values of  $\lambda$ , the share of trades that takes place between unconnected peripheral dealers falls more than proportionally. However, the model still generates significant price dispersion in the peripheral market, driven by transactions between unconnected and connected dealers. For instance, in simulations with  $\lambda = 10\%$ (available upon request), the share of trades between unconnected dealers drops to almost zero, but the coefficients  $\beta^{U,C}$  and  $\beta^{C,U}$  in Table 1 remain qualitatively similar and statistically significant. This observation highlights the fact that heterogeneity in dealers' connectedness and inventory positions is sufficient to generate significant price effects in the interdealer market, even when, in equilibrium, the trading volume between unconnected dealers is small.

# 6 Trading Costs and Efficiency

In this section we analyze how trading frictions among peripheral dealers affect the total trading costs, denoted  $TC^{pe}$ , of their customers. These trading costs are defined as the difference between the total amounts customers pay when they buy from peripheral dealers and the total amounts dealers pay when they buy from customers:

$$TC^{pe}(\Sigma) = (1 - \alpha^{pe})(\lambda ask^U + (1 - \lambda)ask^C) - \alpha^{pe}(\lambda bid^U + (1 - \lambda)bid^C), \qquad (34)$$

where we emphasize that, ultimately, trading costs for peripheral dealers' clients depend on the peripheral dealers' strategy profile,  $\Sigma$ . Using (29) and (30), we show in the proof of Proposition 4 (see below):

$$TC^{pe}(\Sigma) = (1 - \beta)L + \beta IC(\Sigma) - \beta W(\Sigma),$$
(35)

where (i)  $IC(\Sigma)$  is the aggregate inventory holding costs (across all peripheral dealers) in the absence of trading among peripheral dealers and (ii)  $W(\Sigma)$  is equal to the aggregate gains from trade among peripheral dealers. Thus, trading costs for peripheral dealers' clients are inversely related to gains from trade between peripheral dealers,  $W(\Sigma)$ . Indeed, by trading among each other, peripheral dealers effectively reduce their inventory holding costs and pass through a fraction  $\beta$  of these savings to their clients. Moreover, one can show that total gains from trade between peripheral dealers and their clients are equal to  $L+W(\Sigma)-IC(\Sigma)$ . Thus, maximizing total gains from trade or minimizing clients' trading costs require maximizing gains from trade among peripheral dealers.

An interesting question is therefore whether the equilibrium of the peripheral market

is efficient in the sense that it maximizes  $W(\Sigma)$ , holding the market structure (i.e.,  $\lambda$ ) constant.<sup>29</sup> To study this question, we analyze how a social planner would optimally organize trades among peripheral dealers to maximize  $W(\Sigma)$  (or equivalently maximize total gains from trade). In doing so, we assume that the central planner has the same information as that available to peripheral dealers when they make their decisions. Specifically, consider a given ordering of peripheral dealers, i.e., the sequence in which these dealers take actions as described in Section 3.2. We assume that the social planner can choose a strategy profile  $\Sigma$  (i.e., an action for each dealer), but that this profile cannot prescribe actions based on information not available to dealers when it is their turn to make a decision. In particular, the social planner cannot make the action of a specific dealer contingent on the type of the next dealer in the sequence. Thus, any efficiency losses in equilibrium relative to the efficiency level achieved by the social planner only stem from dealers' strategic behavior, and *not* from additional information only available to the social planner.

We say that an equilibrium strategy profile  $\Sigma^*$  is efficient if it maximizes  $W(\Sigma)$ . For tractability, we only consider the case in which  $\kappa \to +\infty$ . We obtain the following result.

**Proposition 4.** If  $\alpha^{pe} \neq 1/2$ , then any strategy profile  $\Sigma \in {\Sigma^{ACD}, \Sigma^{ICB}, \Sigma^{ICS}}$  is inefficient. The efficient strategy profile is  $\Sigma^{FB-} = (1, 1, 1, 0)$  when  $\alpha^{pe} < 1/2$  and  $\Sigma^{FB+} = (1, 1, 0, 1)$  when  $\alpha^{pe} > 1/2$ .

Thus, the equilibrium regime of the peripheral market  $(\Sigma^*)$  is always inefficient when  $\alpha^{pe} \neq 1/2$ . As explained in detail in the Internet Appendix II. D, there are two reasons for this result. First, in the ICS or ICB regimes, some unconnected peripheral dealers choose to make aggressive offers with a low chance of acceptance. As a result of this rent-seeking behavior, there are instances in which mutually profitable trades do not take place and dealers end up paying inefficiently high inventory holding costs ( $C^s$  or  $C^b$ ). Second, in the ACD regime connected dealers on the crowded side choose to make offers to other peripheral dealers. These offers are accepted by unconnected dealers on the uncrowded side, who are in scarce supply and the only possible counterparties for unconnected dealers on the crowded side. Thus, matches between a connected dealer on the crowded side and

<sup>&</sup>lt;sup>29</sup>One can ask the same question for the core market. As all core dealers bear identical inventory holding costs, the optimal allocation is to split core dealers' aggregate inventory equally among them. This is indeed the allocation obtained in equilibrium (see Lemma 1).

an unconnected dealer on the uncrowded side increase the likelihood that unconnected dealers on the crowded side end up paying inefficiently high inventory holding costs.

We also show in the Internet Appendix II. E that total gains from trade among peripheral dealers decrease with  $\lambda \left(\frac{\partial W(\Sigma^*)}{\partial \lambda} < 0\right)$ . One testable implication is that trading costs for dealers' clients should increase when the network of connections between dealers becomes sparser or when some dealers lose their connections to core dealers<sup>30</sup>. This result also suggests that any effort to centralize trading in interdealer markets should reduce trading costs for dealers' clients and improve efficiency.<sup>31</sup>

# 7 Conclusion

Motivated by recent empirical findings, we propose a new model of trading in OTC markets that allows to analyze the joint effects of inventory and network frictions on transaction prices in OTC markets. Dealers accumulate inventories by trading with end-investors and trade together to reduce their inventory holding costs. As in real-world interdealer networks, our model features core and peripheral dealers. We assume that the high level of interconnectedness among core dealers enables them to reach an efficient allocation of inventory holding costs among themselves. In contrast, peripheral dealers trade bilaterally and, as is observed in reality, are heterogeneous in their access to core dealers.

