EUROPEAN CENTRAL BANK
EUROSYSTEM

# Working Paper Series

Grzegorz Hałaj

# Agent-based model of system-wide implications of funding risk

No 2121 / January 2018

ABSTRACT

Liquidity has its systemic aspect that is frequently neglected in research and risk management applications. We build a model that focuses on systemic aspects of liquidity and its links with solvency conditions accounting for pertinent interactions between market participants in an agent-based modelling fashion. The model is confronted with data from the 2014 EU stress test covering all the major banking groups in the EU. The potential amplification role of asset managers is taken into account in a stylised fashion. In particular, we investigate the importance of the channels through which the funding shock to financial institutions can spread across the financial system.

**Keywords**: liquidity, systemic risk, ABM

**JEL Classification**: G11, G21, C61

## Non-technical summary

Liquidity risk is a systemic concept and should be treated in this way. Illiquidity unwinds abruptly and turbulently because of discrete time and immediate calls for cash transfers related to contractual obligations between creditors and debtors and behaviors of the market participants that need to raise cash or strategically try to assure payments in the near future. Agents of various types can act differently given their business models, objectives and regulatory constraints. However, in the stressful market conditions herding behaviours of the agents are likely to amplify the original shocks. The complexity of how the liquidity situation develops and how unpredictably liquidity can dry out renders the Agent-based Modeling (ABM) a legitimate approach to capture the interactions between financial market participants' balance sheets. We follow this ABM avenue.

Liquidity is coupled with solvency, i.e. agents' ability to absorb some significant losses. Financial institutions under solvency stress may have difficulties in paying back their due obligations or may experience elevated funding costs related to creditors' weakening trust that the banks will continue their operations. Moreover, a deterioration in liquidity may imply more frequent and higher liquidation haircuts (e.g. a more severe revaluation of the liquidated assets). In turn, it has an impact on the profit and loss accounts and consequently on the solvency figures. The model we propose takes into account the feedback effects between liquidity and solvency via interactions of agents which are subject to regulatory constraints.

We use the model to build a system-wide stress test tool to assess how a funding shock to financial institutions of two types: banks or asset managers can propagate across the financial system and be amplified via agents' behaviors and their interconnectedness. First, the framework can help to identify the key contagion channels or vulnerable nodes (i.e. those agents that can trigger wide-spread contagion losses). Second, the model can be used to understand effectiveness of policy measures targeting liquidity of solvency conditions.

The model is calibrated to the data collected during the stress test of the 2014 Comprehensive Assessment exercise, covering detailed balance sheet breakdowns and their parameters as well as the P&L items of 130 largest banking groups in the European Union. The balance sheets of Asset Managers (AM) are not calibrated but parameterised in a stylised way; the role of AMs in the model is to study amplification of the initial funding shock via the fire sale channel.

We find that there is a heterogeneity in the importance of the contagion drivers and channels. Specifically, the fire sales and the relationship between solvency and funding costs, as well as the relative size of the asset managers segment of the financial system amplify the initial funding shocks. We show robustness of the mechanics of the model with a sensitivity analysis of the contagion losses to changes of the key parameters and we validate the model applying panel regression techniques. We illustrate the policy-relevance of the framework analysing the effectiveness of some liquidity-based regulatory instruments (LCR limits and the utilisation of most liquid assets under stress).

# 1 Introduction

Liquidity is the ability of an institution to generate cash from its assets to timely meet its obligations. Solvency is such a structure of the balance sheet that the future losses are covered with a high likelihood by the capital of the institution. Liquidity and solvency are usually treated separately. For instance, stress testing activities are split into solvency stress testing (e.g. ECB stress testing[1]) and liquidity stress tests (e.g. Bank (2015)). But liquidity and solvency conditions are closely linked and influence each other by behaviors of market participants, especially under stress. We explore this relationship in our model.

Our model is motivated by the fact that the main measures to understand and control liquidity and funding of banks lack a systemic perspective. Liquidity Coverage Ratio (LCR) and Net Stable Funding Ratio (NSFR) are the instruments to control adequacy of the cash generation potential of the assets to cover short term liquidity outflows and the appropriateness of the balance sheet structure to sustainedly finance the investment strategy. Difficulties are in the implementation: banks may not have enough room to build up the cushion. Consequently, a severe funding shock may be amplified by banks and other market participants trying to tap liquidity from a drying up market. According to a study released by BIS Committee on the Global Financial System (CGFS, 2013), the additional global demand for high quality liquid assets needed to meet the requirement of a sufficiently high LCR may amount to 1.8 trillion USD raising concerns about possible collateral shortages. Moreover, the increase of the liquidity buffers may have macroeconomic side effects of a reduced loan supply. Liquid instruments do not necessarily stimulate financing of the long-term projects in the economy. The complexity of market connections makes it more difficult to unravel the channels of potential contagion spreading. Behaviors and network topology matter as shown in sociology (Jackson and Wolinsky, 1996), epidemiology (Bansal et al., 2007), transportation (de Oliveira and Camponogara, 2010) or economics (Gatti et al., 2006).

In reality, a strong relationship between solvency and liquidity can be observed. One of the most prominent examples was the outburst of the 2007-2008 crises that was mainly driven by liquidity issues but translated into bankruptcies of some largest market players in the market (e.g. Lehman or AIG). There is also a reverse relationship whereby solvency risk translates into funding risk. Poor capitalisation of banks was reflected in the funding cost spreads, aggravated by solvency risk of some sovereigns. This happened in 2010 as the second phase of the recent financial crisis.

Liquidity is in the scope of the macroprudential measures, even though not in the first line of instruments which focus directly on the provision of credit to the economy. However, in the contractionary phase of an economic cycle, as specified by (Claessens et al., 2013), LCR and NFSR limits can be effective but may not be sufficient. As Vítor Constâncio (the vice-president of the European Central Bank) claimed[2] the stress tests – one of the approaches to the macroprudential policy assessment – should embrace "a macro liquidity stress test in the solvency stress testing framework".

The model of the liquidity and solvency interactions includes several parameters that exactly correspond to the known macroprudential instruments. First, it is a requirement of an additional capital buffer from the systemically important institutions. In our context, they can be perceived as those institutions providing liquidity or transmitting, or amplifying liquidity shocks. The capital level expressed in terms of the risk-weighted assets serves as the standardised buffer for the P&L stresses originated to the liquidity or funding situation of banks. Second, these are the limits on the Basel III regulatory liquidity requirements that are potentiometers for the liquidity conditions imposed on banks: LCR and NFSR. The baseline liquidity requirement assumes the one-to-one coverage of the liquidity outflows with the high quality assets. But the limit can be tuned according to the macroprudential needs. By varying the limits in the model it can be assessed how efficient the limits are in dampening the initial shocks and deactivating the contagion channels of the coupled liquidity and solvency conditions.

We contribute to the research on liquidity and solvency in the following way. First, we develop a

[1] https://www.bankingsupervision.europa.eu/banking/comprehensive/html/index.en.html
[2] See speech: https://www.ecb.europa.eu/press/key/date/2015/html/sp151029.en.html

framework of a consistent treatment of coupled liquidity and solvency conditions of banks. We call our ABM a 6-step model. That is because the main features of the framework are: (A) utilisation of direct liquidity buffers; (B) presence of the interbank funding channel; (C) ability to capture the amplification effects of funding shocks via fire sales; (D) relationship between funding cost of the rolled-over debt and changes in solvency ratios; (E) capturing the information (panic) contagion; (F) unwinding of exposures related to direct lending and cross-holding of debt in case of the solvency defaults. As far as the applications are concerned, the models is first used in the field of stress-testing of the financial system as a whole. The recent acceleration in the development of these models is fostered by the financial overseers and regulators.[3] For instance, Hałaj (2013b) mentions the need to capture complex interactions of sophisticated agents. Notably, ABMs brought attention of an expert group created by the Bank of International Settlements studying linkages between liquidity and solvency in stress testing (Anand et al., 2011). Second, we analyse the efficiency of two policy instruments to mitigate the systemic risk. These are capital adequacy and Liquidity Coverage Ratio. Third, the framework is a novel approach to a system-wide stress testing of banks and asset managers subject to some extreme funding conditions. System-wide stress testing exercises have been recently recognised (see e.g. Hałaj and Laliotis (2017) or Turrell (2016)) as one of the most important instruments to understand adverse conditions of the financial system, given its global nature and strong interconnectedness. In fact, Hałaj and Laliotis (2017) show one way the proposed ABM framework can be integrated into the macroprudential liquidity stress tests.

Agent-based models proved to be effective in replicating properties of real economic systems which emerge from interaction between heterogenous agents. The key feature of the agent-based models is that complex properties of the modeled system emerge from the interactions of agents, usually defined with simple rules. Lux (2015) was able to reconstruct a core-periphery structure of the interbank lending network from the liquidity perturbations that force banks to seek liquidity from other banks. Liu et al. (2017) are capable of reproducing features of interbank market dynamics around Lehman default event. Core periphery structure is an emergent property, not an imposed one, observed in a steady state following the skewness in the distribution of banks' total assets which is the only source of heterogeneity imposed at the outset of the horizon of the simulation. Bookstaber and Paddrik (2015) analyse the interactions of market makers, liquidity suppliers and demanders to capture the price formation on the market in an order book setup and to model liquidity dynamics. Giansante et al. (2012) study interactions between liquidity and solvency conditions for economic agents that determine the bilateral flows based on the assessment of counterparty solvency and liquidity scoring index. The main differences of our approach are: more detailed treatment of the balance sheet structures, price formation, individual liquidity characteristics of balance sheet items and spill-over of funding problems with peer groups of banks with similar business models. Klimek et al. (2015) deal with the efficiency of the bank resolution mechanisms, confirming the intuition that a bail-in mechanism may perform better than other closer to the bail-out concept. There are also some more macro-economic applications of the ABM frameworks departing from the constraining DSGE modelling paradigm. For instance, Dosi et al. (2010) study influence of the technological advances on the macroeconomic variables and tries to capture the essence of the micro-behaviours of agents, departing for the representative agent approach taken in the DSGE models. Baptista et al. (2016) show the application of ABMs in a macroprudential policy context to evaluate the effectiveness of the Loan-to-Income ratio instrument. As Farmer and Foley (2009) point out, agent-based models can be powerful enough to foster economic recovery from 2008 financial crisis. Last but not least on our list, Calimani et al. (2017) study the fire sales mechanism stemming from interactions between banks and shadow-banks.

