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On the inflation risks embedded in sovereign bond yields

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Abstract

The purpose of this paper is to study the compensation for inflation risks priced in sovereign bond yields. And we do so by modelling the time-varying dynamics of asset returns and inflation, and then estimating the cost of hedging inflation risks from the perspective of a well diversified portfolio. This allows to disentangle the time-varying compensation for expected and unexpected inflation shocks embedded in sovereign bond yields; and provides estimates of the real risk-free rate. We show that nominal sovereign bond yields for Germany, France, Japan and the United States, reflect, over the more recent years, a low real risk-free rate, as well as low levels of compensation for both expected and unexpected inflation. The simultaneous occurrence of these low contributions is novel, and not encountered previously in our sample. We also find that inflation risks are not necessarily reduced with the inclusion of real estate assets in the minimum variance portfolio. Our analysis also prompts us to suggest that the financial advantage of issuing inflation-linked sovereign debt, and namely saving on the embedded inflation risk premium of issuing nominal debt, appears to be eroded by the liquidity premium charged by investors for holding the less attractive inflation-linked debt asset.

JEL classification: C32, E31, G11, G12.
Keywords: Inflation Risks, Yields, Portfolio Choice.

NON-TECHNICAL SUMMARY

The purpose of this paper is to study the time-varying compensation for inflation risks priced in sovereign bond yields. And we do so by revisiting the work of Bodie (1976), and in particular his method for estimating the cost of hedging inflation risks from the perspective of well diversified portfolio. In doing so, we model explicitly the time-varying nature of asset returns; including also time variation in correlations and volatility. This allows us to disentangle the time-varying compensation for expected and unexpected inflation shocks demanded by holders of sovereign bonds; as well as providing us with estimates of the real risk-free rate.

When modelling asset returns by means of a time-varying VAR model, we adopt the kernel-based estimation method discussed in Giraitis et al. (2018). However, and as we will be employing this VAR model to infer long-term asset returns, it is critical that the estimated VAR model is stable (i.e. polynomial roots outside the unit circle). We enforce stability of the estimated time-varying VAR by integrating the estimation method proposed by Morf et al. (1978) into the kernel based procedure of Giraitis et al. (2018). A simple Monte Carlo study presented in this paper validates this method, and warns against the use of alternative methods which fail to enforce the stability of the VAR.

Over recent years there has been much talk in the financial circles on the Japanization of the West. That is, the extrapolation to the Western economies of the economic environment prevailing in Japan; and by this, it is meant an environment of weak growth, close to zero, or at points even negative, inflation; and perpetually low sovereign bond yields. The results presented in this paper suggest that the current very low nominal sovereign bond yields in Germany, France, Japan and the United States, are all a reflection of a low real risk-free rate, low inflation expectations and a low cost of hedging inflation risks. Taking one at a time, these low values are not abnormally low compared to historical norms. We have tentatively searched for a similar pattern in episodes characterised by recessionary phases of the business cycle coupled with low inflation expectations. However, we failed to robustly associate those episodes neither with low real risk-free rates nor low costs of hedging inflation risks. The simultaneous occurrence of a low real risk-free rate, low inflation expectations and a low cost of hedging inflation risks encountered in the aftermath of the Global Financial Crisis is indeed novel, and there are no other similar episodes in the sample under study.

In contrast with some previous studies, we also find that inflation risks are not necessar-

ily reduced with the inclusion of real estate assets in the minimum variance portfolio. Our results suggest that only in Germany, France and the United States during the 80s, and in France and Japan during the late 90s real estate provided investors with some additional inflation hedging power. However, this additional power came at a cost, as the cost of hedging inflation risks was 50 to 300 basis points higher when employing a diversified portfolio with real estate assets.

As part of our analysis, we also present an evaluation of the benefits of financing by means of inflation-linked debt. Our results suggest that the financial advantage of issuing inflation-linked sovereign debt, and namely avoiding the extra payment for compensating nominal debt holders for the inflation risks, appears to be eroded by the extra payment governments have to provide to inflation-linked-debt holders to compensate them for the relatively poor liquidity of this instrument.

1 Introduction

For many households, their expenses and, often, their salaries or pensions are both directly affected, but in opposite directions, by inflation. However, the purchasing power derived from the value of their wealth may not be immune to inflation. Most economic agents, are equally unlikely to hold balanced exposures to inflation risks. For example, pension funds that guarantee pensions that adjust with the level of inflation, and invest their assets in long-term fixed income debt are subject to inflation risks. Insurance companies offering say life insurance with a payment indexed to a price index, find themselves in a similar position. Certain banks offering savings accounts which are remunerated at rates partly linked to inflation are also potentially exposed to inflation risks.[1] The search for investment strategies that provide a hedge against inflation has thus been an important topic in the Finance literature. With the advent of the great inflation of the 1970s and 80s, not surprisingly, there was a large proliferation of studies in this field.[2] Much of the earlier work, e.g. Johnson et al. (1971); Oudet (1973); Bodie (1976); Fama and Schwert (1977), was devoted to the study of the inflation hedging properties of individual assets. Recent research has focused instead on the hedging properties of well diversified investment portfolios, see e.g. Strongin and Petsch (1997); Attie and Roache (2009); Amenc et al. (2009); Briere and Signori (2012); Martellini and Milhau (2013).

A priori, it is sensible to expect that most financial assets offer some compensation for inflation exposures. For example, *stocks* represent ownership of the physical productive capital of the firm, and the value of such capital (e.g. the machines and properties of the firm) should thus also be immune to inflation. It is also to be expected that investors only agree to hold *bonds*, if the interest paid compensates them for the inflation risks. Rather than locking an investment over a long-term period, rolling *cash* lending over short periods of time, in so far as the nominal charged rate reflects inflation, may also result in an investment strategy that reduces exposures to inflation risks. Beyond these assets, inflation-linked sovereign bonds, and/or inflation-linked swaps provide a '*perfect*' hedge against inflation risks.

The purpose of this paper is to study the time-varying compensation for inflation risks priced in sovereign bond yields since 1950 in Germany, France and Japan, and since 1920

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[1] This is indeed the case of a very popular saving account in France: the Livret A. This saving account is regulated by the French government, is free from income tax and social charges, and, importantly, must offer an enhanced rate of interest which is linked to developments in inflation.

[2] In his presidential address to the American Finance Association in 1974, John Lintner had suggested that '*few matters are of more serious concern to students of finance and members of the financial community than the impacts of inflation [...] and its implications for investment policy*', see Lintner (1975).

in the United States. Most of the recent literature on measuring the compensation for inflation risks embedded in sovereign bond yields, relies on the use of affine term structure models for pricing inflation-linked debt instruments, see e.g. Buraschi and Jiltsov (2005); Gurkaynak et al. (2010); D'Amico et al. (2018) for the United States and Hordahl and Tristani (2012); Camba-Mendez and Werner (2017); D'Amico et al. (2018) for the euro area. However, this is not an option for our study which goes back to times when such inflation link instruments were not traded, or no records of such trading can be found for analysis. As an alternative, we will follow Bodie (1976), and we will measure the cost of hedging inflation risks as the difference between the expected 'real' return of a nominal sovereign bond and the expected real return of a minimum variance portfolio. This approach has also been recently pursued in Amenc et al. (2009); Briere and Signori (2012), and will allow us to disentangle the compensation for expected and unexpected inflation shocks demanded by holders of sovereign bonds; as well as providing us with estimates of the real risk-free rate.

For our empirical analysis we follow the by now well established literature of employing a vector autoregressive (VAR) model to describe the dynamic behaviour of asset returns, see e.g. Campbell et al. (2003); Attie and Roache (2009); Amenc et al. (2009); Briere and Signori (2012). However, and rather than adopting a standard VAR model with fixed parameters, we use a time-varying VAR model. Time variation in asset return correlations has been largely acknowledged in the financial literature, see e.g. Bollerslev et al. (1988), Barsky (1989), or Engle (2002). Our choice of a time-varying VAR is further justified by the recent works of Dangl and Halling (2012), Johannes et al. (2014) and Grassi et al. (2017), which suggest that there are merits in the adoption of time-varying coefficient models to predict asset and portfolio returns; or indeed by the standard practice of evaluating portfolio performance using a rolling window to estimate expected returns and covariances, see e.g. DeMiguel et al. (2009) and Harris et al. (2017). Our modelling of time-varying correlations is also driven by the changing nature of the underlying forces driving inflation. First, monetary policy is now much more focused than in the past on targeting inflation. Second, globalisation, the ongoing trend of deregulation, and the widespread use of information and communication technologies (which has lowered the cost of access to information), have all fostered a more competitive environment which has kept profit margins under watch. Third, the rapid pace of technological innovation continue to contribute to the reduction of production costs.

Our results show the following. First, during the 70s and 80s, the lion's share of nominal bond yields were compensation for expected inflation, while from the mid-1990 this share has remained relatively contained, and broadly aligned with the level of the inflation target of the monetary policy authorities. Second, the cost of hedging inflation risks was larger on average, and more volatile, during the 70s and 80s compared to the sample from mid-1990s onwards. Third, episodes of a negative cost of hedging inflation risks have not been uncommon. Fourth, nominal sovereign bond yields for Germany, France, Japan and the United States in the aftermath of the Global Financial Crisis, reflect a low real risk-free rate, as well as low levels of compensation for both expected and unexpected inflation. The simultaneous occurrence of these low contributions is novel, and not encountered previously in our sample under study.

The paper is organised as follows. Section 2 elaborates on the conceptual framework for our paper. We explain Bodie's concept for the cost of hedging inflation, which we borrow for our analysis. Section 3 discusses our modelling strategy, and in particular, the time-varying VAR model which we employ to compute expected asset returns and covariances. Section 4 presents our main empirical results. Finally, section 5 provides some concluding remarks.

2 The Cost of Hedging Inflation

As indicated above, for some investors there is an asset class which provides a '*perfect*' hedge against inflation risks; namely the inflation-linked sovereign bond, and/or inflation-linked swaps. However, hedging inflation risks with these instruments, is not a viable option for all investors. For once, there may not be trading associated with the inflation index the investor is exposed to. Furthermore, the cost of hedging inflation risks using these products (measured as the foregone expected real return, or the resulting undesirable distribution of the funding ratio) may be very high. In the quest for hedging inflation risk exposures, many investors have thus resorted to investments in portfolios which contain risky assets which may only '*partly*' countered such exposures. Bodie (1976) defined the 'cost of hedging' inflation risks as the difference between the expected real return of the sovereign bond, and that of a minimum-variance portfolio. When this difference is positive, investors pay a premium for holding a portfolio which safeguards against inflation volatility.