In the model, price dispersion in the D2D and D2C markets stems from the interaction between the heterogeneity in peripheral dealers' connectedness and imbalances in their aggregate inventories. In particular, all trades take place at the same price in the interdealer market (as if it were fully centralized) when peripheral dealers' aggregate inventory is balanced, even if dealers are heterogeneous. Moreover, in this case, all dealers trade at the same quotes with their clients. In contrast, when peripheral dealers' aggregate inventory is unbalanced, better connected peripheral dealers trade at better prices on average than less well connected dealers in the interdealer market, and therefore offer better prices to their

<sup>&</sup>lt;sup>30</sup>See Friewald and Nagler (2019) and our Internet Appendix for evidence that dealers' connectedness is time-varying.

<sup>&</sup>lt;sup>31</sup>In particular in our model, total gains from trade are maximal when trading is centralized ( $\lambda = 0$ ). In contrast, Malamud and Rostek (2017) and Duffie, Malamud, and Manso (2014) predict that market centralization does not necessarily improve welfare.

clients. Finally, shocks to dealers' aggregate inventories affect the bargaining power of different dealers, the prices at which they trade in the interdealer market, and the bid-ask spreads they charge to their clients. This result stands in contrast to standard inventory models (e.g., Ho and Stoll (1983) or Biais (1993)), in which only bid-ask midpoints are affected by dealers' aggregate inventory.

# A Main Notations

Variable	Definition				
v	Asset payoff				
$\ell$	Clients' private valuation for the asset $(\ell \in \{-L, L\})$				
$\beta$	Clients' bargaining power				
$\alpha^{pe}$	Fraction of peripheral dealers with a long position				
$\alpha^{co}$	Fraction of core dealers with long position				
$\kappa$	Number of clients per core dealer				
$\lambda$	Fraction of peripheral dealers connected to core dealers				
$z_0^{pe}$	Peripheral dealers' aggregate inventory $(=(2\alpha^{pe}-1))$				
$z_0^{co}$	Core dealers' aggregate inventory $(=\kappa(2\alpha^{co}-1))$				
$z_{it}$	Dealer i's position at date $t$				
$m_{it}$	Dealer i's cash holdings at date $t$				
$q_{it}$	Trade of dealer $i$ at date $t$				
$p_{it}$	Price at which dealer $i$ trades at date $t$				
$C^b$	Cost (per unit) of a short position at date 3				
$C^s$	Cost (per unit) of a long position at date 3				
$\bar{C}$	Expected inventory holding cost of dealer $i$				
$\pi_b$	Probability that a peripheral seller finds a peripheral buyer $(= \min\{\frac{1-\alpha^{pe}}{\alpha^{pe}}, 1\})$				
$\pi_s$	Probability that a peripheral buyer finds a peripheral seller $(= min\{\frac{\alpha^{pe}}{1-\alpha^{pe}},1\})$				
$\epsilon_{i3}$	Inventory shock of dealer $i$ at date 3				
$\Phi(.)$	Cumulative distribution of $\epsilon_i$				
$p^{co}$	Price in the core market				

Variable	Definition
$\theta_k$	Likelihood that a dealer of type $k \in \{b,s\}$ makes an offer at price $p_k = V_{-k}^C$
$\gamma_k$	Likelihood that a dealer of type $k \in \{b,s\}$ does not directly trade in the core market
$\Sigma$	A strategy profile in the peripheral market $(\Sigma = (\theta_s, \theta_b, \gamma_s, \gamma_b))$
$\omega_k$	$=rac{\lambda(1-\pi_k)}{1-\pi_k\lambda(2-\lambda)}  ext{ for } k \in \{b,s\}$
$\mu_s^{co}$	Mass of peripheral dealers with a long position trading in the core market
$\mu_b^{co}$	Mass of peripheral dealers with a short position trading in the core market
$\Delta^*(\alpha^{pe},\lambda,\Sigma)$	Order flow (sales - buys) of connected peripheral dealers trading in the core market
$V_k^i$	Continuation value of a peripheral dealer with type $(k, i) \in \{b, s\} \times \{U, C\}$
$arphi_k$	Probability that an offer of a dealer with type $k \in \{b, s\}$ is accepted
$ ho_k^C$	$=rac{1-\pi_k}{1-\pi_b\pi_s\lambda}  ext{ for } k\in\{b,s\}$
$ ho_k^U$	$= -\lambda \pi_k \rho_{-k}^C$ for $k \in \{b, s\}$
$M_k^i$	Market power of a dealer with type $(k,i) \in \{b,s\} \times \{U,C\}$
$P_k^i$	Price at which a dealer of type $(k,i) \in \{b,s\} \times \{U,C\}$ expects to unwind
	his position
$ar{p}^{i,j}$	the average price at which a seller of type $i \in \{C, U\}$ trades with a buyer
	of type $j \in \{C, U\}$
$S^i$	Bid-ask spread posted to clients by dealer of type $i \in \{C, U, co\}$
$TC^{pe}$	Total trading costs of peripheral dealers' customers
W	Total gains from trade in the periphery

# **B** Figures



Figure 2: Trading in the periphery. This figure depicts the trading process in the peripheral market for a particular sequence of arrivals. Red circles designate buyers while blue circles designate sellers. We denote by  $p_b^j$ , the offer made by a buyer of type  $j \in \{C, U\}$  and by  $p_s^j$ , the offer made by a seller of type  $j \in \{C, U\}$ .



#### ACD Equilibrium

Figure 3: Equilibrium configurations. An arrow going from one type of trader to another indicates that the former makes offers to the latter in equilibrium.

### Figure 4: Equilibrium type as a function of $\alpha^{pe}$ and $\alpha^{co}$ .

Panel A:  $\kappa \to \infty$ ,  $\lambda = 0.3$  and  $\lambda = 0.4$ .

For each pair  $(\alpha^{pe}, \alpha^{co})$ , Panel A provides the corresponding strategy profile obtained in equilibrium in the thick core market case, for two values of  $\lambda$  (0.3 and 0.4).



Panel B:  $\kappa = 2, \lambda = 0.4$ .

For each pair  $(\alpha^{pe}, \alpha^{co})$ , Panel B provides the corresponding strategy profile obtained in equilibrium when the core market is not thick. The colored areas represent the  $(\alpha^{pe}, \alpha^{co})$  pairs for which an ACD, ICS, or ICB equilibrium is obtained. A mixed equilibrium obtains otherwise.





Figure 5: Equilibrium Prices. This figure shows the position of equilibrium prices in the peripheral market relative to the equilibrium core market price in each possible equilibrium regime when  $\alpha^{pe} > 0.5$  and  $\alpha^{pe} < 0.5$ ;

#### Table 1: Dealer-to-dealer prices and dealer types

This table presents estimates of regression (32). Column (1) gives the coefficients estimated on all dealer-todealer transactions. Columns (2) to (5) give the coefficients estimated on four subsamples, corresponding to the four quartiles of  $\alpha^{pe}$ . t-statistics are given in parentheses. \* \* \*, \*\*, and \* denote statistical significance at the 1%, 5%, and 10% level, respectively. Standard errors are clustered at the trading day level. The constant, as well as the coefficients  $\beta^{co,C}$  and  $\beta^{C,co}$  are equal to zero and omitted. There are 1175 trades between unconnected dealers, 4914 trades between a connected and an unconnected dealers, and 10036 trades between connected and core dealers.