Our approach tries to fulfill a postulate of Demekas (2015) to embed elements of ABMs into the macroprudential stress tests. The ABMs allow for *emergence of outcomes that could not have been predicted based on the past behavior of individual agents in the financial system.* The model is similar in its goal and validation steps to Bookstaber et al. (2017). Specifically, both models focus on integrating behavioral interactions of key players of the financial market to capture shock

[3] Some research papers are also supportive of solvency and liquidity interactions, e.g. Gai et al. (2011); Arinaminpathy et al. (2012); Hurd et al. (2014).

amplification mechanism under extreme stress to render stress testing tools more useful in measuring and explaining the systemic consequences of adverse financial shocks and to contribute to a the macroprudential toolbox of models. However, Bookstaber et al. (2017) is more granular in types of agents that play different roles in providing cash and collateral and in consuming both liquidity resources, whereas our model considers much more detailed balance sheets calibrated to real data from stress test reporting. In term of validation, both papers have similar attitude: they try to confirm that sensitivity of the models are consistent with financial theory or empirical evidence documented in other research studies.

The challenges of the ABM approach needs to be recalled. As emphasized by Doyne Farmer in his speech at the ECB conference in 2010, ABMs are capable of reproducing stylised facts but may easily get intractable if they attempt to reflect the whole complexity of the financial system. Moreover, they should carefully consider inclusion certain behavioural rules given the purpose they need to serve.[4] They also require a high-quality and granular set of input data which is not necessarily an easy objective to achieve. The usually insufficient datasets hamper the calibration of the models and lead to inaccurate projections of the system dynamics.[5] A success story in Farmer's view would be to outperform the predictive models developed in the DSGE philosophy. This however requires to have a sound validation strategy which would be determined by data availability and would be constrained by the available computational power. Design and implementation of such a validation process is usually more feasible in ABMs of macroeconomic dynamics and for financial markets in case the emphasis of a model is on the prices and not quantities nor rare default events as in our case.

The rest of the paper is organised as follows: section 2 presents details of the theoretical model; section 4 presents a validation of the model based on randomly simulated systems and as well as for the observed structure of the banking system calibrated with the real banking data; section 5 shows results of simulations of contagion propagation; section 6 discussed applications of the ABM framework for policy analysis and section 7 concludes.

# 2 Model

## 2.1 Overview and motivation

The model is composed of a chain of six steps that are played sequentially in a system of banks and asset managers after a funding shock materialises. The funding shock is understood as the outflow of certain classes of funding or as redemptions experienced by the asset managers. Practically, it can be related to a materialisation of roll-over risk. The chains of the 6 steps can be played sequentially, i.e. after the last step (F) of a given chain is completed the game enters the step (A) assuming all the effects of the proceeding chain are accumulated. Notably, we specify two chains at least to account for the feedbacks from behavioral redemptions to the banking system via the revaluation of the Marked-to-Market (MtM) portfolios. We try to justify the behavioural rules bringing some empirical findings from the literature, in particular about the mechanism of the recent financial crisis.

(0) [For the first chain] 'Step 0' defines the initial shock structure; either to the funding sources of banks (in practice to retail or wholesale deposits, asset-backed instruments, etc.) or to asset managers participations (in practice after redemptions by the participants). This may trigger subsequent steps (A) to (F).
[For the subsequent chains] Significantly large revaluations of the AMs' assets following steps (A) to (F) of the proceeding chains can lead to forced, behavioral redemptions of investors. We assume a certain revaluation threshold above which AMs would experience additional

[4] E.g. in the short term horizon of the presented model profit maximisation based balance sheet optimisation may be ill-suited (Hałaj, 2013a, 2016) contrary to models having a longer term, solvency and ALM perspective. Traditionally, ABMs assume some form of bounded rationality that may be suboptimal (Lux and Zwinkels, 2017).

[5] See also opinions of BlackRock submitted to FSB (http://www.fsb.org/wp-content/uploads/BlackRock1.pdf).

redemptions; this would be treated as additional initial funding shock potentially further deteriorating financial conditions of the agents in the financial market. The fire-sale risk related to the redemptions experienced by the asset managers has already been a concern expressed in the finance literature (Morris et al., 2017).

(A) For shocks hitting the banking system side, banks verify whether they have enough good quality, eligible collateral to engage into the repurchase agreements. Should the redemptions affect the asset managers cash is assumed to first be used to meet the outflows. If banks hold enough eligible collateral and the AMs enough cash then no further steps follow and the shock is contained within the high quality counterbalancing capacity. Bindseil (2013) studied the availability of the eligible collateral in the case of the funding shocks showing its importance as the first buffer and its heterogeneity in terms of effective capability to generate cash (haircuts).

(B) In case the eligible asset buffer is insufficient for a given bank, it resorts in the second step to the interbank assets. The bank cuts the additional short-term interbank lending in an attempt to cover the remaining gap from the first step. Consequently, banks that funded themselves by deposits from that bank need to search for alternative sources. We assume that this induces additional funding spread related to the search cost and the fact that potentially a new funding relationship needs to be established. The spread impacts banks' P&L and capital. Notably, the topology of a network of interbank exposures matters for the directions of how the contagion spreads across the system. We are able to justify this behavioral assumption based on Cappolletti and Mistrulli (2017) who provide with some empirical evidence that a liquidity distress in the banking system is costly for other banks financing their operations through interbank borrowing.

(C) The third, a fire sale step is triggered if the verification fails for banks and there was not enough capacity in their interbank portfolios to generate liquidity covering the initial funding shock. Banks liquidate less liquid assets. This has an impact on their valuation and would impacts banks capital for those recognised at the mark-to-market in banks' balance sheet. Analogously, if a given AM cannot cover the redemptions using cash it resorts to liquidation of some of the assets. Notable, there is a system-wide effect of the liquidation meaning that the price of the assets is a function of the total volume disposed by banks on the market.

    The fire sale story is not an obvious one. Although this mechanism is used in many models of financial system[6] sometimes it is questionable, as argued by why banks would engage in a costly liquidation of assets before other sources of liquidity are utilised. However, there are studies that support a statement that fire sales significantly amplify losses incurred by financial firms during a market wide stress analysing price formation and characteristics (Mitchell and Pulvino, 2012; Helwege and Zhang, 2016). Nevertheless, it is a more frequently observed phenomena and a concern from regulators in case of asset managers (Chernenko and Sunderam, 2004).

(D) Losses accumulated in steps (A)-(C) impact the capital ratio (CAR) of banks. In case the drop in CAR is significant, banks may experience an increase in funding spreads of the wholesale funding to be rolled over since their actual or perceived solvency conditions may deteriorate. We account for this mechanism in the fourth step. The significant relationship between the solvency and funding costs was econometrically confirmed by Babihuga and Spaltro (2014); Schmitz et al. (2017) (for an international sample of banks), Aymanns et al. (2016) (for the US market) and Korsgaard (2017). Interestingly, they point out to some nonlinearities of the relationship with respect to the magnitude of the solvency shock and level of solvency (i.e. wholesale funding cost of less capitalised banks is more sensitive than of the banks with better solvency position). Nevertheless, the studies are ambiguous about the strength of the relationship: roughly ranging from 4 bps to 105 bps for a 100 bp decline of the capital ratio.

[6] see e.g Merrill et al. (2014)

(E) Not only banks that have experienced a significant drop in their capital ratios may be subject to the elevated funding spreads but banks that are similar in their business models to the directly affected banks as well. We model a potential indirect contagion effects in this step. Banks-peers are assumed to pay additional spreads on their maturing wholesale funding. It is difficult to find a direct proof of such a relationship in literature. Nevertheless, some empirical research was able to capture the relationship. Notably, Roengpitya et al. (2014) observed for 2008 a wide-spread increase of the cost-to-income ratio in the peer group of banks directly impaired by the crisis with large trading desks due to persistently high cost base. Bonaldi et al. (2015) demonstrates that stress to funding spreads of large, more central banks will likely be reflected to their peers' funding costs. Pierret (2015) shows the liquidity-solvency nexus that makes the funding cost particularly sensitive to banks' capitalisation under a general market stress, i.e. other banks' impaired solvency conditions exert additional pressure on a given bank's wholesale funding spread.

(F) In the last step we gauge some longer term effects of the initial funding shock with a capital footprint. All losses aggregated from steps (A)-(E) undermine banks solvency. In some cases this may lead to defaults once the capital falls below a required minimum. Consequently, cross-bank exposures are resolved and contagion spreads via interbank lending and cross-holding of bank debt securities. The contagion stemming from banks' defaults has been studied in details e.g. Helwege and Zhang (2016) as event studies and Elsinger et al. (2006) in a stress testing context. We would like to emphasise Helwege and Zhang (2016) to be very comprehensive in terms of micro-level data collection and a successful attempt to separate direct (counterparty) and indirect (information-based) contagion channels.