2.1 Perfect Inflation Hedging with Inflation-Linked Bonds

Think on a sovereign bond offering a one-period nominal return of i; and a sovereign inflation-linked-bond offering a one-period 'real' return of r. The 'real' value of a unit investment in these two asset classes is respectively:

$$\begin{array}{rl} \text{Bond} & 1+i-\bar{\pi}-\varepsilon \\ \text{Inflation-linked bond} & 1+r \end{array}$$

and where $\bar{\pi}$ is the expected inflation rate, ε is an 'unexpected' inflation shock. The nominal return of the bond is eroded by expected and unexpected inflation. In the absence of arbitrage, it follows from standard finance theory that the expected value of the product of the (stochastic) discount factor M and the real return of an asset (in per unit terms) should be equal to one. It must thus follow that:

$$1 = E\left\{M\left(1+i-\bar{\pi}-\varepsilon\right)\right\} = E\left\{M\left(1+r\right)\right\}$$

Noting that r is deterministic, if follows that $E\left\{M\right\} = \left(1+r\right)^{-1}$. Using this result and some standard algebra on the expression for the real return of the nominal bond, it follows that:[3]

$$\left(1+i-\bar{\pi}\right) - \left(1+r\right) = \left(1+r\right)\operatorname{Cov}\left(M,\varepsilon\right) = \text{IRP}$$

What this expression suggests is that the difference between the expected 'real' return of a nominal bond and the real return of the inflation-linked bond is equal to the inflation risk premium (IRP). Inflation risks are fully hedged when investing in the inflation-linked bond. The price to pay, is the reduction in expected real return, i.e. the inflation risk premium. It should be noted, that this premium may not always be positive. The sign and magnitude of that premium is associated with the sign and magnitude of the covariance between the inflation shock and the stochastic discount factor, as further discussed in Camba-Mendez and Werner (2017).

2.2 Partial Inflation Hedging with an Investment Portfolio

We can define the 'real' value of a unit investment in a 'risky asset' offering a expected nominal return of s as being equal to:

$$\text{Risky asset} \quad 1+s-\bar{\pi}-\gamma\varepsilon+\nu$$

where ν is a random shock specific to the risky asset, and γ is a parameter. Once more, in the absence of arbitrage, it follows from standard finance theory that:

$$1 = E\left\{M\left(1+s-\bar{\pi}-\gamma\varepsilon+\nu\right)\right\}$$

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[3] In the derivation of the result, use is made of the fact that for two random variables X and Y it follows that $E\left(XY\right) = E\left(X\right)E\left(Y\right) + \operatorname{Cov}\left(X,Y\right)$.

Using similar standard algebra, but this time operating on the expression for the risky asset, it also follows that:

$$\begin{aligned}(1+s-\bar{\pi})-(1+r) &= \gamma \mathrm{IRP}-(1+r)\operatorname{Cov}(M,\nu)\\ &= \gamma \mathrm{IRP}+\rho_s\end{aligned}$$

Now the excess real return reflects compensation for inflation risk and for stock specific risks, ρ_s.[4] An investment on a portfolio with a weight ω on the risky asset and a weight $(1-\omega)$ on the nominal bond may indeed still provide some partial hedging of the inflation risks. Such portfolio would provide the following real return:

$$\text{Portfolio} \quad 1+i-\bar{\pi}+\omega(s-i)-[1+\omega(1-\gamma)]\,\varepsilon+\omega\nu$$

Once more using the non-arbitrage relationship provides us with an expression for the expected excess real return of the portfolio.

$$(1+s-\bar{\pi})-(1+r)=[1+\omega(1-\gamma)]\,\mathrm{IRP}+\omega\rho_s \tag{1}$$

The expression on the left-hand side of (1) is the expected excess return of the portfolio versus the inflation-linked bond. It is partly a reflection of the inflation risk premium and the premium paid for investing in the risky asset. Bodie (1976) suggested as an inflation hedging strategy to choose as weight, ω for the portfolio, that which minimizes the variance of the expected real return of the portfolio. It is simple to show that that weight should be:

$$\omega=-\frac{1-\gamma}{(1-\gamma)^2+\sigma_\nu^2/\sigma_\varepsilon^2}$$

This portfolio partly isolates (but not fully) for the inflation risks, but it would also incorporate some of the risks specific to the risky asset. The definition for the cost of hedging inflation risks put forward by Bodie, which we adopt in this paper, refers to the difference between the expected real return of the nominal sovereign bond and the expected real return of this minimum variance portfolio.

Had we adopted $\omega=-1/(1-\gamma)$ as weight for the portfolio, this would have resulted in a portfolio 100% immune to fluctuations in unanticipated inflation. Working out the weights for such a portfolio could be accomplished by means of modelling the dependence between

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[4] For the risky stock we follow the standard practice of using the negative of the covariance for defining the risk premium. When the covariance is negative, this means that the return is low when the discount factor (associated with the unwillingness to forego consumption) is high; this would render such an asset unattractive and the investor would demand a premium for holding it. The sign for the covariance term associated with the inflation risk premium is of the opposite sign. This is so because, inflation hampers the real return of an investment.

asset returns and inflation. However, such a portfolio would possibly embed higher risks, and thus offer higher expected return, than a nominal sovereign bond. Building such a portfolio would thus not allow us to identify the 'cost' of hedging inflation risks.

3 Modelling Strategy

For our portfolio analysis, we focus on portfolios of domestic assets denominated in local currencies. The assets comprise: sovereign bonds, cash, equity, commodities, and gold.[5] In line with the literature on return predictability, when modelling asset returns, we also incorporate into our modelling framework other relevant series which help to predict asset returns; and namely the dividend yield, and the term spread, see e.g. Campbell et al. (2003), Cochrane (2008), and Hoevenaars et al. (2008). Given our interest in modelling 'real' returns, we also include inflation in our modelling framework. Our focus is on the ten-year investment horizon, and we take the ten-year zero-coupon sovereign bond yield as the guaranteed 'nominal' return of an investment in *bonds*. From this perspective, sovereign bond yields do not need to be formally included in the model. We employ monthly series from January 1955 to December 2018 for Germany, France, and Japan, and from January 1920 to December 2018 for the United States. The appendix provides full details on the construction of our dataset.

In more precise terms, we need a model for the random vector series $\boldsymbol{x}_t$, which comprises: i) monthly inflation; ii) monthly nominal returns of the various asset classes (but excluding the sovereign bond yield): *cash*, *equity*, *commodities* and *gold*; and iii) the *term spread* and the *dividend yield*. We choose to model the dynamics of $\boldsymbol{x}_t$ by means of a time-varying VAR model. This model will provide time varying estimates of expected 'real' returns (by discounting the projected erosion in value resulting from future inflation), and time-varying covariances of 'real' asset returns. More precise details for the computation of these time-varying estimates are provided in section 3.1 below. Using these estimates at every point t, we compute the minimum variance portfolio, and thus also estimates of the expected real return of this minimum variance portfolio. The difference between the expected real return of the ten-year sovereign bond yield and the real return of this minimum variance portfolio is our measure for the cost of hedging inflation risk. The portfolios are not dynamic portfolios, but those resulting at every point in time t from decisions of buy-to-hold long-term investors.

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[5] We only allow for the inclusion of gold in the portfolio after August 1971. Prior to that date, gold prices were fixed as part of the Bretton Woods agreement in early 1947, and thus the expected nominal return from gold was in effect zero, and would thus not represent a superior asset class to Treasury Bills. The price of gold was fixed at 35 dollars, while other governments agreed to keep the value of their currencies relatively stable vis-a-vis the dollar.

3.1 Time-Varying 'Real' Expected Returns and Covariances

The dynamics of $\boldsymbol{x}_t$ are described by a time-varying VAR model of the form:

$$\boldsymbol{x}_t = \boldsymbol{\delta}_t + \boldsymbol{B}_{1,t}\boldsymbol{x}_{t-1} + \ldots + \boldsymbol{B}_{p,t}\boldsymbol{x}_{t-p} + \boldsymbol{\varepsilon}_t \tag{2}$$

The intercept term, $\boldsymbol{\delta}_t$, as well as the coefficient matrices $\boldsymbol{B}_{i,t}$ are time-varying; while $\boldsymbol{\varepsilon}_t$ follows an n-variate standard normal distribution with zero mean and covariance matrix $\boldsymbol{\Omega}_t$, which is also time-varying (i.e. stochastic volatility). The expected 'real' accumulated returns are computed by simply deducting from the accumulated nominal returns projected from the VAR, the expected accumulated inflation, as also projected by the VAR. The real return of the nominal bond is simply computed by subtracting the expected accumulated inflation (as projected by the VAR) from the zero-coupon bond yield at each time t. The computation of the covariance matrix of the 'real' returns from the computed covariance matrix of the accumulated nominal returns is also easily workable. In particular, the computation of the expected accumulated returns, and covariance matrix of the accumulated returns, follows from standard results on forecasting with VAR models, see e.g. Lutkepohl (1993, Ch. 2). When computing forecasts using our time-varying VAR model, we simply adopt the usual convention of assuming that the parameters $\boldsymbol{\delta}_t$, $\boldsymbol{B}_{i,t}$ and $\boldsymbol{\Omega}_t$ remain fixed out of sample, see e.g. Koop and Korobilis (2013) and Abbate and Marcellino (2018).

We adopt the kernel-based method discussed in Giraitis et al. (2018) for the estimation of $\boldsymbol{\delta}_t$, $\boldsymbol{B}_{i,t}$ and $\boldsymbol{\Omega}_t$. Beyond its computational simplicity, this method is also robust to a broad range of alternative modelling assumptions for the time varying coefficients. When applying this kernel-based method, we depart from the 'standard' implementation discussed in Giraitis et al. (2018) on two main dimensions. First, rather than using a Gaussian kernel for the estimation of the time-varying VAR, we adopt an exponentially weighting filter with a decaying factor equal to 0.97. The use of this kernel is aligned with standard practice in Finance for modelling of stochastic volatility. Furthermore, it relies exclusively on present and past (but not future) information, and thus computes 'truly' expected returns at a point in time. Second, the implementation of Giraitis et al. (2018) amounts to least square estimation on appropriately weighted data. However, least square estimation does not enforce the stability of the estimated VAR system, i.e. the absolute value of the roots of the VAR polynomial defined by the coefficients are not guaranteed to be larger than one. As discussed below, this is very problematic when using the VAR model for forecasting over long horizons. Instead of least squares, we adopt the estimation method proposed by Morf et al. (1978)

which enforces the stability of the VAR.[6,7]

3.2 Monte Carlo Evaluation of our Modelling Strategy

For our Monte Carlo study we simulate a k-dimensional VAR(1) model similar in spirit to that chosen for their Monte Carlo analysis by Giraitis et al. (2018):

$$\boldsymbol{x}_t = \boldsymbol{\delta}_t + \boldsymbol{B}_t \boldsymbol{x}_{t-1} + \boldsymbol{\varepsilon}_t \tag{3}$$

With $\boldsymbol{\varepsilon}_t$ a normally distributed error term with covariance matrix $\boldsymbol{\Omega}_t = \text{diag}\left(\omega_{1,t}, \cdots, \omega_{k,t}\right)$. We further assume independent random walk time variation for the components of $\boldsymbol{\Omega}_t$, implying stochastic volatility for the stochastic process driving $\boldsymbol{x}_t$; the components of $\boldsymbol{\delta}_t$ and $\boldsymbol{B}_t$ are equally assumed to follow random walk processes with stochastic volatility, and where we additionally enforce non-explosive behaviour in $\boldsymbol{B}_t$. The algebra describing the underlying processes for the simulation of the VAR is somehow tedious, and is thus left for the appendix.[8] Four estimation methods are put to the test:

KF method. This refers to the implementation of a standard Kalman filter assuming that the time-varying noise variances are known. This method provides a benchmark for comparing our results. However, this method is not valid for empirical applications, as the noise variances are unknown.