	(1)	(2)	(3)	(4)	(5)
$\beta^{C,U}$	0.193***	0.460***	0.285***	0.010**	0.000
	(9.52)	(11.80)	(12.06)	(2.56)	(0.00)
$\beta^{U,C}$	-0.181***	0.000	-0.000***	-0.182***	-0.498***
	(-8.44)	(0.00)	(-20.71)	(-5.37)	(-17.97)
$\beta^{U,U}$	0.008	0.375***	0.148***	-0.083***	-0.342***
	(0.28)	(5.20)	(4.52)	(-4.85)	(-8.48)
# Obs.	17,375	4,161	4,402	4,674	4,138

#### Table 2: Dealer-to-customer prices and dealer types

This table presents estimates of regression (33). Column (1) gives the coefficients estimated on all dealerto-customer transactions. Columns (2) to (5) give the coefficients estimated on four subsamples, corresponding to the four quartiles of  $\alpha^{pe}$ . *t*-statistics are given in parentheses. \*\*\*, \*\*, and \* denote statistical significance at the 1%, 5%, and 10% level, respectively. Standard errors are clustered at the trading day level. The constant and the coefficient  $\beta^{cu,co}$  are equal to zero and omitted. There are 9849 trades between clients and unconnected dealers, and 14950 trades between clients and connected dealers.

	(1)	(2)	(3)	(4)	(5)
$\beta^{cu,U}$	-0.159***	0.056***	0.020***	-0.129***	-0.368***
	(-9.00)	(7.91)	(8.26)	(-5.53)	(-22.19)
$\beta^{U,cu}$	0.176***	0.364***	0.190***	-0.006	-0.054***
	(9.26)	(14.09)	(11.90)	(-1.56)	(-12.74)
$\beta^{cu,C}$	0.034***	0.140***	0.078***	0.002**	0.000
	(7.44)	(12.39)	(12.00)	(2.32)	(0.00)
$\beta^{C,cu}$	-0.029***	-0.000	0.000	-0.049***	-0.144***
	(-7.17)	(.)	(.)	(-5.74)	(-20.54)
# Obs.	34,738	8,833	8,450	8,817	8,638

# C Proofs

# C.1 Proof of Lemma 1

Follows from the discussion preceding the proposition.

# C.2 Proof of Lemma 2

Follows from the observations that precede the lemma.

# C.3 Proof of Lemma 3

### C.3.1 ACD Regime

We start by deriving the conditions under which the active connected dealers (ACD) regime obtains. In this equilibrium, a connected seller's offer is accepted if and only if the seller is matched with an unconnected buyer. This event has probability  $\lambda \pi_b$ , so that:

$$V_s^C = \lambda \pi_b p_s^C + (1 - \lambda \pi_b) p^{co}.$$
 (C1)

Symmetrically, a connected buyer's offer is accepted with probability  $\lambda \pi_s$ . Therefore:

$$V_b^C = -\lambda \pi_s p_b^C - (1 - \lambda \pi_s) p^{co}.$$
 (C2)

Moreover, an unconnected seller's offer is accepted if and only if he is matched with a buyer, of any type, which gives:

$$V_s^U = \pi_b p_s^U - (1 - \pi_b) C^s.$$
 (C3)

And, symmetrically:

$$V_b^U = -\pi_s p_b^U - (1 - \pi_s) C^b.$$
(C4)

Furthermore, from Lemma 2, we know that  $p_s^C = -V_b^U$ ,  $p_b^C = V_s^U$ ,  $p_s^U = -V_b^C$ , and  $p_b^U = V_s^C$ .

Combining these conditions with (C1)-(C4), we obtain a system of 4 equations with 4 unknowns  $(p_s^C, p_s^U, p_b^C, p_b^U)$ . Solving this system, we obtain:

$$p_b^U = V_s^C = \frac{(1 - \pi_b \lambda)p^{co} + \pi_b \lambda(1 - \pi_s)C^b}{1 - \pi_s \pi_b \lambda}, p_b^C = V_s^U = \frac{\pi_b (1 - \pi_s \lambda)p^{co} - (1 - \pi_b)C^s}{1 - \pi_s \pi_b \lambda}$$
(C5)

$$p_s^U = -V_b^C = \frac{(1 - \pi_s \lambda)p^{co} - \pi_s \lambda(1 - \pi_b)C^s}{1 - \pi_s \pi_b \lambda}, p_s^C = -V_b^U = \frac{\pi_s (1 - \pi_b \lambda)p^{co} + (1 - \pi_s)C^b}{1 - \pi_s \pi_b \lambda}.$$
 (C6)

Equations (C5) and (C6) yield equilibrium prices in the ACD regime.

It remains to derive the conditions under which dealers have no incentive to deviate from their equilibrium behavior in the ACD regime. First, observe that we have  $p_b^C \leq p^{co}$ and  $p_s^C \geq p^{co}$ . Hence,  $\gamma_s = \gamma_b = 1$  is optimal for connected dealers.<sup>32</sup> Second, we have  $p_b^U \leq C^b$  and  $p_s^U \geq -C^s$  so that unconnected dealers are better off trading with another peripheral dealer rather than bearing their inventory holding cost.

Last, we need to check that  $\theta_s = \theta_b = 1$  is optimal for unconnected dealers. Consider an unconnected seller. In equilibrium, he obtains an expected payoff of  $V_s^U$  with an offer at  $p_s^U = -V_b^C$ . As explained in the text, his most profitable deviation is to offer the same price as that of a connected seller, i.e.,  $p_s^C = -V_b^U$ . This offer is accepted only by unconnected buyers, i.e., with probability  $\pi_b \lambda$ . Thus, an unconnected seller is better off not deviating if and only if:

$$V_s^U > -\pi_b \lambda V_b^U - (1 - \pi_b \lambda) C^s.$$
(C7)

Using (C5), we find that this condition is necessarily satisfied if  $\alpha^{pe} \ge 1/2$  and hence  $\pi_s = 1$ . If instead  $\alpha^{pe} < 1/2$  this condition is equivalent to:

$$\frac{p^{co} + C^s}{C^b + C^s} > \frac{\lambda(1 - \pi_s)}{1 - \pi_s \lambda(2 - \pi_b \lambda)}.$$
(C8)

<sup>&</sup>lt;sup>32</sup>When  $\alpha^{pe} \geq 1/2$ , we have  $\pi_s = 1$  and thus  $p_s^C = p^{co}$ . Thus, in this case, connected sellers are indifferent between making an offer and directly trading in the core market. Thus, they could play a mixed strategy between making an offer and trading in the core market. However, this indifference breaks down as soon as the likelihood that a buyer finds a seller is not exactly 1, as studied in Section III. F of the Internet Appendix. Indeed in this case,  $p_s^C > p^{co}$  for all values of  $\alpha^{pe}$  in the ACD regime and a connected seller is therefore strictly better off making an offer. Hence, we discard potential equilibrium regimes in which connected dealers use such a mixed strategy as non-robust.