The technical details of the six steps are explained in the following subsection 2.2.

## 2.2 Details

There are two groups of agents that operate in the modelled financial system: a set of banks (set $\mathcal{B}$ with $K^b$ banks) and a set of asset managers (set $\mathcal{A}$ with $K^a$). Agents have a specific composition of their balance sheets. Balance sheet of a bank $b$ is composed of some $N$ generic classes of assets $(a_1^b, \ldots, a_N^b)$, $M$ liabilities $(l_1^b, \ldots, l_M^b)$ and capital $e^b$ and the balance sheet sum identity holds:

$$\sum_{n=1}^{N} a_n^b = \sum_{m=1}^{M} l_m^b + e^b$$

Similarly, balance sheet of an AM is composed of the same assets which for AM $a$ are denoted $(a_1^a, \ldots, a_N^a)$ and funding is assumed to exclusively be obtained via participation $e^a$

$$\sum_{n=1}^{N} a_n^a = e^a$$

We simplify substantially the funding side of the AMs since usually their funding is quite homogenous (no seniority, no guarantees, unlike in case of banks). The ample link between banks and AMs is established via common exposures to the same asset classes. In fact, the assumption about the same asset classes in banks' and AMs' portfolios is just a technical one; for instance AMs would not hold loans in their assets which means that for categories $n \in N$ $a_n^a = 0$. More importantly, banks and asset managers would likely invest in some common categories of securities (bonds, equities, commodities, etc.). The resulting overlapping portfolios of the two types of agents are crucial in the fire sales mechanism to spread losses through marked-to-market revaluation of assets.

To keep track of the changes to balance sheet composition of agents as the 6-step contagion unwinds we denote the initial structure of the assets by $a^{b,0}$ and $a^{a,0}$ for banks and asset managers respectively.

The following parameters characterise the asset and liability classes (let's denote $\bar{N} = \{1, \dots, N\}$ and $\bar{M} = \{1, \dots, M\}$). In the applications, the parameters are asset-specific, rather than bank-specific.

1. liquidity horizon $T$: usually it would be a short period of several days, for instance 30 days
2. haircut $h_n$: a haircut on the book value of the securities in case of liquidation (usually close to 100% for loans reflecting their high degree of illiquidity and significantly higher than 0 for corporate bonds or asset-backed securities).
3. eligibility $\mathcal{E} \subset \bar{N}$ and cash (or cash equivalent) holdings of AMs $\mathcal{C} \subset \bar{N}$: a set of asset classes that are eligible in central bank operations (mostly high quality securities, therefore implicitly excluding e.g. loans to customers and financial corporations).
4. non-eligible ($\mathcal{NE}$) and non-cash ($\mathcal{NC}$) assets: the scope of the assets under fire sales is different for banks and AMs. Banks are assumed to liquidate non-eligible assets $\mathcal{NE} := \bar{N} - \mathcal{E}$. Asset managers liquidate non-cash assets $\mathcal{NC} \subset \bar{N}$.
5. marked-to-market (MtM) assets of banks $\mathcal{M}$: a subset of assets is recognised at a fair value in banks' profit and loss statement. The rest is insensitive to the market prices and their book value follows the amortised cost principles (Beatty and Liao, 2014). Notably, all assets of the asset managers are fair valued. This has important implications in the fire sales context – the higher the proportion of the MtM assets in banks' books the more contagion amplification potential of the fire sales.
6. outflow parameter $f_m^O$: rate of withdrawal of *non-rolloverability* of the deposits (retail and wholesale)
7. liquidity weights $\lambda^A$, $\lambda^L$: the weights represent the runoff rates for cash outflows associated with funding sources of banks and asset managers. They can be thought of as the weights of the Liquidity Coverage Ratio. LCR is based on the liquidity parameters that determine regulatory inflow and outflow ratios. We use a convention that $\lambda_n^a \geq 0$ and $\lambda_m^l \leq 0$.
8. liquidity threshold $\tau^\lambda$: bank is considered as liquid if and only if

$$\frac{\sum_n (1-h_n) a_n^b}{\sum_n \lambda^A a_n^b + \sum_m \lambda^L l_m^b} > \tau^\lambda$$

9. risk weights (for solvency calculations) $\omega_n$: risk weights for risk-weighted assets ($\Omega^b$) calculation and consequently for the solvency ratio computation

$$CR^b := e^b / \Omega^b = \frac{e^b}{\sum_n \omega_n a_n^b}$$

10. fire sales elasticity $\alpha_n$: this parameter, asset-specific defines the change in market price of assets following the liquidation of portfolios. It can be thought of as a measure of the depth of the market $n$. Specifically, if each bank sells $\Delta a_{\bar{n}}^b$ of a given security $\bar{n}$ then on aggregate the market absorbs $\sum_b \Delta a_{\bar{n}}^b$ which leads to revaluation of $1 - \exp(-\sum_b \alpha_{\bar{n}} \Delta a_{\bar{n}}^b)$. The elasticity $\alpha_n$ would be 0 if there is no impact on the capital of banks, i.e. the accounting rules would exclude the Mark-to-Market (MtM) recognition of changes in valuation of assets (like in case of loans).
11. minimal capital adequacy ratio $\tau^b$: a bank-specific minimum amount of capital to cover the risks on the asset side of the balance sheet measured by the RWA. Bank-specific ratios allow to differentiate between jurisdictions or to set specific thresholds for the systemically important banks.

12. maturity profile of funding $\mu_m$: each liability category matures in a give time horizon. That is important for the repricing risk in case of deterioration of the solvency conditions of a bank.

13. interest rates or remuneration from assets $r_n$ and funding costs $c_m$: asset generate income – either accrued in the P&L or directly recognised on the capital accounts. This is captured synthetically by the asset-specific rates $r_n$. Specifically, loans pay interests and securities pay interest or their book value is amortised or changes in their market valuation impacting the P&L. On the funding side, deposit require interest payments $c_m$.

14. intervals for changes in funding cost implied by a significant decline of the solvency ratios $\mathcal{T}_m :=(-\infty, \tau_m^f]$ for $m \in M$ funding classes and the elasticity $\phi_m$ of funding cost to the changes in capital ratios: a change of the solvency ratio within an interval $(\tau_m^f, +\infty)$ does not imply any change of the funding cost of the rolled-over funding volumes. For a change $x$ of the capital ratio in $\mathcal{T}_m$ the additional cost for funding category $m$ is $\phi_m(\tau_m^f - x)$.
For the peers of the banks with increasing funding cost related to the deterioration of the solvency conditions, it is assumed that the cost of roll-over of funding increases by $\Delta^2 c_m$, i.e. homogenously for all banks.

15. a significant reduction of capital ratios $\Delta^\tau$: this threshold defines a group ($\mathcal{ES}$) of banks with a significant decrease of capital ratios; subsequently, banks similar to any of the banks in the group $\mathcal{ES}$ experience increasing funding cost on the rolled-over wholesale debt.

16. revaluation threshold of the AMs $\rho^a$: the behavioral redemptions happen in a given AM $a$ for which the net asset value (NAV) falls in a previous chain of six steps below $1-\rho^a$ of the NAV at the beginning of that previous chain. The revaluation is assumed to follow the fire sales of assets by all the agents in the market. The induced behavioural redemptions are denoted $R$ and amount to $Re^a$ for the AM $a$.

17. interbank assets and liabilities: for the purpose of the interbank contagion analysis we delimit one special category in funding sources and one in banks' loan portfolios. These are
    (a) interbank deposits class $m^I \in \bar{M}$
    (b) interbank loan class $n^I \in \bar{N}$

    We assume that the system is closed, i.e.

$$\sum_{b \in \bar{K}^b} a^b_{n^I} = \sum_{b \in \bar{K}^b} l^b_{m^I}$$

18. own debt issued: similar to the interbank market of direct lending and borrowing we specify one class of funding sources that correspond to the debt issued in a form of bonds that may be cross-held by other banks. These are:
    (a) own debt issued $m^B \in \bar{M}$, $m^B \neq m^I$
    (b) bank bonds in securities portfolios of banks $n^B \in \bar{N}$, $n^B \neq n^I$

    In the case of debt issued the market may not be closed since some of the bonds issued by banks may be held by other market participants not captured by the interbank system modelled in the paper (asset managers and investment funds, but also pension funds and insurance companies, or simply banks outside the system at stake),

$$\sum_{b \in \bar{K}^b} a^b_{n^O} \leq \sum_{b \in \bar{K}^b} l^b_{m^O}$$

19. interbank market represented by a $K^b \times K^b$-matrix $I$: it has the following properties:

- $I_{k_1 k_2}$ means an exposure of banks $k_2$ to $k_1$, i.e. $k_2$ expects to receive at the end of the contract $I_{k_1 k_2}$ from bank $k_1$
- $\sum_k I_{kk_2} = a^{k_2}_{n^I}$ and $\sum_k I_{k_1 k} = l^{k_1}_{m^I}$

20. network of the cross-holding of the debt issued represented by $K^b \times K^b$-matrix $B$: properties:
    - $B_{k_1 k_2}$ means that bank $k_2$ holds $B_{k_1 k_2}$ volume of the debt issued by banks $k_1$
    - $\sum_k B_{kk_2} \le a^{k_2}_{n^B}$ and $\sum_k B_{k_1 k} = l^{k_1}_{m^B}$

21. interbank replacement cost: $c^I$ is a cost (relative to volume) that a bank incurs should it replace a given volume of interbank funding, i.e. if the volume to be replaced amounts to $v$ then the cost (and a capital impact) equals to $c^I v$.