KK method. Koop and Korobilis (2013) proposed a parsimonious and computationally light estimation approach which relies on the use of forgetting factors in combination with the Kalman filter. Interestingly, this alternative approach has also been shown suitable for forecasting, see Abbate and Marcellino (2018). However, this method does not enforce the

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[6] While the Yule-Walker equations also enforce the stability of the VAR, and should thus have been a more standard choice, Poskitt (1994) has shown that the procedure of Morf et al. (1978) is less prone to suffer from the significant bias of the Yule-Walker method in finite samples.

[7] The dynamics of the parameters, and the representation of the full model in (2) is most commonly completed by assuming that the parameter matrices $\boldsymbol{\delta}_t$ and $\boldsymbol{B}_{i,t}$ follow stochastic random walks. Modern methods to estimate these time-varying VAR models would then rely on the use of Markov-Chain Monte Carlo methods (MCMC), see e.g. Cogley and Sargent (2005) and Primiceri (2005). The use of MCMC methods remains, however, computationally challenging when the dimension of the VAR exceeds 4 or 5. The most challenging issue when employing Bayesian methods, is to constrain the simulated coefficients to a set which enforces a non-explosive VAR model. Koop and Potter (2011) provide an MCMC algorithm which enforces the simulations of non-explosive VAR coefficients. However, the acceptance probabilities employed in their Metropolis-Hastings algorithm cannot be evaluated analytically, and must be evaluated via simulations which renders the method once more, as very challenging for large VAR models.

[8] Monte Carlo simulations for more simple assumptions on the data generating process for $\boldsymbol{x}_t$ (e.g. simple random walk assumptions for the elements of $\boldsymbol{B}_t$, that is with no stochastic volatility, or homogeneous error term for $\boldsymbol{\varepsilon}_t$) were also conducted and provided similar results.

estimation of a stable (non-explosive) VAR model.[9]

LS method. This refers to the kernel-based method proposed by Giraitis et al. (2018), but as discussed above, using an exponentially weighted filter.

MVLK method. This relates to the kernel based method of Giraitis et al. (2018), but using the Yule-Walker type of estimator proposed by Morf et al. (1978), and not the least squares rolling window approach of the LS method above; and once more employing an exponentially weighted filter.

Table 1 reports the relative root mean square error (RMSE) of the various methods with respect to the MVLK benchmark. Values larger than one suggest the MVLK method is better Values for the MVLK method are thus left unreported. The table also reports the instances in percent when explosive behaviour is encountered. The KF and KK methods are very prone indeed to estimate explosive VAR models. For the KF method, knowledge of the true data generation process for time varying coefficients is not sufficient to guarantee the estimation of a stable VAR. The LS method, while less prone to estimate explosive VARs, still fails to come with a stable VAR in 25% to 40% of the simulations. Unsurprisingly, the KF method is best to estimate $\boldsymbol{B}_t$. However the forecasting performance of the KF method (which fails to enforce stability) is far from optimal when forecasting over medium and long-term horizons. The forecasting advantage of the KF is lost beyond the $t + 12$ horizon (one year when using monthly data). The same can be said for the KK method. However, the KK method is not always at an advantage to estimate $\boldsymbol{\Omega}_t$, which also plays an important role in portfolio analysis. The table suggests that the performance of the LS method is very similar to that of the MVLK. We are thus inclined to employ the MVLK method in our empirical application, as this nonetheless completely avoids the potential dangers of employing an explosive VAR model, which the average reported results in our Table may not easily identify.

4 Main Empirical Results

4.1 Inflation Risks Embedded in Yields

We employ the time-varying VAR to construct the minimum variance real-return portfolio. We will refer to this portfolio, as the ‘inflation hedging’ (IH) portfolio. We choose to discuss

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[9] We also fix the parameters λ and κ with the standard values of 0.94 and 0.975 respectively as suggested by Koop and Korobilis (2013).

our empirical results by breaking down the long-run nominal return of a 10-year sovereign bond into the sum of three main components:

$$y = r + \bar{\pi} + CHI \tag{4}$$

where y is the 10-year sovereign bond yield; r is the expected real return of the IH portfolio, which we will alternatively refer to as the real risk-free return; $\bar{\pi}$ is expected inflation, thus measuring compensation for the 'expected' inflation risks embedded in the yield; and CHI is the cost of hedging inflation risks defined in section 2, which measures compensation for the 'unexpected' inflation risks embedded in the yield. Note that the minimum variance portfolio provides only partial hedging of inflation risks.[10] From this perspective, the estimate of the CHI is not to be understood as being 'fully' equivalent with the term associated with the inflation risk premium, i.e. $[1 + \omega\,(1 - \gamma)]\,\mathrm{IRP}$ in equation (1). The estimate of CHI may contain also compensation for other residual risks embedded in the minimum variance portfolio, $\omega\rho_s$ in equation (1). In a similar vein, the expected real return of the minimum variance portfolio, r in equation (4), may be seen as a good proxy for the real rate of return formulated in equation (1), in so far as the volatility associated with the minimum variance portfolio is sufficiently small. In our empirical results below we will show that the volatility reduction in real returns achieved by the minimum variance portfolio is very significant, albeit not full.

Our main results are shown in Figures 1 to 4. The top panel of these figures shows the decomposition of the sovereign bond yield, while the lower three panels display the individual components; both its expected value (inferred from the IH portfolio), and the realised value after 10 years. Needless to say, 10-year forecasts are not known for being very accurate, and thus the gap between the expected and realised values may be large at times.[11]

On the real risk-free rate. The 'expected' long-run risk-free rate has fluctuated over a broad range; has been on a slightly declining trend since 1990; and has stood at low values over the most recent period, 2010-2018. For Germany, one could argue that the very low risk-free rate recorded over 2010-2018 is at odds with historical regularities. However, for the United States, France and Japan, the current estimate is not abnormally low. The estimates for the 'expected' real risk-free rate have been below 3% on average over the full sample; and over the more recent period 2010-2018 have stood on average below 0.35%, see

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[10] See the discussion in section 2 above.

[11] The reported gaps are helpful to evaluate the uncertainty surrounding the estimation of the real risk-free rate, the expected inflation and the cost of hedging inflation risks.

Table 2.[12] These low estimates bring to mind the old debate about the so-called '*risk-free rate puzzle*' which claimed that very low real risk-free rates are difficult to reconcile with common estimates of agents aversion to intertemporal substitution of consumption, see e.g. Mehra and Prescott (1985) and Weil (1989). One resolution to the puzzle was to account for the possibility of rare but severe market crashes, see Rietz (1988) and more recently Nakamura et al. (2013), which seems appropriate to recall after the recent Global Financial Crisis. Our figures also show that the estimate for the 'expected' real risk-free rate grossly underestimated the 'realised' real risk-free rate from the early 70s to the late 80s. For the US, our estimate of the 'realised' real risk-free rate, at 3.59%, is broadly aligned with that reported by Siegel (1992) when claiming to solve the risk-free rate puzzle by simply reporting estimates over a longer time span than in the earlier study of Mehra and Prescott (1985). Our 'realised' real risk-free rate is also slightly above that reported in Jorda et al. (2017) for the post-1950 period. For example, for the United States our estimate amounts to 4.47% for the post-1950 period, while Jorda et al. (2017) reported an estimate of 2.64%.[13] However, Jorda et al. (2017) reported the 'realised' real return of a sovereign bond, while we document the 'realised' real return of the IH portfolio.

On inflation expectations. The top panel of Figures 1 to 4 equally suggests that during the 70s and 80s the compensation for expected inflation represented the lion's share of nominal bond yields. In contrast, since the mid-1990s, the compensation for expected inflation embedded in yields has remained relatively contained, and broadly aligned with the level of the inflation target of the monetary policy authorities. During the period that goes from the early 1960s to the mid-1970s inflation was largely underestimated; while the opposite was the case for the period from the mid-1970s to the mid-1980s. As results in Table 2 also show, in spite of gross errors made in inflation forecasts over these periods, there was not a clear bias in inflation forecasts over the full sample.

On the cost of hedging inflation risks. The cost of hedging inflation risks has fluctuated over a relatively broad range throughout the sample. The cost of hedging inflation was larger on average, and more volatile, during the 70s and 80s, compared to the more recent years of the Global Financial Crisis. Episodes of a negative cost of hedging inflation are not uncommon. Our average estimates of the cost of hedging inflation for the United

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[12] Our estimate of the 'expected' real risk-free rate for the US over the full sample, at 1.70%, is broadly aligned with that reported by Briere and Signori (2012) for the real return of their minimum variance portfolio.

[13] Our 'realised' real risk-free rate for the post 1950 period amounts to 4.5%, 4.98%, 3.53% and 4.47% respectively for DE, FR, JP and US respectively, compared to 3.69%, 2.96%, 2.83% and 2.64% in Jorda et al. (2017).

States, Germany and France are broadly aligned with estimates of the inflation risk premium reported in other studies. But we cannot say the same for Japan. For example, D'Amico et al. (2018) report an average inflation risk premium of 29 basis points in the United States for the period 1990-2013, while our average estimate for that period is 46 basis points. In the same vein, Buraschi and Jiltsov (2005) report an estimate of 70 basis points for the period 1960-2000, while our average CHI for that same period is 89 basis points. Hordahl and Tristani (2012) report an average inflation risk premium of between 0 and 50 basis points for the euro area for the period 1999-2007, while our average estimate for Germany is 50 basis points and for France 5 basis points. Our estimates of the cost of hedging inflation risks are also more volatile than the estimates reported in these studies, which relied on the use of affine term structure models. Once more, we need to bear in mind that the estimate of the CHI is not 'fully' equivalent with the inflation risk premium captured by these affine term structure models, as the estimate of CHI may also embed compensation for other residual risks in the portfolio. While negative risk premiums cannot be excluded over some special episodes in the sample under study, we can possibly safely assumed that on average, the estimate of CHI may be slightly contaminated upwards by some residual positive risk premia. To the extent that the reduction in the volatility of real returns achieved by the minimum variance portfolio since the 1980s is very significant, the magnitude of that bias should be small. For Japan our estimates do not appear aligned with the estimates reported in the literature which are available to us. In particular, Imakubo and Nakajima (2015) reported an inflation risk premium which fluctuated between 0 and -60 basis points over the period 2007-2015, while in our study the average for this period is 65 basis points.