Symmetrically, an unconnected buyer is better off not deviating if and only if:

$$V_b^U > -\pi_s \lambda V_s^U - (1 - \pi_s \lambda) C^b.$$
(C9)

Using (C6), this condition is necessarily satisfied if  $\alpha^{pe} \leq 1/2$ . Otherwise, it is equivalent to:

$$\frac{C^b - p^{co}}{C^b + C^s} > \frac{\lambda(1 - \pi_b)}{1 - \pi_b \lambda(2 - \pi_s \lambda)}.$$
(C10)

After straightforward manipulations, (C8) and (C10) can be shown to be equivalent to Condition (19) in Lemma 3.

We now derive conditions under which the other regimes are obtained in equilibrium.

### C.3.2 Inactive connected sellers (ICS) regime

In this equilibrium, connected buyers make offers that are accepted by unconnected sellers only, i.e.,  $p_b^C = V_s^U$ . Moreover, connected sellers reject offers from buyers (whether unconnected or connected) and trade with a core seller with probability 1 ( $\gamma_s = 0$ ). Hence, in this equilibrium, we have:

$$V_s^C = p^{co} \tag{C11}$$

$$V_b^C = -\lambda \pi_s V_s^U - (1 - \lambda \pi_s) p^{co}.$$
 (C12)

In addition, in this equilibrium, unconnected sellers make offers that are accepted by all buyers, i.e.,  $p_s^U = -V_b^C$ , while unconnected buyers make offers that are accepted by unconnected sellers only, i.e.,  $p_b^U = V_s^U$ . Thus, we have:

$$V_s^U = -\pi_b V_b^C - (1 - \pi_b) C^s$$
(C13)

$$V_b^U = -\pi_s \lambda V_s^U - (1 - \pi_s \lambda) C^b.$$
(C14)

Solving the previous system of equations for  $V_s^U$ ,  $V_b^U$ , and  $V_b^C$ , we obtain that in an ICS regime:

$$p_b^U = p_b^C = V_s^U = \frac{(1 - \pi_s \lambda)\pi_b p^{co} - (1 - \pi_b)C^s}{1 - \lambda \pi_s \pi_b},$$
(C15)

$$V_b^U = -\left(\frac{\lambda \pi_s \pi_b (1 - \pi_s \lambda) p^{co} - \pi_s \lambda (1 - \pi_b) C^s}{1 - \lambda \pi_s \pi_b} + (1 - \pi_s \lambda) C^b\right),$$
(C16)

$$p_{s}^{U} = -V_{b}^{C} = \frac{(1 - \pi_{s}\lambda)p^{co} - \pi_{s}\lambda(1 - \pi_{b})C^{s}}{1 - \lambda\pi_{s}\pi_{b}},$$
(C17)

$$V_s^C = p^{co}. (C18)$$

We now establish show that no peripheral dealer has an incentive to deviate from the ICS equilibrium if and only if  $p^{co} > (1 - \omega_b)C^b - \omega_bC^s$ . We first check that this is the case for buyers.

From (C17) in the Internet Appendix, we deduce that  $V_b^C > -p^{co}$ , so that connected buyers are better off making an offer at rate  $p_b^C$  to another peripheral dealer when they reject an offer rather than directly contacting a core dealer. This implies  $\gamma_b = 1$  is optimal for a connected buyer, as it should in the ICS regime.

In the ICS regime, unconnected buyers offer prices that are accepted by unconnected sellers only. As explained in the text, their best deviation is to offer a price  $V_s^C$  that is accepted by all types of sellers. This deviation is not optimal when other dealers behave as in the ICS regime iff:

$$V_b^U > -\pi_s V_s^C - (1 - \pi_s) C^b.$$
(C19)

When  $\alpha^{pe} < \frac{1}{2}$ ,  $\pi_b = 1$  and  $\pi_s < 1$ . Thus, substituting  $V_b^U$  and  $V_s^C$  by their expressions in (C16) and (C18) into (C19), we observe that (C19) requires  $p^{co} > C_b$ , which is impossible. Thus, the ICS regime cannot obtain when  $\alpha^{pe} < \frac{1}{2}$ . When  $\alpha^{pe} > \frac{1}{2}$ ,  $\pi_s = 1$  and  $\pi_b < 1$ . Thus, substituting into Condition (C19)  $V_b^U$  and  $V_s^C$  by their expressions in (C16) and (C18), we can rewrite (C19) as:

$$\frac{C^b - p^{co}}{C^b + C^s} < \frac{\lambda(1 - \pi_b)}{1 - \lambda \pi_b (2 - \lambda)},\tag{C20}$$

which is equivalent to  $p^{co} > (1 - \omega_b)C^b - \omega_bC^s$ . Thus,  $p^{co} > (1 - \omega_b)C^b - \omega_bC^s$  and  $\alpha^{pe} > \frac{1}{2}$ 

are necessary conditions to obtain the ICS equilibrium.

Now, we show that peripheral sellers have no incentive to deviate from their strategy profile in an ICS equilibrium if  $p^{co} > (1 - \omega_b)C^b - \omega_bC^s$ . In the ICS regime, a connected seller directly contacts a core dealer if he rejects an offer ( $\gamma_s = 0$ ). His best deviation (see the discussion preceding Lemma 2) is to make an offer at the highest price that unconnected buyers are willing to accept, i.e.,  $-V_b^U$ . This deviation cannot be optimal if  $p^{co} > -V_b^U$ . Substituting  $V_b^U$  by its expression in (C16), we find that when  $\alpha^{pe} > \frac{1}{2}$ , this condition is satisfied if (C20), or equivalently  $p^{co} > (1 - \omega_b)C^b - \omega_bC^s$ , is satisfied. Thus, when  $\alpha^{pe} > \frac{1}{2}$  (which, as just explained, is a necessary condition for the ICS regime) and  $p^{co} > (1 - \omega_b)C^b - \omega_bC^s$ ,  $\gamma_s = 0$  is a best response for connected sellers.