Banks and asset managers may experience a liquidity shock which can be either:

- (for banks) outflow of deposits or funding in general; (for AMs) redemption by participants or
- deterioration of the liquidity buffer in terms of marketability and haircuts.

There are six steps of a chain of events in the system that we consider. Therefore, we call the model a 'six step' model. If the initial shock is small enough not all steps may be activated. More specifically, a given step is conditional on the outcomes of the proceeding one. For example, fire sales can only be triggered if a bank have insufficient amount of eligible assets or interbank assets to use immediately to cover the outflow. We describe the steps in the following subsections.

## Step 0: Funding shock

The chain of reactions is triggered by a liquidity shock which is understood as an outflow of deposits, which is a bank-specific vector $s^b$, s.t. $s^b_m :\, = f^b_m,\ l^b_m \in [0, l^b_m]$.

If the chain of steps is repeated, we account for a behavioral redemptions following a significant devaluation of AMs assets in the proceeding chains. Specifically, if

$$\frac{\sum_{n \in \bar{N}} a^a_n}{\sum_{n \in \bar{N}} a^{a,0}_n} < 1 - \rho^a$$

then the AM $a$ experiences redemptions that amounts to $R^a e^a$ and the new stock of participations is

$$e^a \to (1 - R^a) e^a$$

Since the redemptions rates are not calibrated in the model, in the applications presented in sections 4 and 5 we assume for simplicity that $R^a \equiv R$ is equal across all asset managers.

## Step A: Sufficiency of a liquidity buffer

If

$$\sum_{n \in \mathcal{E}} (1 - h_n) a^b_n \ge \sum_{m \in \bar{M}} s^b_m \tag{1}$$

the banks $b$ has enough buffer to counterbalance the shock. However, if 1 does not hold then there is a liquidity deficiency and the need for remitting liquidity via cutting of interbank money. Let's define a subset of banks that do not have enough buffer:

$$\mathcal{NB} :\, = \left\{ b \in \mathcal{B} \middle| \sum_{n \in \mathcal{E}} (1 - h_n) a^b_n < \sum_{m \in \bar{M}} s^b_m \right\} \tag{2}$$

## Step B: Replacing interbank funding

Defaults that are related to insufficient amount of liquid (or partly liquid) buffer spill over to the interbank market – defaulting banks stop rolling over their credit to the interbank market. This means that the debtors need to search for other sources of funding which is a costly process. This additional cost reflected in the P&L is quantified in the following way.

$$e^{is,b} :\, = e^b - \sum_{k \in \mathcal{NB}} c^I I_{bk}$$

## Step C: Fire sales

Let's define a quantity $D^b$ that for each bank $b$ measure the size of the deficiency of eligible assets to cover the initial outflow of funding sources. $D^b :\, = \sum_{n \in \bar{M}} s^b_m - \sum_{n \in \mathcal{E}} (1 - h_n) a^b_n$ which needs to be supplemented by selling not necessarily liquid assets ($\mathcal{NE}$).
We assume that the order of selling is defined by the haircuts – starting from those securities of lowest haircut (most liquid) to the largest ones (most illiquid).

Analogously, the asset managers liquidate assets in case the redemption shock surpasses the available cash equivalents. Formally, $D^a :\, = \sum_{n \in \bar{M}} s^a_m - \sum_{n \in \mathcal{C}} (1 - h_n) a^a_n$.

The assumption on the pecking order reflects the actual process of liquidity risk management. Banks want to use their liquidity buffers in a most effective way. However, there are two caveats of the approach. Releasing the highest quality buffers may hamper the liquidity ratios. Liquid assets have low haircuts and high liquidity weights. Therefore, in practice there would be a trade-off between selling the highest quality buffer to supplement liquidity the most and exposing itself to breaching the liquidity requirement since the highest quality buffers would have the highest liquidity weights for LCR ratio calculation.

The amount needed is computed sequentially (in an algorithmic way for all banks). Banks with insufficient amount of assets (or rather good quality assets in terms of liquidity) to cover the liquidity gap are included into a default-on-liquidity set $\mathcal{DL} :\, = \emptyset$. Selling of some securities transforms the liquidated securities into cash which gets a new, apparently zero risk weight. We denote the remaining asset classes $a_n^{fs,x}$, $x \in \{a, b\}$.

**Algorithm 1:** Usage of the liquidity buffer for bank $b$ or asset manager $a$ (repeated for all banks and AMs)

**Data:** Prepare the buffer to be liquidated (composed of the non-eligible assets)
– $x == b \implies \mathcal{X} :=\mathcal{NE}$;   $x == a \implies \mathcal{X} := \mathcal{NC}$;
– sort $\mathcal{X}$ by the haircuts, from the smallest to the largest $\implies$ sequence $I$ of permutation of indices of elements of $\mathcal{X}$;
– initialisation: $D_1^x := D^x$, $t := 1$;
– bank-specific set of pairs $\mathcal{L}^b$: index corresponding to liquidated asset category and the liquidated amount $\implies \mathcal{L} := \emptyset$;
– initialise the new volumes of assets: for all $n \in \bar{N}$ $a_n^{fs,x} := a_n^x$;
**while** $D_t^x > 0$ ***and*** $t \leq \#\mathcal{X}$ **do**
    $D_{t+1}^x := \max\{0, D_t^x - a_{I_t}^x(1 - h_{I_t})\}$;
    **if** $D_t^x > a_{I_t}^x(1 - h_{I_t})$ **then**
        $\mathcal{L}^x := \mathcal{L}^x \cup \{(I_t, a_{I_t}^x)\}$;
        $a_{I_t}^{fs,x} := 0$;
    **else**
        $\mathcal{L}^x := \mathcal{L}^x \cup \{(I_t, \frac{D_{t+1}^x}{1-h_{I_t}})\}$;
        $a_{I_t}^{fs,x} := a_{I_t}^{fs,x} - \frac{D_{t+1}^x}{1-h_{I_t}}$;
    $t := t + 1$;
**if** $t == \#\mathcal{X} + 1$ ***and*** $D_t^x > 0$ **then**
    $\mathcal{DL} := \mathcal{DL} \cup \{x\}$;

The *else* part of the *if-clause* accounts for the cases of insufficient amount of assets of type $I_t$ in their balance sheet to cover the liquidity gap $D_t^x$. Agents liquidate the assets sequentially moving from asset class $I_t$ to $I_{t+1}$ if the whole volume of class $I_t$ is sold and $D_{t+1}^x$ is still positive. A bank receives a liquidity default flag, i.e. its index is included into the set $\mathcal{DL}$ if and only if $D_{t+1}^x$ is positive and it does not hold asset classes $I_s$ for $s > t$. Notably, the default situation only applies to banks; asset managers cannot default since they are assumed to be fully funded by participations. However, haircuts and revaluation of AMs' assets are reflected one-to-one into the net asset value of the AMs.

The liquidation procedure has a direct and an indirect implication. Fire-sales directly impact the market prices of securities which translate into losses via the MtM revaluation. The indirect consequences of defaults and the contagion spreading via the network of bilateral exposures are considered as the next step of the default chain.

According to assumption 10 the MtM revaluation of securities impacts the capital position of banks in the following way:

$$e^{fs*,b} := e^{is,b} - \sum_{n \in \mathcal{M}} a_n^b \big(1 - \exp(-\sum_{y \in \mathcal{A} \cup \mathcal{B}} \sum_{(k,x) \in \mathcal{L}^y} \mathbb{I}_{\{k=n\}} \alpha_k x)\big)$$

The formal representation is more complicated that the meaning of the expression: each bank is affected by repricing of a given asset class by the aggregate volume of liquidated assets (by all agents in $\mathcal{A}$ and $\mathcal{B}$) on the market depending on the depth of the market ($\alpha_k$).[7]

Similarly,

$$e^{fs*,a} := e^{is,a} - \sum_{n \in \bar{N}} a_n^a \big(1 - \exp(-\sum_{y \in \mathcal{A} \cup \mathcal{B}} \sum_{(k,x) \in \mathcal{L}^y} \mathbb{I}_{\{k=n\}} \alpha_k x)\big)$$

The summation runs across all the asset types since we assume that all assets are recognised at a fair value.

[7] We use a convention of $\sum_{x \in \emptyset} w(x) = 0$ for some function $w$.

Let us define a set of banks $\mathcal{ES}$ with a *significantly* affected capital ratio:

$$\mathcal{ES} = \left\{ b \Big| \frac{e^b}{\Omega_b} - \frac{e^{fs*,b}}{\Omega_b} > \Delta^\tau \right\}$$

## Step D: Direct effects on funding costs

Banks incurring losses due to fire sales and interbank costs are becoming more vulnerable to solvency problems since their capital adequacy figures deteriorate. Even is still above regulatory minimum a sharp enough drop of the CR may produce a signal to the funding market that the risk of uninterrupted debt servicing increases. Counterparties providing funding to the effected bank may revise the risk premia demanded for the rolled-over funding. The magnitude of an impact would depend on the maturity profile of the funding sources. These increasing costs may likely have further indirect consequences on the funding costs of other banks that run a similar business model to the directly affected banks. More specifically, the participants of the funding market may assess that banks with unaffected CRs but having a similar funding an investment strategy are likely to experience similar solvency shocks and would took precautionary measures also demanding higher premium for the rollover of the debt. This would be a spill-over across the funding markets. Summarizing, banks with solvency problems would directly face increasing funding costs as well as indirectly via the similarity of business models.