On recessions and the cost of hedging inflation. David and Veronesi (2013) and D'Amico et al. (2018) suggest that inflation risk premiums are likely to turn negative during periods of deflationary risks. In table 3 we report the median estimate of the real risk-free rate and of the cost of hedging inflation over periods characterised by different economic regimes. Deflationary risks should be highest during recessionary periods with low inflation expectations.[14] Our median estimates reported in Table 3 do not allow us to suggest that a negative cost of hedging inflation is characteristic of recessionary episodes with low inflation, or indeed simply associated with episodes of low inflation expectations. While for Japan and the US our estimates would be broadly aligned with such an assessment, this pattern does not hold for Germany and France. Furthermore, the cost of hedging inflation in Japan and the US was lower during periods which equally displayed low inflation, but were expansionary.

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[14] The episodes characterised by being on a recession and predicting very low inflation may not have too many observations (16, 29, 46 and 88 respectively for DE, FR, JP and US), and for this reason we choose to report the median, which should be more robust to the effect of outliers.

4.2 Weights and Variance of IH Portfolio

Figure 5 shows some of the characteristics of the IH portfolio for the United States.[15] The top panel displays the weights of the portfolio. The second panel from the top shows the volatility (standard deviation) of the real return of the IH portfolio; and, for comparison, we also show in the bottom two panels the volatility of the real return of the sovereign bond, and the volatility of the real return of an investment in Cash. Once more, the figure shows both the expected volatility and the realised volatility.[16]

The asset class with the highest weight is usually cash. At times, the portfolio is exclusively composed of cash. Most often the IH portfolio is primarily a portfolio of cash and sovereign bonds with time-varying weights. Commodities and equities are also sometimes included, but with much smaller weights; and furthermore, for the past twenty years, with the possible exception of Japan (not shown in the figure), their weights in the IH portfolio have been almost negligible. As shown in the second panel, the reduction in volatility achieved by the IH portfolio is very large, this result also extends to the other countries. A large share of the inflation risks embedded in the sovereign debt bond asset is diversified away in the IH portfolio. This is so in expected terms, when looking at the true realised estimate for the volatility of the IH portfolio, the variability of the real return was higher than that expected and realised for an investment in the sovereign bond for the period mid-70 to late 80s, and this was so across all four countries. Incidentally, during those years, the higher volatility of the IH portfolio resulted from much larger than expected real returns. In retrospect, the model had envisaged both higher inflation, and higher cost of hedging inflation than those that later materialised. Finally, the lower panel shows that the IH portfolio also provides large efficiency gains, in terms of reducing volatility, with respect to an investment in Cash. This result is in contrast with that reported in Bekaert and Wang (2010), who had gone to report inflation hedging portfolios which gave a weight very close to one for the Cash asset. From their perspective they suggested that this illustrated more the variance reducing properties of the asset, rather than the inflation hedging properties per se. However, the portfolio weights reported in Bekaert and Wang (2010) were computed using expected real returns and covariances estimated for the full sample of available real returns, and was not exploiting the time-varying analysis of this paper.

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[15] For the sake of space, we have chosen not to report similar figures for Germany, France and Japan. However, the main broad results discussed in this section usually extend to those countries.

[16] We compute the realised volatility by means of the root of the exponentially weighted average of the square of the forecasting errors.

4.3 IH Portfolio with Real Estate

In this section we replicate the results reported in the previous section but, i) starting the analysis in 1970; and ii) allowing for the inclusion of real estate assets.[17] The good properties of real estate to hedge against inflation risks in the United States was already documented in Amenc et al. (2009) and Bekaert and Wang (2010). Bekaert and Wang (2010) were less conclusive about the hedging properties of real estate for other world regions, and in particular the European Union.[18] The analysis reported in those papers did not, however, take into account the time-varying dynamics of asset returns and inflation. In Figure 6 we report estimates of the Cost of Hedging Inflation for two alternative portfolios: one which may contain real estate assets, and one without real estate assets and labelled in the figure as a 'Standard' portfolio.[19] For most of the sample, the cost of hedging inflation risks is the same for both portfolios, and thus suggests that there is no gain from including real estate assets into the portfolio. Real estate presented investors with a good inflation hedging option only during the 80s in DE, FR and US, and also during the late 90s in FR and JP. However, this advantage came at a cost, as the cost of hedging inflation risks was 50 to 300 basis points higher when employing the 'Real Estate' portfolio.

A somehow more intriguing result, relates to those periods when the Real Estate Portfolio provided a cheaper option for hedging inflation risks. The instances when this happened relate to those years which preceded the collapse of a housing bubble in FR (house price expansion of late 2000s), JP (house and asset price bubble of late 80s), and US (house price expansion of first half of 2000). Interestingly, the estimates for DE also suggest that the cost of hedging inflation risks with a real estate portfolio have been cheaper over the more recent years when house prices have been increasing at levels above historical norms. However, when reviewing the 'realised' performance of these real-estate portfolios during housing booms, it transpires, that the true cost of hedging inflation turned out to be much higher, as losses from the (unanticipated) burst of the house bubble materialised.

4.4 Break Evens and Liquidity Premiums

The theoretical framework presented in section 2 suggested that an investment in inflation-linked debt provided a perfect hedge for inflation risks. The yield of sovereign inflation-linked

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[17] Data on real estate asset returns available to us only start in January 1970.

[18] Bekaert and Wang (2010) reported no good hedging properties for either expected or unexpected inflation risks in the European Union.

[19] We rely on the same time-varying VAR system for constructing these estimates. The two IH portfolios are then computed by either allowing or disallowing for the inclusion of real estate assets.

debt instruments are thus void of compensation from inflation risks. However, in the analysis presented in section 2 we ignored the fact that inflation-linked debt instruments usually command a liquidity premium with respect to the more liquid nominal debt instruments. It could thus be argued that the real yield of an inflation-linked bond is the sum of the real risk-free rate and the liquidity premium, that is:

$$y^L = r + LP \tag{5}$$

where we use LP to denote the liquidity premium. The so called break-even inflation rate (BEIR) for a maturity of say 10 years, is the spread between the 10-year yield of the nominal sovereign bond instrument and the 10-year yield of the inflation-linked instrument. From equations (4) and (5) this is simply $BEIR = \bar{\pi} + CHI - LP$. Using recent data for the BEIRs, and our estimates of $\bar{\pi}$ and CHI, we can provide an implicit measure for the liquidity premium.[20] We could use this decomposition to evaluate the benefit for the sovereigns of financing via inflation-linked debt. Governments should profit when issuing inflation-linked debt as opposed to nominal debt, when the extra payment for compensating nominal debt holders for inflation risks, is more than the extra payment they have to provide to holders of inflation-linked debt for the relatively more poor liquidity of that instrument. We conduct this analysis using BEIR data for the United States over the period 1997-2018, and for France over the period 2003-2018.[21]

The estimates for the liquidity premium are reported in Figure 7. For the United States, and but for one 'not' minor difference, these estimates are broadly aligned with estimates reported in Gurkaynak et al. (2010), Fleckenstein et al. (2014) and D'Amico et al. (2018). As in those studies, the liquidity premium declined from levels of around 70 basis points in the early 2000s to around 15 basis points in 2005, only to climbed to high values in 2009 during the financial crisis years, when investors demanded a very high premium to hold TIPS. Our estimate displays a similar conclusion, but for 2006, when it was much higher.

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[20] Note that our estimate of CHI and r have been identified from the results of our minimum variance portfolio. As stated above, their reliability as good proxies for the inflation risk premium and the real rate of return respectively, is thus dependent on whether the minimum variance portfolio is sufficiently void of risk, or, in other words, on whether their magnitudes are large relative to the magnitude of the compensation for residual risk. The presence of liquidity premia in inflation-linked bonds does not alter this assessment.

[21] Germany has also issued inflation-linked debt, but the reference price index is the inflation price index for the eurozone, rather than domestic inflation which is the focused on this paper. For Japan, issuance of inflation-linked debt resumed in 2004, however, the issuance has focus on the 10-year instrument, and the number of debt instruments currently traded make it difficult to estimate a real yield curve. Data on BEIR for the United States are taken from Haver Analytics. For France, we use the reported data on BEIR for the period 2003-2014 from Haver Analytics, and construct our own estimate of the BEIR using real 10-year rates from OATis provided by the Agence France Tresor.

Figure 7 also shows that very often the liquidity premium exceeded the cost of hedging inflation, and the magnitude of the LP was non-negligible. Over the sample shown in Figure 7 the cost of hedging inflation averaged 40 and 25 basis points for the United States and France respectively, and the magnitude of the liquidity premium in linkers was broadly aligned on average with those figures. The net average loss for the Debt Management offices, as measured by the spread between the cost of hedging inflation and the liquidity premium came to -1 and -2 basis points for the United States and France respectively. From this perspective, and over the sample shown in Figure 7, our results suggest that debt issuance in the form of inflation-linked bonds has not been cheaper (although it has not been significantly more expensive) compared to financing with standard nominal debt instruments. For the United States, this result is indeed aligned, and equivalent to the results reported in Fleckenstein et al. (2014) and D'Amico et al. (2018), although in this paper we have come to this result via a different modelling route.[22]

Of course, the final cost of servicing inflation-linked debt will depend on realised inflation. From this perspective, positive inflation surprises, that is realised inflation turning out to be higher than previously expected, will result in a higher cost of financing via inflation-linked debt than standard nominal debt; and the opposite holds true in the event of negative inflation surprises. However, inflation surprises are unlikely to be biased over the long-term, and thus this issue could be safely neglected when taking a long-term perspective.

5 Conclusions

Over recent years there has been much talk in the financial circles on the Japanization of the West. That is, the extrapolation to the Western economies of the economic environment prevailing in Japan; and by this, it is meant an environment of weak growth, close to zero, or at points even negative, inflation; and perpetually low sovereign bond yields. The results presented in this paper suggest that the very low nominal sovereign bond yields in Germany, France, Japan and the United States, recorded during the late 2010s are all a reflection of a low real risk-free rate, low inflation expectations and a low cost of hedging inflation risks. Taking one at a time, these low values are not abnormally low compared to historical norms.

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[22] In computing a measure of the liquidity premium of TIPs, Fleckenstein et al. (2014) rely on the observation that the cash flows of a nominal sovereign bond can be replicated by the following investment strategy: i) being long on an inflation-linked bond, ii) paying inflation (and receiving the fixed agreed rate) on an inflation-linked swap; and iii) going short or long on STRIPS for special, easily workable amounts. They found that the price of nominal bonds usually exceeds the price of this investment strategy. Meanwhile, D'Amico et al. (2018) compute the liquidity premium by means of an affine term structure model comprising nominal yields, inflation-linked debt yields, and inflation; and where they allow for an inflation-linked specific factor to capture the liquidity premium.

We have tentatively searched for a similar pattern in episodes characterised by recessionary phases of the business cycle coupled with low inflation expectations. However, we failed to robustly associate those episodes neither with low real risk-free rates, nor with low costs of hedging inflation risks. The simultaneous occurrence of a low real risk-free rate, low inflation expectations and a low cost of hedging inflation risks encountered in the aftermath of the Global Financial Crisis is indeed novel, and there are no other similar episodes in the sample under study.