Finally, unconnected sellers offer a price that is accepted by all types of buyers, i.e., a price equal to  $p_s^U = -V_b^C$ . Their best deviation is to offer a price  $-V_b^U$  that is accepted by unconnected buyers only (see the discussion preceding Lemma 2). This deviation is not optimal if and only if:

$$V_s^U > -\pi_b \lambda V_b^U - (1 - \pi_b \lambda) C^s.$$
(C21)

Substituting  $V_s^U$  and  $V_b^U$  by their expressions, we deduce that, when  $\alpha^{pe} > \frac{1}{2}$ , (C21) is equivalent to:

$$\frac{C^b - p^{co}}{C^b + C^s} < \frac{1 - \lambda}{1 - \lambda^2 \pi_b}.$$
(C22)

When  $\lambda < 1/2$ , (C22) is satisfied if (C20) (or equivalently  $p^{co} > (1 - \omega_b)C^b - \omega_bC^s$ ) is satisfied.

We deduce from this analysis that  $\alpha^{pe} > \frac{1}{2}$  and  $p^{co} > (1 - \omega_b)C^b - \omega_bC^s$  are necessary and sufficient conditions for obtaining the ICS regime. As  $\alpha^{pe} > \frac{1}{2}$  is a necessary condition for  $p^{co} > (1 - \omega_b)C^b - \omega_bC^s$ , we deduce that  $p^{co} > (1 - \omega_b)C^b - \omega_bC^s$  is a necessary and sufficient condition an ICS equilibrium.

## C.3.3 Inactive connected buyers (ICB) regime

We can show that this equilibrium obtains if and only if  $\alpha^{pe} < \frac{1}{2}$  and:

$$\frac{p^{co} + C^s}{C^b + C^s} < \frac{\lambda(1 - \pi_s)}{1 - \lambda \pi_s(2 - \lambda)},\tag{C23}$$

by proceeding exactly as we did for ICS regime. This condition is equivalent to the condition on the third line of (19).

## C.4 Proof of Lemma 4

See the Internet Appendix II. C.

## C.5 Proof of Proposition 1

The threshold values are defined by  $\alpha^+_{ACD} = \alpha^+(\Sigma^{ACD}), \ \alpha^+_{ICB} = \alpha^+(\Sigma^{ICB}), \ \alpha^-_{ACD} = \alpha^-(\Sigma^{ACD}), \ \alpha^-_{ICS} = \alpha^-(\Sigma^{ICS}), \ \text{with:}$ 

$$\alpha^{+}(\Sigma) = \frac{1}{2} \left( 1 - \left[ \Phi^{-1}(\omega_s) + \frac{\Delta^{*}(\alpha^{pe}, \lambda, \Sigma)}{\kappa} \right] \right), \qquad (C24)$$

$$\alpha^{-}(\Sigma) = \frac{1}{2} \left( 1 - \left[ \Phi^{-1} \left( 1 - \omega_b \right) + \frac{\Delta^*(\alpha^{pe}, \lambda, \Sigma)}{\kappa} \right] \right), \tag{C25}$$

where, for brevity, we omit the fact that  $\alpha^+(\Sigma)$  and  $\alpha^-(\Sigma)$  are ultimately functions of the exogenous parameters  $\lambda$  and  $\alpha^{pe}$ . Suppose that  $\alpha^{pe} < \frac{1}{2}$ . In this case, we know from Lemma 3 that the only possible equilibrium strategy profiles are  $\Sigma^* = \Sigma^{ACD}$  or  $\Sigma^* = \Sigma^{ICB}$ . A full equilibrium in which  $\Sigma^* = \Sigma^{ICB}$  obtains if and only if the condition on the third line of (19) is satisfied for  $p^{co} = p^{co*}(z_0^{co}, \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICB}))$ . As  $p^{co*}(z_0^{co}, \Delta) =$  $\Phi(-\kappa^{-1}(z_0^{co} + \Delta))C^b - C^s(1 - \Phi(-\kappa^{-1}(z_0^{co} + \Delta)))$  and  $z_0^{co} = \kappa(2\alpha^{co} - 1)$ , this is equivalent to:

$$\alpha^{co} > \alpha^+ (\Sigma^{ICB}). \tag{C26}$$

A full equilibrium in which  $\Sigma^* = \Sigma^{ACD}$  obtains if and only if the condition on the second line of (19) is satisfied for  $p^{co} = p^{co*}(z_0^{co}, \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD}))$ . As  $p^{co*}(z_0^{co}, \Delta) = \Phi(-\kappa^{-1}(z_0^{co} + \Delta))C^b - C^s(1 - \Phi(-\kappa^{-1}(z_0^{co} + \Delta)))$  and  $z_0^{co} = \kappa(2\alpha^{co} - 1)$ , this is equivalent to:

$$\alpha^{co} < \alpha^+ (\Sigma^{ACD}). \tag{C27}$$

For  $\alpha^{pe} < \frac{1}{2}$ ,  $\Delta^*(\alpha^{pe}, \lambda, \Sigma^*) \leq 0$ , since  $\Sigma^* = \Sigma^{ACD}$  or  $\Sigma^* = \Sigma^{ICB}$  (see Lemma 4). Moreover,  $\omega_s < 1/2$  and therefore  $\Phi^{-1}(\omega_s) < 0$  (since  $\Phi(\frac{1}{2}) = 0$ ).<sup>33</sup> Lastly, as shown in Lemma 4,  $\Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICB}) < \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD}) < 0$ . It follows that:  $\frac{1}{2} < \alpha^+(\Sigma^{ACD}) < \alpha^+(\Sigma^{ICB})$ . The proof for the other cases  $(\alpha^{pe} > \frac{1}{2}$  and  $\alpha^{pe} = \frac{1}{2})$  is similar and is therefore skipped for brevity.

# C.6 Proof of Proposition 2

The proof of the proposition follows directly from the expressions for equilibrium prices in the peripheral market obtained in the proof of Lemma 3.

# C.7 Proof of Proposition 3

We consider the ACD Equilibrium and the ICS Equilibrium separately (the ICB Equilibrium is symmetric to ICS).

### C.7.1 ACD Equilibrium

We first derive the values of the  $M_i^k$ . Consider unconnected buyers. Using Proposition 2 we have:

$$-V_b^U = \pi_s p_b^U + (1 - \pi_s) C^b = \pi_s [p^{co*} - \rho_b^U (C^b - p^{co*})] + (1 - \pi_s) C^b.$$
(C28)

Using (27) and simplifying we obtain:

$$M_b^U = \pi_s (1 + \rho_b^U).$$
 (C29)

Symmetrically, for unconnected sellers we have:

$$M_s^U = \pi_b (1 + \rho_s^U).$$
 (C30)

<sup>&</sup>lt;sup>33</sup>We have  $\omega_s < 1/2$  because  $\lambda \leq \frac{1}{2}$ .