We are ultimately interested in the impact of the cost on the capital position of a bank. Therefore, we proceed with the mathematical formality of the impact. Should there be no sensitivity of the funding costs to changing solvency conditions of a bank following the consequences of the liquidation of assets (fire sales price impact with the potential interbank effects), the net income has the following impact on the capital position:

$$e^{Inc,b} = e^{fs*,b} + \sum_{n \in \bar{N}} r_n a_n^b - \sum_{m \in \bar{M}} c_m l_m^b \tag{3}$$

Accounting for the relationship between funding cost and solvency and denoting the change in solvency $\Delta^b_{solv} : = \frac{e^{fs*,b}}{\sum_n \omega_n a_n^{fs*,b}} - \frac{e^b}{\sum_n \omega_n a_n^b}$ lead to:

$$e^{IncS,b} \quad = \quad e^{Inc,b} - \sum_{m \in \bar{M}} \mathbb{I}_{\{\Delta^b_{solv} \in \mathcal{T}_m\}} \phi_m \left( \tau^f_m - \Delta^b_{solv} \right) l^b_m \mu^b_m \tag{4}$$

## Step E: Indirect effects on funding

The peer groups are defined as follows: for a given tolerance $\tau$, bank's $\bar{b}$ peer group $\mathcal{P}\bar{b}$ contains all banks $b$ such that:

$$\frac{\sum_n a_n^b a_n^{\bar{b}} + \sum_m l_m^b l_m^{\bar{b}}}{\sqrt{\sum_n (a_n^b)^2 + \sum_m (l_m^b)^2} \sqrt{\sum_n (a_n^{\bar{b}})^2 + \sum_m (l_m^{\bar{b}})^2}} > 1 - \tau \tag{5}$$

It means that the structure of asset portfolios and their funding is very similar between banks (measured by the cosine similarity). We collect all the banks which are peers of any of the banks with significantly affected capital ratios ($\mathcal{ES}$) and denote $\mathcal{P}(\mathcal{ES}) : = \{b \in \bar{K}^b | \exists \bar{b} \in \mathcal{ES} \, [b \in \mathcal{P}\bar{b}]\} - \mathcal{ES}$. For $b \in \mathcal{P}(\mathcal{ES})$

$$e^{Peer,b} = e^{IncS,b} - \sum_{m \in \bar{M}} \Delta^2 c_m l^b_m \mu^b_m \tag{6}$$

Technically, equation 5 is a cosine function of an angle between two vectors representing the balances sheet structures of two given banks. Clearly, the closer the cosine to 1 the more aligned are the vectors, so the greater similarity of the structures.

### Step F: Solvency driven defaults and transmission via interbank and cross holding of debt

If for any reason the capital ratio falls below a regulatory threshold $\tau^b$ the bank defaults. We assume that is means default of payment of the interbank liabilities (with a given LGD, set uniformly for the interbank market) and defaults on the bonds issues that are hold across the market. These two layers of interconnectedness transfer the shocks of the solvency defaults throughout the interbank market. Let us suppose that the two networks of connections between banks in both layers. First trivially, but importantly for the simulation-based methods we use to derive the connections usually not observed on a bilateral basis, the structure of connections needs to be compatible with the marginal, total banks' interbank lending and borrowing and the debt issued by each bank and other banks' total holding of bank bonds. Second, there are many possible structures that fulfill the marginal constraints. Therefore, we generate many structures following methodology of Hałaj and Kok (2013), not to be biased by one specific topology of the interbank market.

The default-related shocks propagate in a cascade fashion. The specification of the cascade algorithm is postponed to the Appendix A since it is nothing more than technical. In broad terms, the losses related directly to the default of agents are first transmitted to the first line creditors. This means a reduction of these creditors' capital ratios and in case they end up below the minimum capital requirement they default as well and become the propagators of the initial shock to creditors of the creditors. The procedure continues until no new defaults occur.

* * *

As a bottom line, all the mechanisms described above magnify the initial funding shock via the 6-step channel to result in some liquidity and, more importantly solvency effects.

## 3 Data and calibrations

The Comprehensive Assessment exercise of 2014 conducted by EBA and ECB in 2014 provide us with a rich dataset that can be used for parametrisation of the model and gives an opportunity to conduct interesting simulations of contagion risk related to some stylised liquidity shocks.[8] Balance sheet data of 130 largest European banking groups available in the sample are broken down by risk categories, currency and country of activities. We use a subset of three types of templates. First, we select templates that provide us with information about funding and investment structure that contain also maturity profile and interest rates paid or earned respectively from liabilities and assets. Second, we pick information of banks capitalisation; in particular the level of capital, total risk weighted assets. Third, we collect risk weighted assets broken down by aggregate portfolios which are granular in the credit risk related templates. The breakdown of portfolios is broadly consistent between funding and the credit risk templates. To have a full picture of the risk weighted assets we supplement the data with aggregate information on sovereign risk weights and operational and market risk related risk weighted assets.

Given confidentiality issues we are able to only report aggregate and anonymised input. However, this is not a major limitation since anyway, given the high granularity of data we would need to perform aggregations to present the output in a readable fashion.

In terms of the currency breakdown EUR and USD are the two dominant components. The combined two currencies account for 95.7% of total assets in the analysed sample of banks, with 85.2% EUR and 10.5% USD. Given such an overwhelming majority we present detailed breakdown of asset and liability categories only for these two currency buckets (see Tables 1 and 2). Notably, there is some degree of bank-by-bank heterogeneity of the share of USD assets in total assets; there are 7 banks in the sample with the share larger than 30% while the vast majority have the share below 5%. The remaining currencies are: GBP, CHF, SEK, PLN, CZK and RUB.

[8] Details on the exercise and its stress testing component that included the data collection that we use can be found under: https://www.bankingsupervision.europa.eu/banking/comprehensive/2014/html/index.en.html.

Table 1: Real banking system: structure of the input data (EUR-denominated exposures)

| | Balance sheet side | Volume | | | | Interest rate | Risk weight | Liquidity risk weight |
|---|---|---|---|---|---|---|---|---|
| | | 25th prct | median | mean | 75th prct | mean (wght) | mean (wght) | mean (wght) |
| EQUITY | E | 1595 | 3131 | 8318 | 1595 | 0 | 0 | 0 |
| CB_CENT_GVMT | A | 369 | 1575 | 8642 | 369 | 0.03 | 6 | 25 |
| INST | A | 725 | 3987 | 25311 | 725 | 0.02 | 23 | 2 |
| CORP | A | 1844 | 9118 | 23651 | 1844 | 0.06 | 106 | 4 |
| RETAIL_SEC_RE | A | 266 | 7751 | 27324 | 266 | 0.06 | 48 | 6 |
| RETAIL_QUAL_REVOLV | A | 0 | 4 | 602 | 0 | 0.14 | 59 | 1 |
| RETAIL_OTH | A | 216 | 2249 | 10195 | 216 | 0.08 | 56 | 2 |
| SECURITISATION | A | 0 | 0 | 750 | 0 | 0.02 | 26 | 0 |
| OTH_NON_CREDIT | A | 0 | 0 | 1248 | 0 | 0.02 | 31 | 0 |
| HTM___SOV | A | 0 | 108 | 2489 | 0 | 0.05 | 7 | 0 |
| HTM___OTH | A | 0 | 422 | 4746 | 0 | 0.04 | 7 | 0 |
| AFS___SOV | A | 299 | 2963 | 7182 | 299 | 0.04 | 8 | 0 |
| AFS___OTH | A | 33 | 755 | 3978 | 33 | 0.04 | 8 | 0 |
| FVTPL___SOV | A | 0 | 17 | 2002 | 0 | 0.03 | 8 | 0 |
| FVTPL_OTH | A | 0 | 36 | 3132 | 0 | 0.03 | 7 | 0 |
| UNSEC_IB_A | A | 47 | 1095 | 6793 | 47 | 0.02 | 39 | 0 |
| SEC_IB_A | A | 0 | 270 | 5652 | 0 | 0.01 | 34 | 0 |
| NON_BANK_CORP_DEP_SIGHT | L | 273 | 2453 | 9023 | 273 | 0.01 | *na* | 6 |
| NON_BANK_CORP_DEP_TERM | L | 180 | 1148 | 7026 | 180 | 0.02 | *na* | 7 |
| RETAIL_DEP___SIGHT | L | 208 | 7455 | 26728 | 208 | 0.01 | *na* | 3 |
| RETAIL_DEP___TERM | L | 163 | 4551 | 15184 | 163 | 0.03 | *na* | 4 |
| GVMT_DEP___SIGHT | L | 0 | 83 | 722 | 0 | 0 | *na* | 2 |
| GVMT_DEP___TERM | L | 0 | 138 | 1214 | 0 | 0.02 | *na* | 3 |
| UNSEC_IB_L | L | 467 | 2065 | 9518 | 467 | 0.02 | *na* | 10 |
| SEC_IB_L | L | 0 | 2227 | 7273 | 0 | 0.01 | *na* | 8 |
| SNR_UNSEC_DEBT | L | 34 | 2778 | 11877 | 34 | 0.04 | *na* | 10 |
| COV_BONDS | L | 0 | 1698 | 8640 | 0 | 0.04 | *na* | 5 |
| OTH_OWN_DEBT | L | 0 | 503 | 4661 | 0 | 0.05 | *na* | 10 |
| CERT_DEPOSIT | L | 0 | 0 | 2523 | 0 | 0.01 | *na* | 10 |
| COMM_PAPER | L | 0 | 0 | 607 | 0 | 0.01 | *na* | 10 |
| STRUCT_PRODUCTS | L | 0 | 0 | 1888 | 0 | 0.01 | *na* | 25 |
| ABS | L | 0 | 0 | 1390 | 0 | 0.02 | *na* | 50 |
| ELA | L | 0 | 0 | 293 | 0 | 0.01 | *na* | 0 |
| OTH_CB_L | L | 0 | 810 | 4913 | 0 | 0 | *na* | 0 |
| CB_DEPOSIT | L | 63 | 425 | 2509 | 63 | 0 | *na* | 0 |