While recent studies have offered various explanations for the declining trend in inflation expectations (e.g. monetary policy, globalisation and technological changes), less attention has been paid to the decline in the cost of hedging inflation risks. We offer two very tentative explanations, which we deem important to pursue in future research. First, the trend decline in the cost of inflation hedging since the 1970's is most likely explained by the fact that monetary policy became more successful at targeting inflation. In turn, inflation became easier to forecast over long horizons and unexpected inflation shocks became smaller. Second, the further decline in the cost of hedging inflation since the outbreak of the great financial crisis, most likely reflects a shift in investor sentiments away from inflation concerns to higher awareness of potential deflation risks.

While the estimates of the cost of hedging inflation risks have been rather volatile, they have been most commonly positive and relatively sizeable. This would suggest scope for gains for Debt Management Offices. However, the cost of hedging inflation risks has not usually been sizeable enough to compensate for the liquidity premium charged for holding inflation linked debt. In spite of this, there might be merits of issuing inflation-linked debt. First, debt management offices could implement strategies that reduced the liquidity premium. For example, switching more issuance towards inflation-linked debt instruments would render these instruments less scarce. Second, there are important benefits associated with the issuance of inflation debt instruments. For example, i) broadening the investor base for debt management offices; ii) being a unique tool for mobilising financing in an environment of hyperinflation; and iii) providing welfare gains for households, as this instrument offers a true insurance against inflation risks, see e.g. Price (1997).

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Technical Appendix

A The Data

Inflation: Consumer Price Index (CPI) inflation.

- *Germany.* For the more recent data (from 1997 onwards), use is made of seasonally adjusted data for the Harmonised index of consumer prices published by the Eurostat. From January 1991 until February 2012 the source is the CPI all items index (non-seasonally adjusted) from the BIS macroeconomic Series Database. This series has been backtracked from January 1991 to January 1955 (using growth rates) with the CPI index (non-seasonally adjusted) for West Germany also published in the BIS Macroeconomic Series Database.[23]
- *France.* For the more recent data (from 1980 onwards), use is made of seasonally adjusted data for the Harmonised index of consumer prices published by the Eurostat. For earlier years, use is made of the CPI (non-seasonally adjusted) from the BIS Macroeconomic Series Database.
- *Japan.* For the more recent period (from 2006 onwards) we rely on seasonally adjusted data for the CPI published in the BIS Macroeconomic series database. Earlier data relates to the CPI index (non-seasonally adjusted) published in the International Financial Statistics of the IMF.
- *United States.* Use is made of the CPI data from the Bureau of Labour Statistics for the period 1947 to present (code CUSR0000SA0). Data from 1920 to 1947 is equally taken from the Bureau of Labor Statistics, but for this period use is made of the non-seasonally adjusted CPI data (code CUUR0000AA0).

Sovereign Bond Yield: Ten-year zero-coupon yield.

- *Germany.* Data from 1973 onwards, we use the 10-year zero-coupon yield published by the Bundesbank. Data prior to 1972 are projected backwards using as main regressors, the 10-year benchmark yield from Global Financial Data and the three-month Treasury Bill rate described above. The estimated coefficients were computed using the sample 1973-2018.
- *France.* Data from 1987 refer to the 10-year sovereign benchmark bond yield from BIS macroeconomic database. Earlier data relate to the 10-year benchmark yield published in Global Financial Data.

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[23] When data from original sources was not seasonally adjusted we use the X-12 ARIMA tool provided by the US Census Bureau for filtering the series for seasonality, see Findley et al. (1998) for technical details.

- *Japan.* We use the 10-year zero coupon yield from Bloomberg for the period 1999-2018. Data for the 1986-1998 are taken from the 10-year yield from the estimated yield curve published by the Ministry of Finance of Japan. Earlier data relate to the 10-year benchmark sovereign bond yield published by Bloomberg and Global Financial Data.
- *United States.* We employ the 10-year zero coupon bond yield series published by Gurkaynak et al. (2007) for the period 1961 to 2018. Data prior to 1961 is projected backwards using as main regressors, the 10-year benchmark yield from Blomberg and the three-month Treasury Bill rate. The estimated coefficients were computed using the sample 1961-2018.

Cash: Three-month Treasury Bill yield.

- *Germany.* Data from 1973 to 2018 relate to the zero coupon 6-month yield published by the Bundesbank. Prior to 1973 we use the Treasury Bill data from the Global Financial Database.
- *France.* Data from 1960 are 3-months Treasury bill data collected from Datastream; from 1952 to 1959 use is made of the daily interbank rate data from the BIS Macroeconomic Series Database.
- *Japan.* Use is made of the 3-month repo on bonds from the BIS for the period 1969 to 1979; from 1980 onwards until mid 2017, use is made of the 3-month Treasury bill reported in the International Financial Statistics of the IMF. For the more recent data, we use the one-year estimated yield from the Ministry of Finance in Japan.
- *United States.* Data from 1954 relate to the three-month Treasury bill yield collected from Datastream. From 1920 to 1953 data relate to the three-month Treasury Bill series published by the Board of Governors of the Federal Reserve System.

Equity: Equity return indexes from Datastream.

Commodities. We use the S&P GSCI (Goldman Sachs Commodity Index) Total return index from DataStream for data from 1969 onwards. Earlier observations are constructed by means of backtracking this series using the growth rate of the commodity futures price index from Thomson Reuters: CRB Index (TR/CC CRB). Dollar prices are transformed into domestic prices using the foreign exchange rate.

Gold. Use is made of the end of month PM Fixing price for gold in New York. Historical prices for the United States are taken from the Commodity Research Bureau, Commodity

Yearbook, Chicago: CRB since 1933. Dollar prices are transformed into domestic prices using the foreign exchange rate.

Foreign Exchange Rates. The Japanese yen exchange rate is taken from Thomsom Reuters Datastream from 1994 to present, and earlier data from Global Financial Data. For Germany and France we use euro/dollar exchange rate from 1999 onwards from Thomsom Reuters Datastream; earlier data relate to the French Franc and the Deutsche Mark series from Global Financial Data, but transformed into euros using the irrevocable conversion rates adopted by the EU Council on 31 December 1998.

Real Estate. Real estate returns are computed from the house price series reported by the BIS. These series are quarterly and non-seasonally adjusted. We once more employ the X-12 ARIMA tool for computing a seasonally adjusted quarterly series. This quarterly series is then mechanically converted into a monthly series by employing cubic splines.

B On the Monte Carlo Simulations

The model employed for the Monte Carlo simulations is:

$$\boldsymbol{x}_t = \boldsymbol{\delta}_t + \boldsymbol{B}_t \boldsymbol{x}_{t-1} + \boldsymbol{\varepsilon}_t \tag{B-1}$$

With $\boldsymbol{\varepsilon}_t$ a normally distributed error term with covariance matrix $\boldsymbol{\Omega}_t$.

Simulation for $\boldsymbol{B}_t$. We define $\operatorname{vec}(\boldsymbol{B}_t) = (0.99 b_{1,t}/\lambda, \cdots, 0.99 b_{kk,t}/\lambda)$, with $b_{i,t} = b_{i,t-1} + \nu_{i,t}$ and $\nu_{i,t} \sim N\left(0, n_{i,t}^2/t\right)$, with $n_{i,t} = n_{i,t-1} + \bar{\nu}_{i,t}$ and $\bar{\nu}_{i,t} \sim N(0, \sigma^2)$. And where λ is the largest, (among the $t = 1, \cdots, T$) absolute value of the eigenvalue of the simulated matrices $\mathcal{B}_t$ defined by $\operatorname{vec}(\mathcal{B}_t) = (b_{1,t}, \cdots, b_{kk,t})$. It follows from results in Lutkepohl (1996, sec. 5.2.1, result 13), that this formulation enforces all simulated matrices $\mathcal{B}_t$ to be non-explosive.

Simulation for $\boldsymbol{\Omega}_t$. We define $\boldsymbol{\Omega}_t = \operatorname{diag}\left(o_{1,t}^2/t, \cdots, o_{k,t}^2/t\right)$, with $o_{i,t} = o_{i,t-1} + s_{i,t}$ and $s_{i,t} \sim N(0, \sigma_e^2)$.

Simulation for $\boldsymbol{\delta}_t$. We define $\boldsymbol{\delta}_t = (d_{1,t}, \cdots, d_{k,t})$, with $d_{i,t} = d_{i,t-1} + \mu_{i,t}$ and $\mu_{i,t} \sim N\left(0, m_{i,t}^2/t\right)$, with . $m_{i,t} = m_{i,t-1} + \bar{\mu}_{i,t}$ and $\bar{\mu}_{i,t} \sim N(0, \sigma^2)$.

Table 1: Root Mean Square Error Relative to MVLK Method.

VAR dimension		Signal noise: $\sigma_s^2 = 1$ & $\sigma^2 = 3$			Signal Noise $\sigma_s^2 = 3$ & $\sigma^2 = 1$		
		KF	KK	LS	KF	KK	LS
5	#	98.7%	100%	32.6%	96.9%	99.9%	25.4%
	$\boldsymbol{A}_t$	0.69	0.94	1.00	0.79	1.00	1.00
	$\boldsymbol{\Omega}_t$	-	1.02	0.99	-	0.99	0.99
	$\tilde{\boldsymbol{y}}_{t+1}$	0.05	0.24	0.97	0.11	0.21	0.97
	$\tilde{\boldsymbol{y}}_{t+6}$	0.50	0.36	0.97	0.45	0.36	0.97
	$\tilde{\boldsymbol{y}}_{t+12}$	1.06	0.62	0.97	0.84	0.60	0.97
	$\tilde{\boldsymbol{y}}_{t+24}$	636.88	16.63	0.97	4.52e+03	5.13	0.97
	$\tilde{\boldsymbol{y}}_{t+60}$	1.08e+15	6.97e+11	0.98	8.71e+19	4.47e+10	0.98
10	#	87.7%	100%	46.9%	83.4%	100%	38.1%
	$\boldsymbol{A}_t$	0.57	0.88	1.00	0.65	0.94	1.00
	$\boldsymbol{\Omega}_t$	-	1.52	0.99	-	1.43	0.99
	$\tilde{\boldsymbol{y}}_{t+1}$	0.01	0.10	0.98	0.03	0.08	0.98
	$\tilde{\boldsymbol{y}}_{t+6}$	0.46	0.62	0.98	0.50	0.66	0.98
	$\tilde{\boldsymbol{y}}_{t+12}$	0.61	1.56	0.98	0.73	1.12	0.98
	$\tilde{\boldsymbol{y}}_{t+24}$	4.77	8.71e+04	0.98	114.30	42.90	0.98
	$\tilde{\boldsymbol{y}}_{t+60}$	1.76e+07	7.27e+21	0.98	5.03e+11	4.68e+12	0.98

NOTE: Values reported under # relate to the percentage of instances when a explosive VAR was estimated, all remaining values are relative root mean square errors (RMSE), and these are relative to the errors made by the MVLK estimation method which is used as the benchmark. RMSE reported for $\boldsymbol{A}_t$ relate to the error made in the estimation of the time varying coefficient matrices, those reported for $\boldsymbol{\Omega}_t$ relate to the estimates of the covariance matrix of the error term; those reported for $\tilde{\boldsymbol{y}}_{t+p}$ relate to the forecast errors made at the $t+p$ horizon.