Consider connected buyers. Proposition 2 gives:

$$-V_b^C = \lambda \pi_s p_b^C + (1 - \lambda \pi_s) p^{co*} = p^{co*} - \lambda \pi_s \rho_b^C (C^s + p^{co*}).$$
(C31)

Using (27) and simplifying we obtain:

$$M_b^C = 1 + \lambda \pi_s \rho_b^C \frac{C^s + p^{co*}}{C^b - p^{co*}},$$
 (C32)

and symmetrically for connected sellers:

$$M_s^C = 1 + \lambda \pi_b \rho_s^C \frac{C^b - p^{co*}}{C^s + p^{co*}}.$$
 (C33)

To prove the remainder of the Proposition we focus on the case  $\alpha^{pe} > 1/2$ , the other case being symmetric. We then have  $\pi_s = 1$  and  $\rho_s^C = \rho_b^U = 0$ , so that  $M_b^U = M_s^C = 1$ ,  $M_b^C > 1$ and  $M_s^U < 1$ . This implies that  $M_b^k > M_s^k$  for  $k \in \{C, U\}$  and  $M_i^C > M_i^U$  for  $i \in \{b, s\}$ . The comparative statics with respect to  $\alpha^{pe}$  follow straightforwardly from the analytical expressions derived above.

## C.7.2 ICS Equilibrium

The values we obtain for the  $M_i^k$  in the ICS equilibrium (which obtains only when  $\alpha^{pe} > 1/2$ ) are the following:

$$M_b^U = \lambda \left( 1 + \rho_b^C \frac{C^s + p^{co*}}{C^b - p^{co*}} \right) \quad M_s^U = \pi_b (1 + \rho_s^U)$$
(C34)

$$M_b^C = 1 + \lambda \rho_b^C \frac{C^s + p^{co*}}{C^b - p^{co*}} \qquad M_s^C = 1$$
(C35)

It is easy to see that  $M_b^C > 1$  and  $M_s^U < 1$ . For  $M_b^U$  consider that an unconnected buyer could have made an offer at price  $p^{co*}$ , accepted by both connected and unconnected sellers and hence accepted with probability one. This implies that  $V_b^U \ge -p^{co*}$ , which is equivalent to  $M_b^U \ge 1$ . We deduce that  $M_b^k \ge M_s^k$  for  $k \in \{C, U\}$ . The expressions above also clearly imply that  $M_i^C > M_i^U$  for  $i \in \{b, s\}$ . The comparative statics with respect to  $\alpha^{pe}$  follow straightforwardly from the analytical expressions of the  $M_i^k$ .

## C.8 Bid and Ask Prices

We prove that the equilibrium bid and ask prices charged to customers are given by (29) and (30) and provide the analytical expressions of these prices.

### C.8.1 Peripheral dealers

Consider a client who wants to buy the asset from a dealer of type  $k \in \{C, U\}$ . The client's expected payoff is nil if he does not trade and  $(L - ask^k)$  if he does (remember that the expected payoff of the asset is normalized to zero). If the trade takes place, the dealer sells the asset to the client and therefore becomes a buyer in the interdealer market. The expected price he will have to pay for the asset is  $P_b^k$ , and thus his expected payoff is  $ask^k - P_b^k$  if he sells the asset to his client, and is nil otherwise. Thus, Nash bargaining between the dealer and his client yields the following ask price:

$$ask^k = L - \beta [L - P_b^k]. \tag{C36}$$

If the dealer's client wants to sell the asset, a similar reasoning implies that the dealer's bid price is:

$$bid^k = -L + \beta [L + P_s^k]. \tag{C37}$$

It is easily checked that the dealer and his client are always better off trading at these prices than not trading because  $L > \max(C_s, C_b)$ . Thus, the proportion of sellers among dealers in the interdealer market (either the peripheral market or the core market) is indeed equal to the proportion of clients who want to sell the asset, as is assumed in the construction of the equilibrium.

To obtain the values of the dealers' bid and ask quotes, we compute the values of  $P_b^k$ and  $P_s^k$  for all types of dealers, using the stationary probabilities  $\mu$  derived in the Internet Appendix II. B.2.

Consider connected sellers. In a given subperiod  $\tau$ , there is a probability  $\mu_1$  that a connected seller makes an offer. With probability  $\varphi_s(p_s^C)$  this offer is accepted and the dealer sells at price  $p_s^C$ , while with probability  $1 - \varphi_s(p_s^C)$  the offer is rejected and the dealer sells the asset to core dealers at price  $p^{co*}$ . The other possibility is that the connected seller accepts an offer from an unconnected buyer. There is a probability  $\mu_7$  in each period that an unconnected buyer makes an offer, and with probability  $\theta_b(1-\lambda)\pi_s$  it is accepted by a connected seller. We thus obtain that:

$$P_s^C = \frac{\mu_1[\varphi_s(p_s^C)p_s^C + (1 - \varphi_s(p_s^C))p^{co*}] + \mu_7\theta_b(1 - \lambda)\pi_s p_b^U}{\mu_1 + \mu_7\theta_b(1 - \lambda)}.$$
 (C38)

Note that by definition we must have  $\mu_1 + \mu_7 \theta_b (1 - \lambda) = \alpha^{pe} (1 - \lambda)$ : the probability of having an action by a connected seller in a given subperiod  $\tau$  must be equal to the proportion of connected sellers in the peripheral market (this equality can of course be checked analytically as well). We can thus rewrite:

$$P_s^C = \frac{\mu_1[\varphi_s(p_s^C)p_s^C + (1 - \varphi_s(p_s^C))p^{co*}] + \mu_7\theta_b(1 - \lambda)\pi_s p_b^U}{\alpha^{pe}(1 - \lambda)}.$$
 (C39)

Using a similar reasoning, we can solve for the other prices at which peripheral dealers expect to unwind their position following a trade with a client and therefore their bid and ask prices in the D2C market:

$$P_s^U = \frac{\mu_3[\varphi_s(p_s^U)p_s^U - (1 - \varphi_s(p_s^U))C^s] + \mu_5\gamma_b\lambda\pi_sp_b^C + \mu_7\lambda\pi_sp_b^U}{\alpha^{pe}\lambda}, \qquad (C40)$$

$$P_b^C = \frac{\mu_5[\varphi_b(p_b^C)p_b^C + (1 - \varphi_b(p_b^C))p^{co*}] + \mu_3\theta_s(1 - \lambda)\pi_b p_s^U}{(1 - \alpha^{pe})(1 - \lambda)},$$
(C41)

$$P_{b}^{U} = \frac{\mu_{7}[\varphi_{b}(p_{b}^{U})p_{b}^{U} + (1 - \varphi_{b}(p_{b}^{U}))C^{b}] + \mu_{1}\gamma_{s}\lambda\pi_{b}p_{s}^{C} + \mu_{3}\lambda\pi_{b}p_{s}^{U}}{(1 - \alpha^{pe})\lambda}.$$
 (C42)