Note: $na \equiv$ not applicable
Source: own calculation based on ECB stress testing data, 2014; [exception] 'Liquidity risk weight' – author's calibration

Table 2: Real banking system: structure of the input data (USD-denominated exposures)

| | Balance sheet side | Volume | | | | Interest rate | Risk weight | Liquidity risk weight |
|---|---|---|---|---|---|---|---|---|
| | | 25th prct | median | mean | 75th prct | mean (wght) | mean (wght) | mean (wght) |
| EQUITY | E | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| CB_CENT_GVMT | A | 0 | 0 | 2259 | 0 | 0.01 | 11 | 25 |
| INST | A | 0 | 0 | 3338 | 0 | 0.02 | 40 | 2 |
| CORP | A | 0 | 0 | 4036 | 0 | 0.06 | 91 | 4 |
| RETAIL_SEC_RE | A | 0 | 0 | 299 | 0 | 0.06 | 85 | 6 |
| RETAIL_QUAL_REVOLV | A | 0 | 0 | 10 | 0 | 0.16 | 67 | 1 |
| RETAIL_OTH | A | 0 | 0 | 321 | 0 | 0.05 | 97 | 2 |
| SECURITISATION | A | 0 | 0 | 73 | 0 | 0.04 | 21 | 0 |
| OTH_NON_CREDIT | A | 0 | 0 | 48 | 0 | 0 | 29 | 0 |
| HTM___SOV | A | 0 | 0 | 92 | 0 | 0.06 | 4 | 0 |
| HTM___OTH | A | 0 | 0 | 433 | 0 | 0.03 | 8 | 0 |
| AFS___SOV | A | 0 | 0 | 318 | 0 | 0.04 | 9 | 0 |
| AFS___OTH | A | 0 | 0 | 427 | 0 | 0.03 | 10 | 0 |
| FVTPL___SOV | A | 0 | 0 | 234 | 0 | 0.04 | 7 | 0 |
| FVTPL_OTH | A | 0 | 0 | 557 | 0 | 0.09 | 7 | 0 |
| UNSEC_IB_A | A | 0 | 0 | 870 | 0 | 0.01 | 38 | 0 |
| SEC_IB_A | A | 0 | 0 | 2022 | 0 | 0 | 47 | 0 |
| NON_BANK_CORP_DEP_SIGHT | L | 0 | 0 | 1210 | 0 | 0 | *na* | 6 |
| NON_BANK_CORP_DEP_TERM | L | 0 | 0 | 1601 | 0 | 0.01 | *na* | 7 |
| RETAIL_DEP___SIGHT | L | 0 | 0 | 650 | 0 | 0 | *na* | 3 |
| RETAIL_DEP___TERM | L | 0 | 0 | 316 | 0 | 0.01 | *na* | 4 |
| GVMT_DEP___SIGHT | L | 0 | 0 | 30 | 0 | 0 | *na* | 2 |
| GVMT_DEP___TERM | L | 0 | 0 | 89 | 0 | 0.01 | *na* | 3 |
| UNSEC_IB_L | L | 0 | 0 | 2700 | 0 | 0.01 | *na* | 10 |
| SEC_IB_L | L | 0 | 0 | 1756 | 0 | 0 | *na* | 8 |
| SNR_UNSEC_DEBT | L | 0 | 0 | 1532 | 0 | 0.03 | *na* | 10 |
| COV_BONDS | L | 0 | 0 | 116 | 0 | 0.02 | *na* | 5 |
| OTH_OWN_DEBT | L | 0 | 0 | 582 | 0 | 0.06 | *na* | 10 |
| CERT_DEPOSIT | L | 0 | 0 | 1228 | 0 | 0 | *na* | 10 |
| COMM_PAPER | L | 0 | 0 | 590 | 0 | 0 | *na* | 10 |
| STRUCT_PRODUCTS | L | 0 | 0 | 71 | 0 | 0.02 | *na* | 25 |
| ABS | L | 0 | 0 | 48 | 0 | 0.01 | *na* | 50 |
| ELA | L | 0 | 0 | 0 | 0 | 0 | *na* | 0 |
| OTH_CB_L | L | 0 | 0 | 773 | 0 | 0 | *na* | 0 |
| CB_DEPOSIT | L | 0 | 0 | 2202 | 0 | 0 | *na* | 0 |

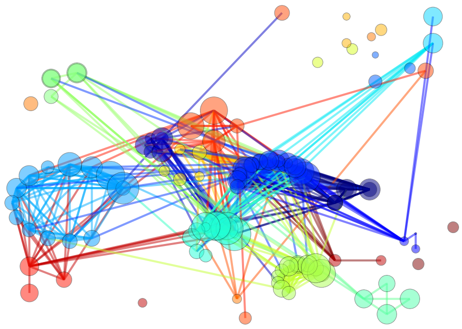
Note: $na \equiv$ not applicable
Source: own calculation based on ECB stress testing data, 2014; [exception] 'Liquidity risk weight' – author's calibration

Table 3: Simulated asset manager system: structure of the input data (EUR-denominated exposures)

| | Balance sheet side | Volume | | | | Interest rate | Risk weight | Liquidity risk weight |
|---|---|---|---|---|---|---|---|---|
| | | 25th prct | median | mean | 75th prct | mean (wght) | mean (wght) | mean (wght) |
| EQUITY | E | 84760 | 101866 | 101081 | 116343 | 0 | na | na |
| FVTPL___SOV | A | 25195 | 45745 | 46877 | 64099 | 0.03 | na | na |
| FVTPL_OTH | A | 35755 | 54700 | 54205 | 72637 | 0.03 | na | na |

Note: $na \equiv$ not applicable
Source: based on assumption that the size of AM sub-system accounts for 20% of the total system (banks and AMs)

Figure 1: Graphical representation of the (simulated) EUR unsecured interbank lending



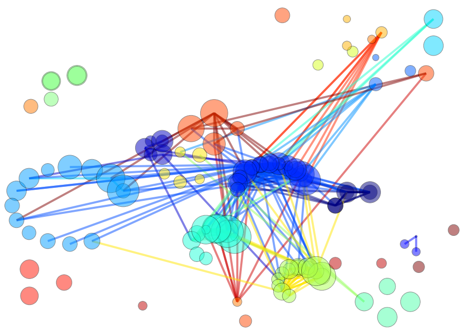
Note: probability map of Hałaj and Kok (2013) used to draw random connections between banks and the figure represents one realisation of the simulated networks algorithm; circles represent banks with the size proportional to log(total assets), edges show exposures, colours represent countries
Source: own calculation based on ECB stress testing data (2014) and NetworkX in Python

The data set does not allow for observing the structure of the bilateral exposures on the interbank market and of the cross-holding of bank debt. We use simulation methods to disaggregate the information to parameterise matrices $I$ and $B$ of the model. Specifically, the algorithm of Hałaj and Kok (2013) was used to randomly generate the relevant networks based on the banks' aggregate figures on interbank lending and borrowing and a matrix of prior probabilities that given two banks are connected. The probability map is based on geographical breakdown of banks' exposures. Figures 1 and 2 present examples of the generated networks.

In the absence of data to calibrate the asset managers we make some stylised assumptions about their balance sheet structures and the size of the system (see Table 3). The assumed composition of their balance sheets is very simplified; there are two asset classes which overlap with the fair-valued portfolios of banks and AMs are funded by a homogenous class of participations. The size of the AM segment is assumed to equal to 20% of the combined bank and asset manager sectors. The impact of this assumption on the dynamics and results of the model is verified in the sensitivity analysis section 4.

Moreover, we make several assumptions about some of the key parameters of the model. They are collected in Table 4. Research literature gives some guidance about reasonable values to be

Figure 2: Graphical representation of the (simulated) EUR-denominated own debt



Note: probability map of Hałaj and Kok (2013) used to draw random connections between banks and the figure represents one realisation of the simulated networks algorithm; circles represent banks with the size proportional to log(total assets), edges show exposures, colours represent countries
Source: own calculation based on ECB stress testing data (2014) and NetworkX in Python

Table 4: Key assumptions on parameters of the ABM

| | | |
|---|---|---|
| $\tau^b \equiv 7.5\%$ | $\rho^a \equiv 6\%$ | $R = 3\%$ |
| $h_n \equiv 0.01$ | $c^I = 20\text{bps}$ | $\tau = 5\%$ |
| $\tau^f_m = 100\text{bps}$ | $\phi = 1.0$ | $\Delta^2 c_m \equiv 50\text{bps}$ |
| LGD=40% | $\alpha = 0.0000005$ | |

Note: $m \in \{\text{wholesale funding categories}\}$
Source: own calibration

considered.