Table 2: Cost of Hedging Inflation, risk-free rate and inflation (Mean).

	period	Expected				Realised			
		DE	FR	JP	US	DE	FR	JP	US
r	1920-2009	-	-	-	1.83	-	-	-	3.59
	1950-2009	3.17	2.46	0.86	1.74	4.50	4.98	3.53	4.47
	1920-2018	-	-	-	1.70	-	-	-	-
	1950-2018	2.70	2.13	0.71	1.56	-	-	-	-
	2010-2018	-0.22	-0.03	-0.14	0.34	-	-	-	-
$\bar{\pi}$	1920-2009	-	-	-	2.76	-	-	-	2.95
	1950-2009	2.70	4.47	3.36	3.69	2.63	4.41	3.08	3.65
	1920-2018	-	-	-	2.66	-	-	-	-
	1950-2018	2.51	4.02	2.93	3.43	-	-	-	-
	2010-2018	1.29	1.26	0.35	1.71	-	-	-	-
CHI	1920-2009	-	-	-	0.59	-	-	-	-1.33
	1950-2009	0.85	0.48	1.76	0.76	-0.35	-2.04	-0.55	-1.89
	1920-2018	-	-	-	0.58	-	-	-	-
	1950-2018	0.77	0.49	1.57	0.72	-	-	-	-
	2010-2018	0.22	0.52	0.46	0.46	-	-	-	-

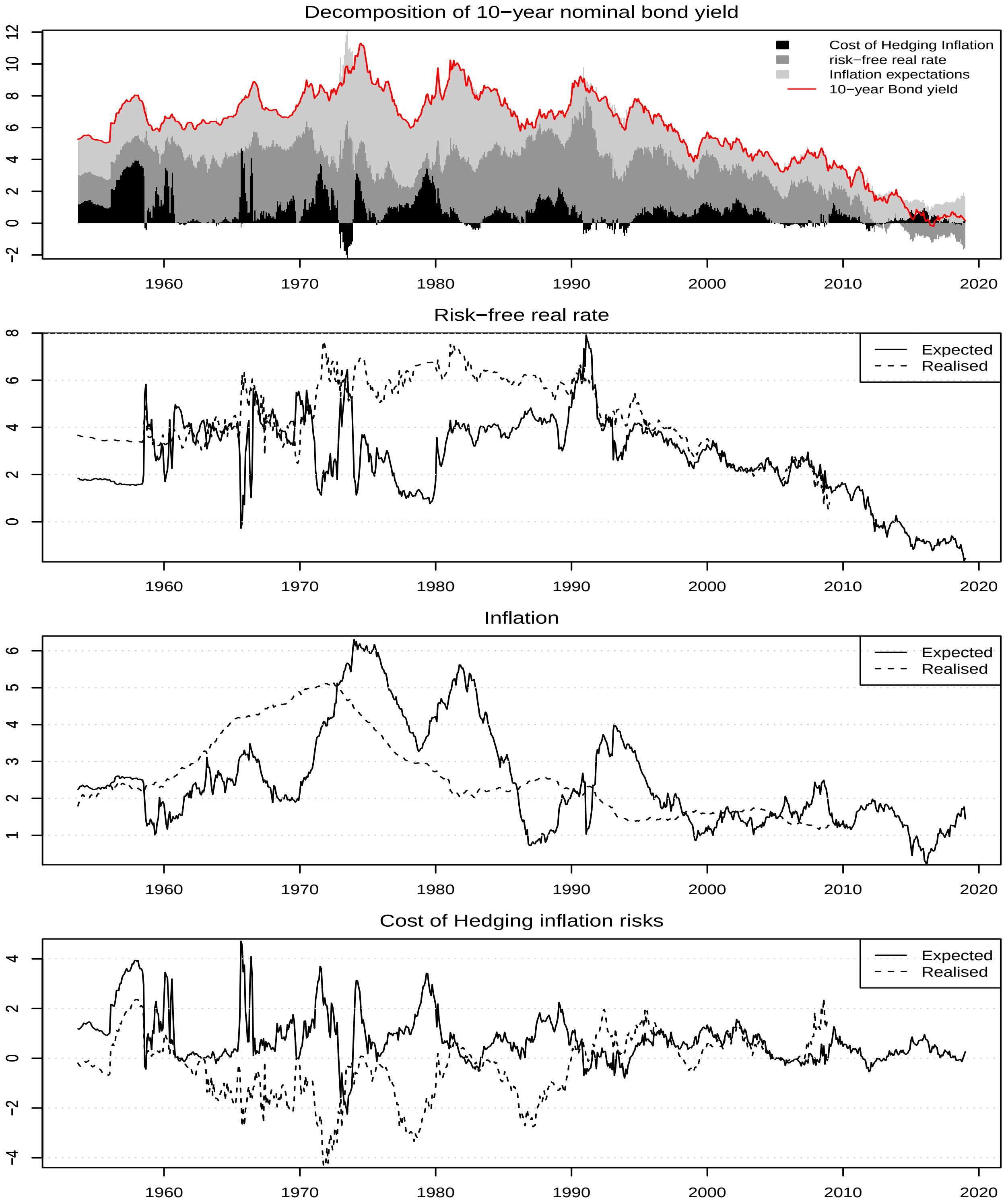
NOTE: We employ r, $\bar{\pi}$ and CHI to refer to the risk-free real rate, expected inflation and the cost of hedging inflation respectively. Values reported are the mean over the selected period. Realised returns can only be computed for investments which were completed 10-years before the end of our sample period, that is December 2009. For the United States the sample goes back to 1920, but for comparing estimates with other countries, sample periods starting in 1950 are also reported. Values reported are in percent.

Table 3: Real risk-free rate and CHI (Median).

		r				CHI			
Economic Regime		DE	FR	JP	US	DE	FR	JP	US
Low Inflation	recession	-0.67	-0.63	0.64	4.04	0.69	0.83	1.04	0.41
	expansion	-0.77	3.86	0.74	4.00	0.74	0.44	0.89	0.12
	all	-0.72	2.81	0.69	4.02	0.72	0.46	0.89	0.16
High Inflation	recession	3.51	1.13	-0.08	1.23	0.63	0.60	2.45	0.54
	expansion	2.66	1.30	0.14	-0.81	0.62	0.48	1.80	0.80
	all	3.02	1.16	0.10	0.18	0.63	0.54	2.05	0.77

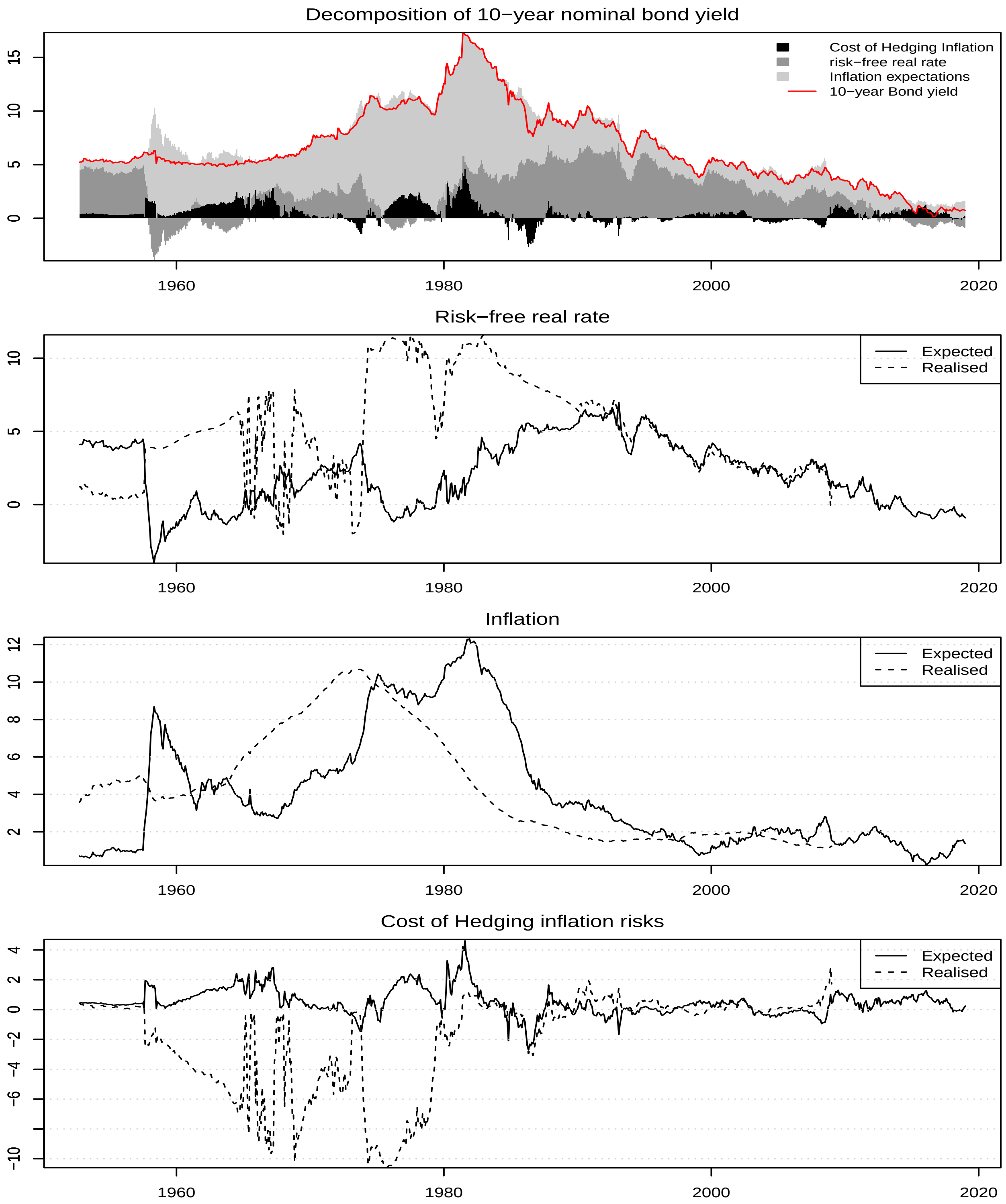
Note: We employ r and CHI to refer to the real risk-free rate and the cost of hedging inflation respectively. Values reported are the median over the selected period. For the United States the sample goes back to 1920. Recessionary periods for the United States refer to those identified by the NBER dating committee; for Germany, France and Japan we take those recessionary periods identified by the OECD. Low inflation periods refer to periods where inflation expectations over the 10 year horizon were below 1% (annualised), while periods of High inflation relate to periods when inflation expectations over the 10 year horizon were above 1%. *all* makes no distinction on whether the economy is on recession or expansion.