#### C.8.2 Core dealers

We now derive the ask and bid prices at which core dealers trade with their clients. The analysis is slightly different than that for peripheral dealers because core dealers' expected payoff in the interdealer market is different from zero even if they do not trade with their clients. Let  $\Pi^{co}(z_{i0}, m_{i0})$  be a core dealer's equilibrium expected payoff if he has a position  $z_{i0}$  and a cash endowment  $m_{i0}$  in period 1 (after trading with a client). After trading in period 2, each core dealer has the same position in equilibrium, equal to  $z_{i0}+q_{i2}^{co*}(p^{co*}, z_{i0}) = z^*$ , due to the expressions for  $q_{i2}^{co*}$  and  $p^{co*}$  derived in Section 4.1. Thus, core dealers *i*' final position at date 3 is  $z_{i3} = z^* + \epsilon_{i3}$ . Replacing  $z_{i3}$  by this expression in the core dealer's

expected payoff (6) and using the expression for  $p^{co*}$  in (13), we obtain:

$$\Pi^{co}(z_{i0}, m_{i0}) = m_{i0} + z_{i0}p^{co*} + C^b \int_{-\infty}^{-z^*} \epsilon_{i3} d\Phi(\kappa^{-1}\epsilon_{i3}) - C^s \int_{-z^*}^{+\infty} \epsilon_{i3} d\Phi(\kappa^{-1}\epsilon_{i3}).$$
(C43)

If a core dealer sells the asset to a client at price  $ask^{co}$ , he obtains an expected profit equal to  $\Pi^{co}(-1, ask^{co})$ . If instead he doesn't sell, his expected profit is  $\Pi^{co}(0, 0)$ . Using (C43), the difference reduces to  $\Pi^{co}(-1, ask^{co}) - \Pi^{co}(0, 0) = ask^{co} - p^{co*}$ . Intuitively, the dealer simply sells one unit to the client and buys it in the core market at the core price, independently of the other trades conducted to share inventory costs among core dealers. Thus, Nash bargaining between a core dealer and his client yields the following ask price:

$$(1-\beta)(L-ask^{co}) = \beta(ask^{co} - p^{co*}), \tag{C44}$$

that is,  $ask^{co} = L - \beta(L - p^{co*})$ . Since  $p^{co*} = P_b^{co}$  we obtain that equation (30) is also valid for core dealers. The reasoning is symmetric for dealers buying from a client.

## C.9 Proofs of Implications 1 to 5

These proofs rely on tedious computations based on the results from Lemma 3 and Proposition 2. We included them in the Internet Appendix II. G.

## C.10 Proof of Proposition 4

We first derive the expression for peripheral dealers clients' total trading costs given in (35). Remember that, by definition,

$$TC^{pe}(\Sigma) = (1 - \alpha^{pe})(\lambda ask^U + (1 - \lambda)ask^C) - \alpha^{pe}(\lambda bid^U + (1 - \lambda)bid^C).$$
(C45)

Using (29) and (30), we obtain:

$$TC^{pe}(\Sigma) = (1 - \beta)L - \beta[\alpha^{pe}(\lambda P_s^U + (1 - \lambda)P_s^C) - (1 - \alpha^{pe})(\lambda P_b^U + (1 - \lambda)P_b^C)].$$
(C46)

Straightforward manipulations show that this can be rewritten as:

$$TC^{pe}(\Sigma) = (1 - \beta)L + \beta[IC(\Sigma) - W(\Sigma)],$$
(C47)

where

$$IC(\Sigma) = \lambda(\alpha^{pe}C^{s} + (1 - \alpha^{pe})C^{b}) + (1 - \lambda)(1 - 2\alpha^{pe})p^{co}$$
(C48)

and

$$W(\Sigma) = \alpha^{pe} [\lambda(P_s^U + C^s) + (1 - \lambda)(P_s^C - p^{co})] + (1 - \alpha^{pe}) [\lambda(C^b - P_b^U) + (1 - \lambda)(p^{co} - P_b^C)].$$
(C49)

Thus,  $IC(\Sigma)$  is the total inventory holding cost of peripheral dealers (given the trades with their clients) if they could not trade among each other while  $W(\Sigma)$  is the aggregate surplus from trading between peripheral sellers and buyers. This means that  $W(\Sigma)$  is equal to the total gains from trade among peripheral dealers. One can therefore interpret  $[IC(\Sigma) - W(\Sigma)]$  as the effective inventory holding costs borne by peripheral dealers (i.e., after accounting for the possibility of interdealer trades). For brevity, we derive  $W(\Sigma)$  as a function of the parameters and prove the statements of Proposition 4 in the Internet Appendix II. D.

# References

- Afonso, Gara, and Ricardo Lagos, 2015, Trade dynamics in the market for federal funds, *Econometrica* 83, 263–313.
- Amstad, Marlene, and Zhiguo He, 2019, Chinese bond market and interbank market, Working paper.
- Anderson, Christopher, and Weiling Liu, 2019, Inferring intermediary risk exposure from trade, Working paper.
- Atkeson, Andrew, Andrea Eisfeldt, and Pierre-Olivier Weill, 2015, Entry and exit in otc derivatives markets, *Econometrica* 83, 2231–2292.
- Babus, Ana, and Peter Kondor, 2018, Trading and information diffusion in over the counter-markets, *Econometrica* 86, 1727–1769.
- Babus, Ana, and Cecilia Parlatore, 2018, Strategic fragmented markets, Working paper.
- Biais, Bruno, 1993, Price formation and equilibrium liquidity in fragmented and centralized markets, *The Journal of Finance* 48, 157–185.
- Caballero, Ricardo J, and Alp Simsek, 2013, Fire sales in a model of complexity, The Journal of Finance 68, 2549–2587.
- Chang, Briana, and Shengxing Zhang, 2016, Endogenous market making and network formation, Working paper.
- Collin-Dufresne, Pierre, Benjamin Junge, and Anders Trolle, 2020, Market structure and transaction costs of index CDSs, *The Journal of Finance* 75, 2719–2763.
- Craig, Ben, and Goetz von Peter, 2014, Interbank tiering and money center banks, *Journal* of Financial Intermediation 23, 322–347.
- de Roure, Calebe, Emmanuel Moench, Loriana Pelizzon, and Michael Schneider, 2018, OTC discount, Working paper.