$\tau^b$: The bankruptcy threshold depends on the definition of capital to be the measure of loss absorption capacity. Common Equity Tier 1 (CET1) is considered to be at a safe level above 4.5% of risk-weighted assets, Tier 1 capital ratio should be above 6% and the total capital buffer (i.e. total capital comprising Tier 1 and Tier 2 capital) should exceed 8% of risk-weighted assets. Moreover, banks may be required to hold even more capital for systemic risk or macroprudential purposes and frequently maintain excess buffers to demonstrate financial soundness to account for a severe market stress (European Banking Authority, 2015; Cetina et al., 2017). Consequently, we choose to define the required level of capital at 7.5% of CET1.

$\rho$, $R$: The seminal work of Sirri and Tufano (1998) provides evidence of the (mutual) fund flows related to past performance of funds, overcoming the survivorship bias.[9] Interestingly, the estimates suggest some convexity of the flows with respect to the returns; highest return funds receive disproportionately large flow. The asymmetry of the relationship has been confirmed by Ben-David et al. (2012) showing additionally that in the crisis period the sensitivity of redemptions to performance weakens. This convexity is considered as a puzzle. Berk and Tonks (2007)[10] attributes the asymmetry to the horizon of performance measurement and find out that funds' flows are sensitive to NAV changes in the first year of their poor performance. The convexity is not homogenous across countries; it is even violated for some developed countries while comparing lowest quintile with mid quintile performance (Ferreira et al., 2012). In other words, sufficiently large negative returns trigger substantial redemptions (e.g. Belgium, Finland, France, Germany, Netherlands but less so for Ireland and US and not for UK). Summarising, we implement the relationship of redemptions and NAV devaluation in a threshold manner to account for the nonlinearity. The first quintile value of the negative returns is slightly below -5% (rounded to -6%) and the corresponding negative flow (redemption) amounts to about 3%.

$h_n$: We inform the parameterisations of the haircuts based on the standards developed by BSBC and IOSCO (2015). The 1% haircut is from the range of high-quality government and central bank securities; from 0.5% for residual maturities less than one year to 4% for maturities higher then five years.

$c^I$: Bräuning and Fecht (2017) use Target 2 data to identify the role of relationship lending on the interbank market in reducing the costs of interbank liquidity. They quantify the spread reduction power of the relationship at the level of not more than 20 bps.

$\tau$: The similarity thresholds are very much specific to the peer group classification problem to be tackled with. In general (Zadeh and Goel, 2013), peer groups composition is very sensitive to the assumed thresholds for the very small values and then stabilises above 10%. We take 5% threshold and run the sensitivity analysis around this value.

$\phi$, $\tau^f_m$: Beau et al. (2014) estimate the nonlinear relationship between market based capitalisation and the CDS premia that can be used as a proxy for the wholesale funding spread. It is level dependent and steepens as the capitalisation falls. In principle, there is a 50bp change for 100bp decline in capital. Moreover, Aikman et al. (2009) suggest that the funding cost respond to rating downgrades of financial institutions. This implies that wholesale funding costs change after a significantly large erosion of capital. We combine these findings in a functional relationship of changes in capitalisation and funding costs that assumes no impact for capital ratio dropping less than 100 bps and a linear increase of the costs for CAR changes exceeding 100 bps.

[9] In fact, there is a large number of research papers trying to understand performance of funds and investors' behaviors, e.g.

[10] It is based on a theoretical model of Berk and Green (2004).

$\Delta^2 c_m$: There is not much research on the transmission of shocks among members of peer groups. Lee et al. (2017), based on sound econometric analysis taking care of potential endogeneity issues, find *inter alia* relationship between leverage ratio and peers' net interest margin, thus indirectly funding costs via interest expenses. To parameterise the funding shock to peers of banks exhibiting a significant decline in capital ratios we make a shortcut in reasoning and refer to Afonso et al. (2011). They document the market reaction after the Lehman collapse noticing an asymmetric patterns of spikes in the cost of funding. Only smaller banks experienced a sharp increase of the interbank funding costs (95 bps) suggesting that larger ones were cut off the funding market. Nevertheless, the impact of the crises event on the funding conditions seems to have been substantial even if not measurable for the larger institutions. We took half of the measured impact as the funding cost markup for peers of banks directly affected by a funding shock.

LGD: Loss-given default, a reciprocal of a recovery ratio, is computed based on the European Banking Authority (2016). We took median of the reported median LGD risk parameters in corporate portfolios of banks from the EU and main non-EU countries amounting to 42% as of the end of December 2016 and we rounded it to 40%.

$\alpha$: We take a conservative value of the sensitivity parameter for the fire sales mechanism inspired by the empirical work in this field. The conservative approach can be justified by the fact that we treat the sovereign bonds as eligible in the repo operations or having no impact on pricing and thus no impact on revaluation of agents' portfolios and the fire sales applies to less liquid instruments (corporate bonds, etc.). Additionally, Feldhütter (2012) depicts that "disentangling selling pressure effects from information effects is at best challenging". Moreover, empirical studies usually focus on specific segments of the market because of data availability constraints (Lakonishok et al., 1992) or a specific research question (e.g. Ellul et al. (2011) on the price impact around bond downgrade events or Eser and Schwaab (2016) on the Securities Markets Programme of the ECB). All in all, Greenwood et al. (2015) accept a 10bp decline of prices following a EUR 10 billion liquidation of a generic asset class admitting it can underestimate the impact for less liquid classes and we assume a baseline value of $\alpha$ corresponding to 50bp impact.

# 4 Sensitivity analysis

## 4.1 Randomly generated systems

The complexity of the ABM setup necessitates validation of the dynamics of the model to prove that the ABM has some intuitive properties. The intuition can be build around dynamics of the observed, real financial system hit by a funding shock. In this case a validation would mean a consistency between evolution of aggregate figures of the model (e.g. total liquidated securities, price dynamics, etc.) and the real market indicators (e.g. OTC and exchange traded volumes, price indices respectively).[11] Another interesting approach, although weaker than the first one, is to verify that the outcomes of the model correlate with the calibrated parameters and that the correlation has an intuitively correct sign. The second approach is particularly useful if market data for validation are not available and for that reason we apply this approach.

To understand the key drivers of the dynamics of the modelled system we study randomly simulated systems. Simulated balance sheets of the agents are composed of asset and liability categories listed in the Table 1 or Table 2.

Notably, random systems allow for studying how shock propagation depend on market structure and, consequently how the structure can be influenced to yield the most resilient configuration. In practice, we can manipulate with the parameters of the generated, fictitious systems, i.e. hetero-

[11] Lux and Zwinkels (2017) provide with a survey of empirical validation techniques.

geneity of the system (in terms of the sizes and balance sheet structures), liquidity and solvency constraints. Specifically:

- The number of banks is equal to the one in the data sample, i.e. 130;
- Banks' balance sheets are composed of the same asset and liability categories. The total assets of the sample of banks are drawn from a gamma distribution. It has two parameters: scale ($\kappa$) and shape ($\theta$) parameters that decide about the skewness of the sizes of the banks in the system. Application of the gamma distribution provides a convenient way to control dispersion with a fixed mean. Namely, for a given $\gamma > 0$, by setting $\kappa = \gamma$ and $\theta = \frac{1}{\gamma}$ the mean is equal to $\kappa\theta = 1$ and variance equals to $\kappa\theta^2 = \frac{1}{\gamma}$. This increases comparability of results of contagion simulation;
- An accept-reject algorithm is adopted: a given bank's volumes of asset and liability classes are drawn and the structure is accepted only if the liquidity and solvency requirements are met. Therefore, sampling of the structures for a given bank is repeated until the conditions are satisfied. We simulate 3000 random structures;
- For each simulated structure we draw a funding shock and run the 6-step model.

As commonly observed in the ABMs, there is a large number of parameters that drive the dynamics of the contagion spreading in our model. We identify 10 key parameters (enumerated 1 to 10 on the list below) and one simulation parameter (enumerated 11) that refer to the relative size of the AM sector comparing with the banking sector. Therefore, a sensitivity analysis is warranted to help understanding the directionality of the system dynamics if the 10 key parameters or one simulation parameter change and to prove that the properties of the modelled system change in an intuitive way.

We focused on 11 parameters of the models which are:

1. (LGD) loss given default on interbank exposures after a default event of bank;
2. ($\alpha$) sensitivity of market prices of assets to the volume of liquidated securities in the fire sales circumstances;
3. ($c^I$) interbank search cost;
4. ($\phi$) sensitivity of funding cost to the changes of solvency ratios;
5. ($\rho$) behavioral redemption threshold;
6. ($R$) behavioural redemption rate;
7. ($\tau$) tolerance defining similarity between banks' balance sheets;
8. ($\Delta^2 c$) additional cost of funding for banks similar in business model to those experiencing increased funding cost related to a deteriorated solvency positions;
9. ($\tau^b$) capital requirement ratio;
10. ($\tau^f_m$) threshold of the change in the solvency ratio above which funding cost on rolled-over funding volumes increases (driven by the sensitivity $\phi$);
11. (SCALEAGENTTYPEACTOR) relative size of the AM sector comparing with the banking system.

Moreover, the contagion propagation may be related to agents' balance sheet ratios (as a share in total assets) gauging banks' buffers against the funding shock: capitalisation, non-eligible securities, Marked-to-Market securities and eligible securities.

Table 5 presents an output of two panel regressions that were run to verify the statistical significance of the relationship between agents capital ratios and the key variables and parameters of the model. Seven parameters are significant (six in the pooling specification of the model). Similarity tolerance ($\tau$), interbank search cost ($c^I$), additional funding cost ($\Delta^2 c$), threshold of solvency ratio activating the roll-over cost of funding ($\tau_m^f$), funding cost elasticity ($\phi$) and size of the AM sector are significant drivers of the solvency ratio after the unwinding of 6-step contagion. Moreover, their signs are as expected which is shown in brackets next to the variables in the first column of the table. The only counterintuitive sign of a significant variable was obtained for the LGD of the defaulting interbank exposures. In section 4.2 we verify the sensitivities based on real banking system data.