Figure 1: Yield, Expected Real Return and Cost of Hedging Inflation (Germany).



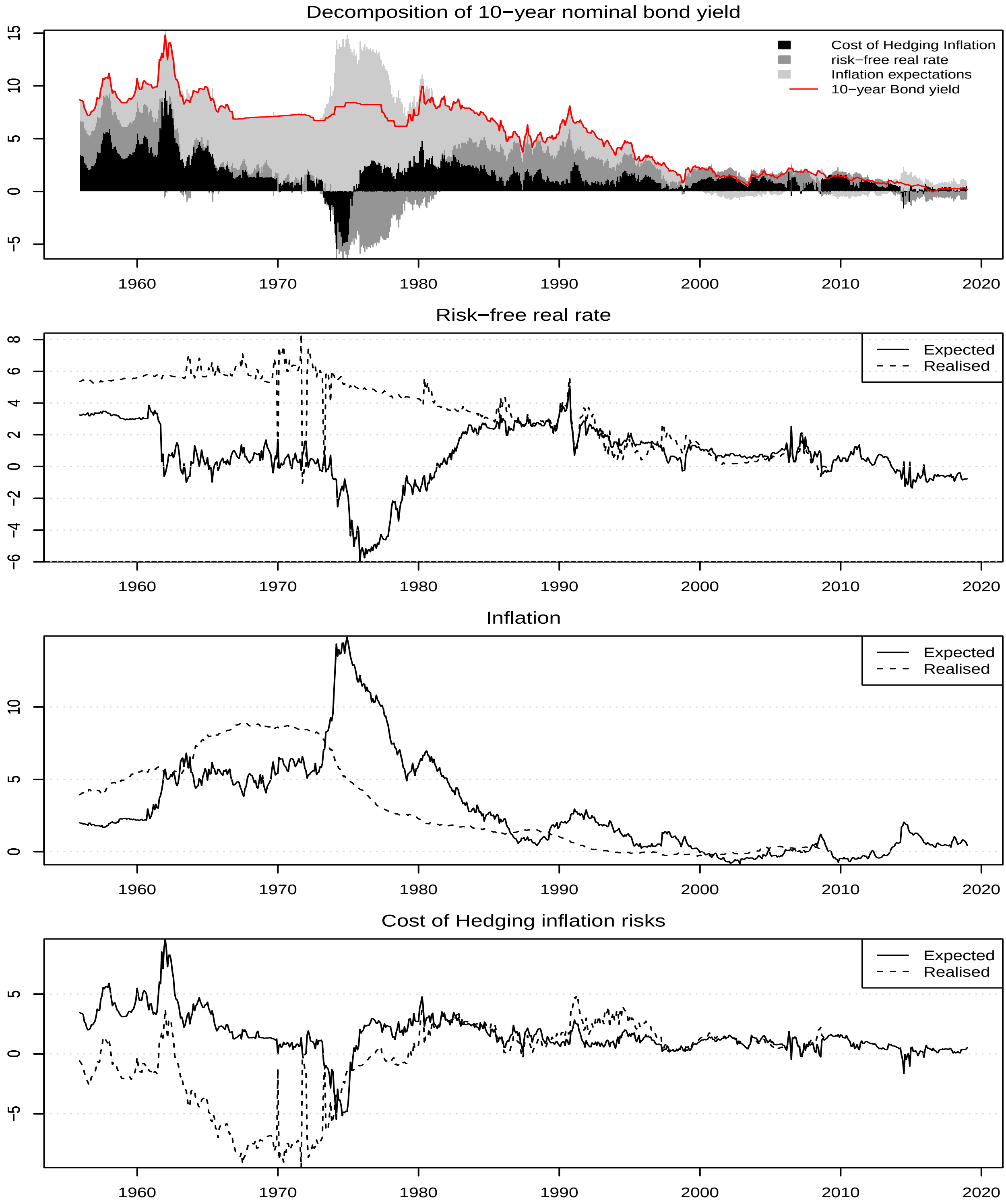
Note: Monthly data. The last observation is December 2018 for all series but for 'realised' series. For the latter the last reported observation is December 2008. This means that the series of December 2008 shows both the expected value at that point in time, together with the realised value for that portfolio executed in December 2008 ten years later, i.e. December 2018.

Figure 2: Yield, Expected Real Return and Cost of Hedging Inflation (France).



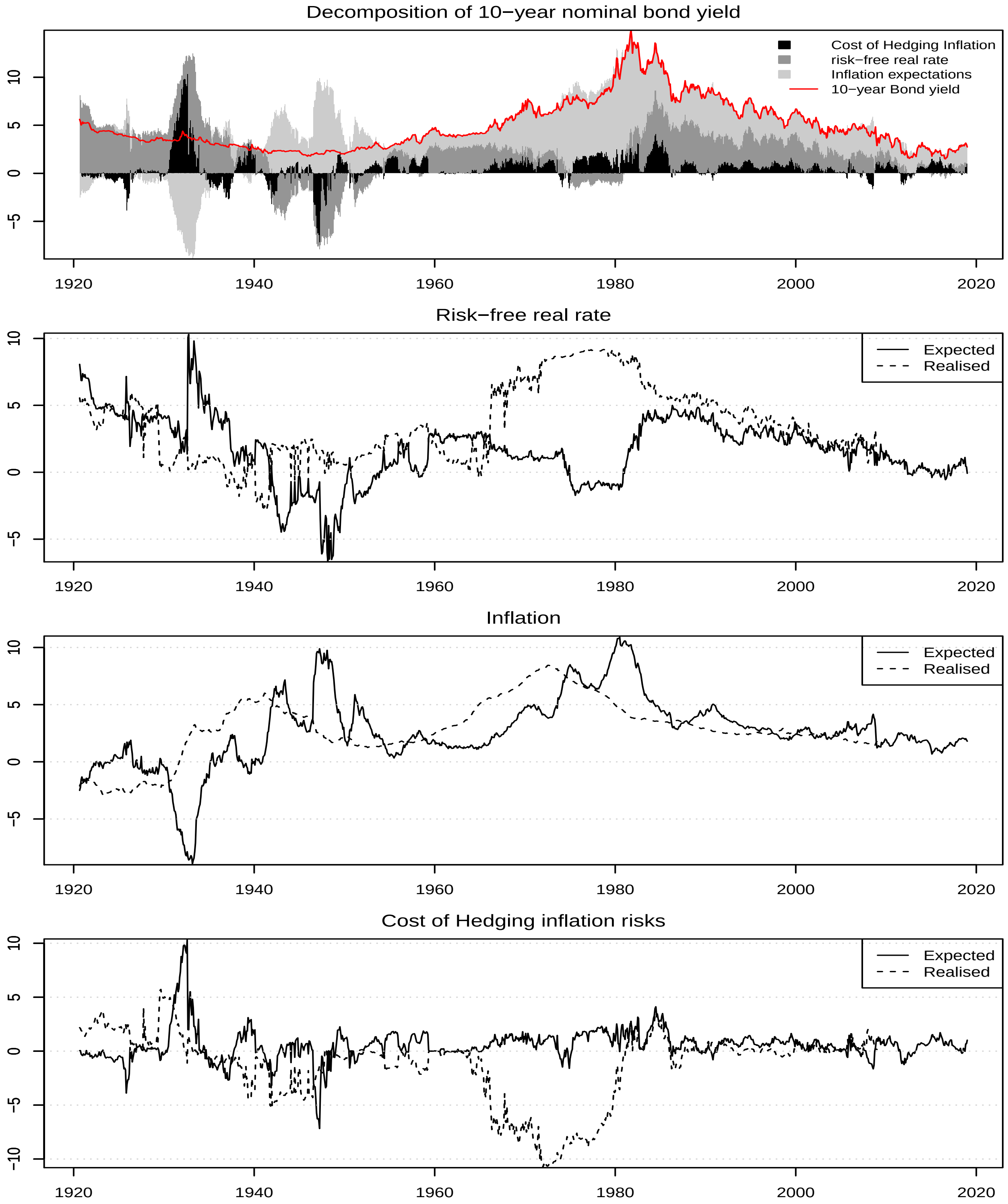
NOTE: Monthly data. The last observation is December 2018 for all series but for 'realised' series. For the latter the last reported observation is December 2008. This means that the series of December 2008 shows both the expected value at that point in time, together with the realised value for that portfolio executed in December 2008 ten years later, i.e. December 2018.

Figure 3: Yield, Expected Real Return and Cost of Hedging Inflation (Japan).



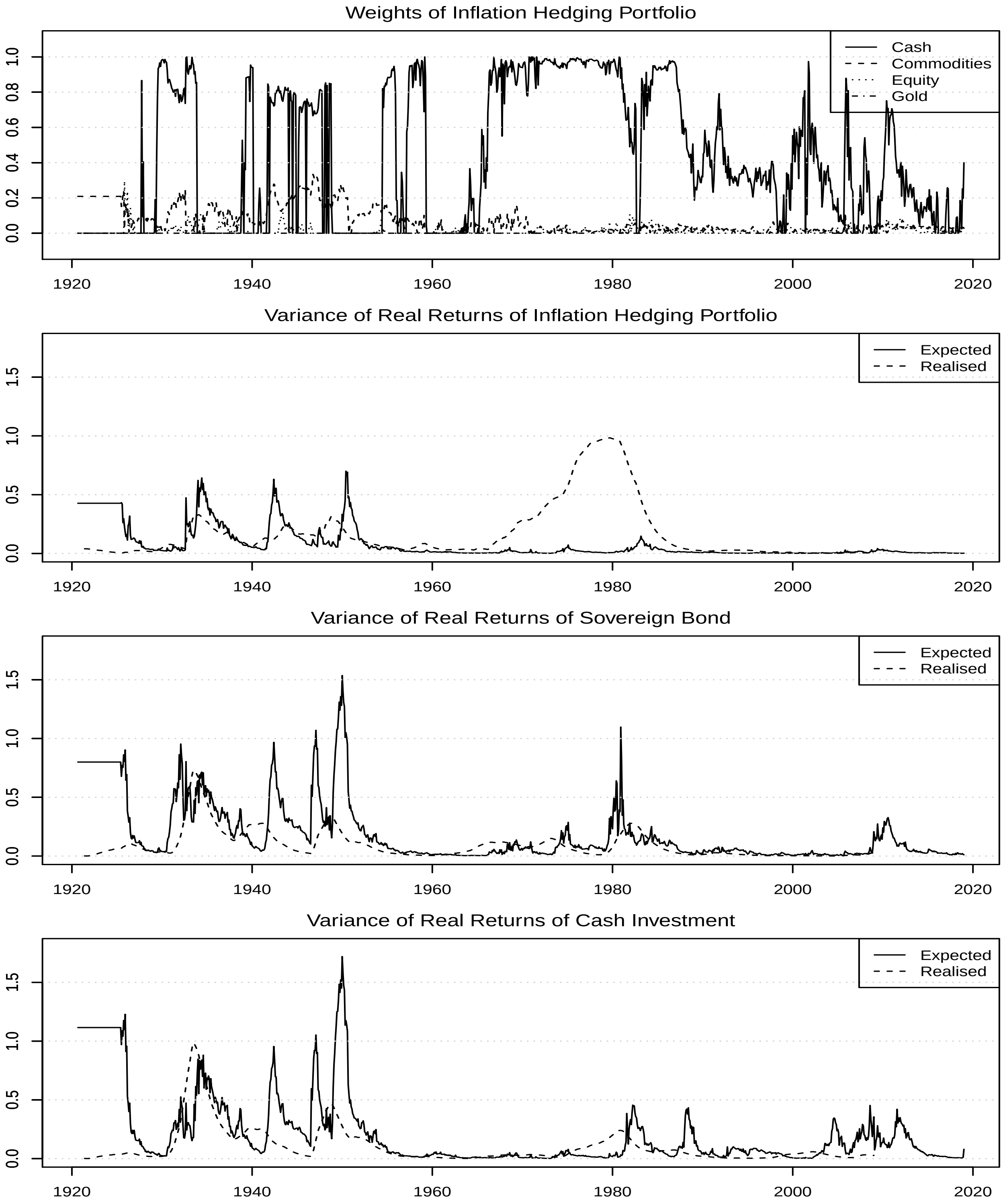
NOTE: Monthly data. The last observation is December 2018 for all series but for 'realised' series. For the latter the last reported observation is December 2008. This means that the series of December 2008 shows both the expected value at that point in time, together with the realised value for that portfolio executed in December 2008 ten years later, i.e. December 2018.