- Di Maggio, Marco, Amir Kermani, and Zhaogang Song, 2017, The value of trading relations in turbulent times, *Journal of Financial Economics* 124, 266 – 284.
- Duffie, Darrell, Nicolae Garleanu, and Lasse Heje Pedersen, 2005, Over-the-counter markets, *Econometrica* 73, 1815–1847.
- Duffie, Darrell, Semyon Malamud, and Gustavo Manso, 2014, Information percolation in segmented markets, *Journal of Economic Theory* 153, 1 – 32.
- Dugast, Jerome, Pierre-Olivier Weill, and Semih Uslu, 2018, Platform Trading with an OTC Market Fringe, Working paper.
- Dunne, Peter G., Harald Hau, and Michael J. Moore, 2015, Dealer intermediation between markets, Journal of the European Economic Association 13, 770–804.
- Eisfeldt, Andrea L, Bernard Herskovic, Sriram Rajan, and Emil Siriwardane, 2018, OTC intermediaries, Working paper.
- Farboodi, Maryam, Gregor Jarosch, Guido Menzio, and Ursula Wiriadinata, 2019, Intermediation as rent extraction, Working paper.
- Farboodi, Maryam, Gregor Jarosh, and Robert Shimer, 2017, The emergence of market structure, Working paper.
- Friewald, Nils, and Florian Nagler, 2019, Over-the-counter market frictions and yield spread changes, *The Journal of Finance* 74, 3217–3257.
- Gallien, Florent, Serge Kassibrakis, Semyon Malamud, Nataliya Klimenko, and Alberto Teguia, 2018, Liquidity provision in the foreign exchange market, Working paper.
- Gofman, Michael, 2014, A network based analysis of over-the-counter markets, Working paper.
- Grossman, Stanford, and Merton Miller, 1988, Liquidity and market structure, The Journal of Finance 43, 617–633.
- Hendershott, Terrence, and Albert J. Menkveld, 2014, Price pressures, Journal of Financial Economics 114, 405 – 423.

- Ho, Thomas S Y, and Hans R Stoll, 1983, The Dynamics of Dealer Markets under Competition, *The Journal of Finance* 38, 1053–1074.
- Hollifield, Burton, Artem Neklyudov, and Chester Spatt, 2017, Bid-ask spreads, trading networks and the pricing of securitizations, *Review of Financial Studies* 30, 3048–3085.
- Hugonnier, Julien, Benjamin Lester, and Pierre-Olivier Weill, 2019, Frictional Intermediation in Over-the-Counter Markets, *The Review of Economic Studies* 87, 1432–1469.
- Huh, Yesol, and Sebastian Infante, 2018, Bond market intermediation and the role of repo, Working paper.
- Li, Dan, and Norman Schürhoff, 2019, Dealer networks, The Journal of Finance 74, 91– 144.
- Malamud, Seymion, and Marzena Rostek, 2017, Decentralized exchange, American Economic Review 107, 3320–3362.
- Neklyudov, Artem, 2019, Bid-ask spreads and the over-the-counter interdealer markets: Core and peripheral dealers, *Review of Economic Dynamics* 33, 57 – 84.
- Randall, Oliver, 2015a, How Inventory Costs Affect Dealer Behavior in the US Corporate Bond Market, Working paper.
- Randall, Oliver, 2015b, Pricing and Liquidity in Over-the-Counter Markets, Working paper.
- Sambalaibat, Batchimeg, 2018, Endogenous specialization and dealer networks, Working paper.
- Schultz, Paul, 2017, Inventory management by corporate bond dealers, Working paper.
- Stoll, Hans R., 1978, The supply of dealer services in securities markets, The Journal of Finance 33, 1133–1151.
- Uslu, Semih, 2019, Pricing and liquidity in decentralized asset markets, *Econometrica* 87, 2079–2140.

Vogler, Karl-Hubert, 1997, Risk allocation and inter-dealer trading, *European Economic Review* 41, 1615–1634.

Wang, Chaojun, 2017, Core-periphery trading networks, Working paper.

Yang, Ming, and Yao Zeng, 2019, The coordination of intermediation, Working paper.

#### Acknowledgements

We are indebted to Philip Bond, an anonymous associate editor, and two anonymous referees for very helpful feedback and suggestions. Our thanks for helpful discussions and comments also go to Ana Babus, Vincent Bignon, Regis Breton, Briana Chang, Hans Degryse, Hugues Dastarac, Peter Feldhutter, Nils Friewald, Rod Garratt, Sergey Glebkin, Co-Pierre Georg, Denis Gromb, Burton Hollifield, Cornelia Holthausen, Cyril Monnet, Stefan Nagler, Artem Neklyudov, Miklos Vari, Mao Ye, Adam Zawadowski, and audiences at various conferences and seminars. We are grateful to Ben Craig and Goetz von Peter for sharing their Matlab codes, and to Markus Bak-Hansen and Yapei Zhang for excellent research assistance. We received financial support from the French National Research Agency (F-STAR ANR-17-CE26-0007-01) and the Investissements d'Avenir Labex (ANR-11-IDEX-0003/Labex Ecodec/ANR-11-LABX-0047) for this project. Colliard gratefully acknowledges the Chair ACPR/Risk Foundation: Regulation and Systemic Risk for its support. The views expressed in this paper are the authors' and do not necessarily reflect those of the European Central Bank or the Europystem.

#### Jean-Edouard Colliard

HEC Paris, Jouy-en-Josas, France; email: colliard@hec.fr

#### **Thierry Foucault**

HEC Paris, Jouy-en-Josas, France; email: foucault@hec.fr

#### Peter Hoffmann

European Central Bank, Frankfurt am Main. Germany; email: peter.hoffmann@ecb.europa.eu

#### © European Central Bank, 2021

Postal address60640 Frankfurt am Main, GermanyTelephone+49 69 1344 0Websitewww.ecb.europa.eu

All rights reserved. Any reproduction, publication and reprint in the form of a different publication, whether printed or produced electronically, in whole or in part, is permitted only with the explicit written authorisation of the ECB or the authors.

This paper can be downloaded without charge from www.ecb.europa.eu, from the Social Science Research Network electronic library or from RePEc: Research Papers in Economics. Information on all of the papers published in the ECB Working Paper Series can be found on the ECB's website.

FDF ISDN 970-92-099-4529-5 ISSN 1725-2000 U01.10.2000/205055 QD-AN-21-020-1	PDF	ISBN 978-92-89	9-4529-5	ISSN 1725-2806	doi:10.2866/263633	QB-AR-21-020-EN
---	-----	----------------	----------	----------------	--------------------	-----------------