Figure 4: Yield, Expected Real Return and Cost of Hedging Inflation (United States).



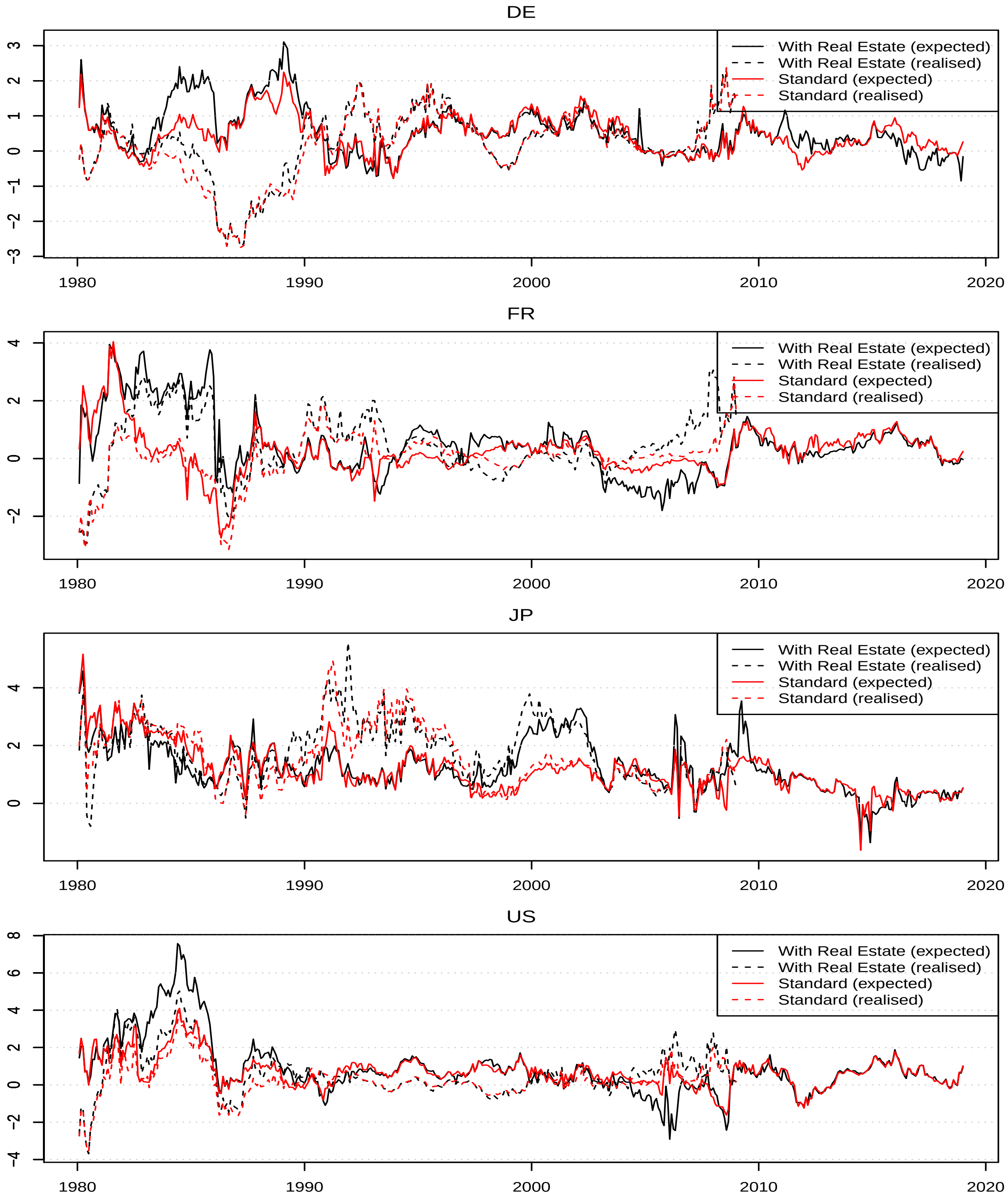
Note: Monthly data. The last observation is December 2018 for all series but for ‘realised’ series. For the latter the last reported observation is December 2008. This means that the series of December 2008 shows both the expected value at that point in time, together with the realised value for that portfolio executed in December 2008 ten years later, i.e. December 2018.

Figure 5: Weights and Volatility of Inflation Hedging Portfolio (United States).



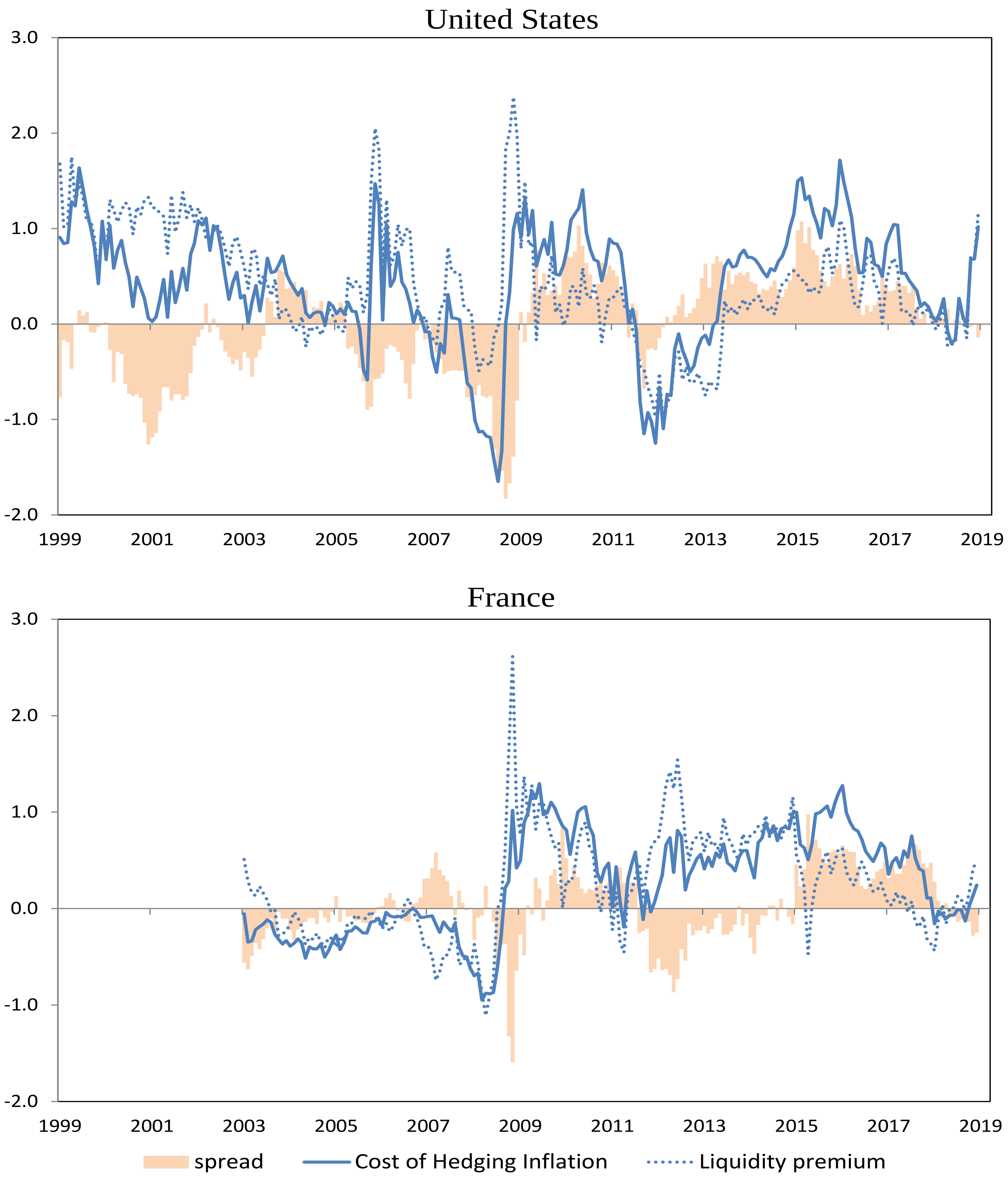
NOTE: Monthly data. For simplicity, the weight of the sovereign bond asset in the portfolio is not shown. It should be understood that the difference between one and the sum of the weights displayed in the figure is the weight of the sovereign bond in the portfolio. The last observation is December 2018 for all series but for 'realised' series. For the latter the last reported observation is December 2008. This means that the series of December 2008 shows both the expected value at that point in time, together with the realised value for that portfolio executed in December 2008 ten years later, i.e. December 2018.

Figure 6: CHI of Standard Portfolio and Extended Portfolio with Real Estate.



NOTE: Monthly data. Values show refer to the cost of hedging inflation risks (CHI). The last observation is December 2018 for all series but for 'realised' series. For the latter the last reported observation is December 2008. This means that the series of December 2008 shows both the expected value at that point in time, together with the realised value for that portfolio executed in December 2008 ten years later, i.e. December 2018.

Figure 7: Cost of Hedging Inflation and Liquidity Premium of linkers.



NOTE: Monthly data. The liquidity premium is computed from data on BEIR as $LP = \bar{\pi} + CHI - BEIR$, where CHI and $\bar{\pi}$ are the cost of hedging inflation and the inflation expectations as defined in the main text.

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