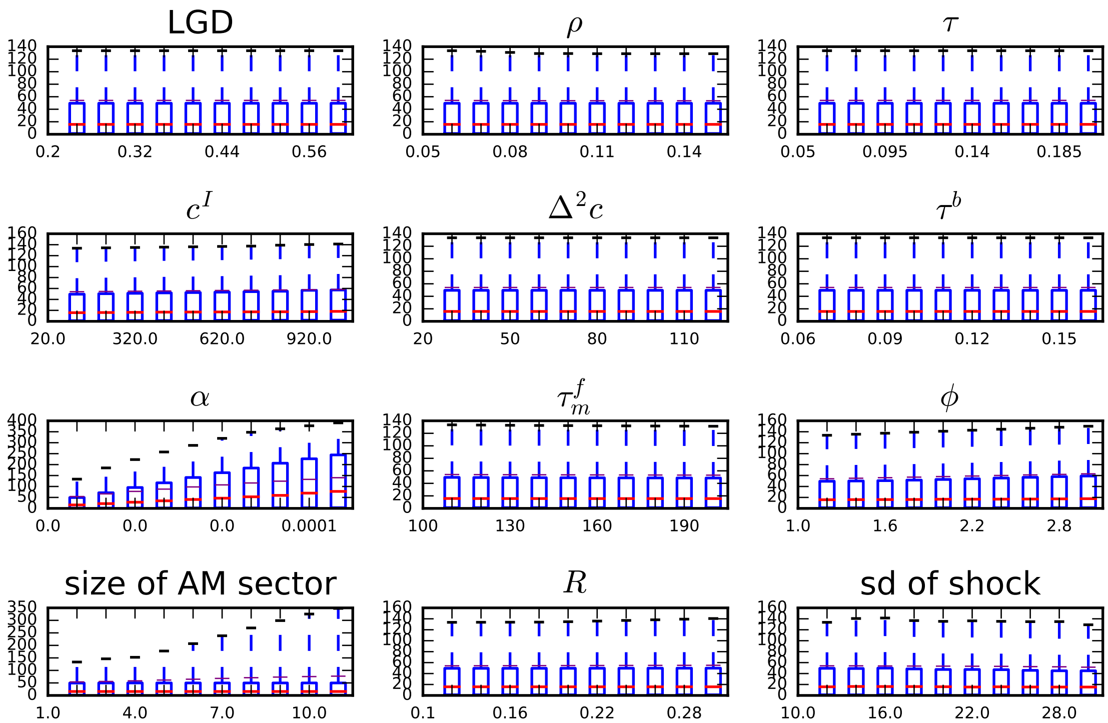
Table 5: Panel regression of the model sensitivity to key balance sheet and ABM parameters based on the randomly generated systems of banks and AMs

| | *Dependent variable:* | |
|---|---|---|
| | Capital Adequacy Ratio | |
| | (pooling) | (within) |
| LGD(-) | 0.025684$^{**}$ | 0.026030$^{**}$ |
| | (*2.105130*) | (*2.367974*) |
| $\rho$(+) | 0.031816 | 0.032467 |
| | (*0.486583*) | (*0.551136*) |
| $\tau$(-) | −0.215172$^{***}$ | −0.215035$^{***}$ |
| | (*-7.442242*) | (*-8.255511*) |
| $c^I$(-) | 0.005583$^{*}$ | 0.005578$^{**}$ |
| | (*1.906945*) | (*2.114496*) |
| $\Delta^2 c$(-) | −0.078644$^{***}$ | −0.078632$^{***}$ |
| | (*-13.019000*) | (*-14.448500*) |
| $\tau^b$(-) | 0.156951 | 0.155356 |
| | (*1.394320*) | (*1.531893*) |
| $\alpha$(-) | −0.062310 | −0.062702 |
| | (*-1.065707*) | (*-1.190348*) |
| $\tau_m^f$(+) | 0.007644$^{**}$ | 0.007748$^{**}$ |
| | (*2.006848*) | (*2.257796*) |
| $\phi$(-) | −1.038174$^{***}$ | −1.016824$^{***}$ |
| | (*-2.629390*) | (*-2.857761*) |
| SCALEAGENTTYPEFACTOR)(-) | 0.100601 | 0.098216$^{*}$ |
| | (*1.555868*) | (*1.685804*) |
| $R$(-) | 0.015347 | 0.015647 |
| | (*0.252789*) | (*0.286064*) |
| cap0 | 0.898920$^{***}$ | 0.849618$^{***}$ |
| | (*4.101708*) | (*4.281303*) |
| nls | 0.057046 | 0.155733 |
| | (*0.349227*) | (*1.054752*) |
| mtm | 0.293958 | 0.137016 |
| | (*1.388221*) | (*0.715163*) |
| elg | 0.355163$^{**}$ | 0.340534$^{**}$ |
| | (*2.027337*) | (*2.148708*) |
| ibdeg | −0.038279 | −0.147016$^{*}$ |
| | (*-0.522399*) | (*-1.944312*) |
| owdeg | 0.064174 | −0.072729 |
| | (*1.080381*) | (*-0.985029*) |
| Constant | 1.292545 | |
| | (*0.633280*) | |
| Observations | 14,136 | 14,136 |
| $R^2$ | 0.022057 | 0.027289 |
| Adjusted $R^2$ | 0.022029 | 0.027036 |
| F Statistic | 18.731050$^{***}$ (df = 17; 14118) | 23.111640$^{***}$ (df = 17; 14005) |
| *Note:* | | $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01 |
| | | cap0 – banks' initial capital; nls – non-eligible securities (less liquid securities); |
| | | mtm – MtM securities portfolios; elg – eligible securities |
| | | ([sign]) – expected sign of a coefficient |

## 4.2 Results for data-driven model

Analogously to the validation of the model the panel regression techniques offer an insight into the structure of drivers of contagion transmission in the model calibrated to the observed structure of

Figure 3: Sensitivity analysis to some key parameters – graphical presentation




Note: box plots, based on simulation of the 6-step chain with a triggering funding outflow sampled from a normal distribution with mean 50% and standard deviation 20% affecting a random sample of banks (drawn from a Poisson distribution with mean equal to 3) and a randomly selected funding category (a uniform distribution on the set of funding categories); red line – median of the distribution across the simulated shocks; purple line – mean of the distribution across the simulated shocks; for each of the 12 box plots x-axis – parameter; y-axis – impact on the average CAR in the system, in bps; 'size of AM sector' – ratio of the sum of assets of all AMs and the sum of assets of all banks.
Source: own calculation based on ECB stress testing data 2014

the banking system. We append the standard deviation of the funding shock (DSHOCK) capturing the structure of the shocks to the list of the 11 parameters which role was tested in section 4.1.

Clearly, as Figure 3 illustrates and consistently with the section 4.1, sensitivity of the model outcomes differs across parameters. Fire-sales can lead to a very different depletion of solvency buffers of banks depending on the parametrisation of the fire sales elasticity. Similarly, the relative size of the asset manager part of the modelled system influences the amplification of the funding shocks stemming from the banking system. This underpins the necessity to integrate the asset management segment into the model and to calibrate this segment accurately because of the significant fire sales mechanism stimulated by the interactions of banks' and asset managers' balance sheets. There is also a visible relationship between the behavioral redemption rate and the contagion spreading. Intuitively, a higher redemption rate implies bigger losses for banks via the fire sales channel. Moreover, there is some degree of relationship between contagion losses and either interbank search costs, sensitivity of funding costs to changes of capital ratio or a threshold of redemption rates. Notably, the relationship goes into an intuitively correct direction. Conversely, the contagion spreading in the model does not seem to depend on the solvency threshold and the threshold for the funding costs to start depending on the changes of the capital ratios, nor the loss given default on interbank exposures in case of banks' insolvency.

Separately, we wanted to verify statistically that the identified relationship holds. We applied a simple panel regression approach to regress the changes in capital ratios on the changes of the key parameters of the model and some centrality measures of the banks in the system, i.e. degree measure of the interbank market and the cross-holding of debt market capturing connectivity of the agents with the rest of the market. We took the following approach:

(i) ranges of variation for the 11 parameters of the model are set (for the key ABM parameters around the baseline calibrations presented in Table 4 and around SCALEAGENTTYPEFACTOR)=1.0 and DSHOCK=20%);

(ii) a value of each parameter in the set of key parameters is sampled from a uniform distribution on the respective range of variation defined in (i) and the 6-step model is run (2 chains to capture amplification effects via a channel of interactions between balance sheets of banks and AMs);

(iii) step (ii) is repeated 1000 times and the impact on capital adequacy ratios of banks is recorded;

(iv) consequently, a data panel with respect to (1000 simulations)*(130 banks) is generated.

Table 6 presents the outcomes of the estimation of the panel regressions. We used to control variables to capture both the size of a bank and its importance in two network layers: 'ibdeg' is a degree number of a bank in the direct interbank lending layer (number of in- and out-links with other banks); 'owdeg' is a degree number in the layer of the own debt issued by the banks in the sample. Moreover, we included fixed individual effects to account for specificity in banks' reactions (significant with an F-test against a pooling model). To avoid the initially ill-conditioned model we normalise values of the variables by their mean in the sample. First, most of the estimated sensitivity parameters are statistically significant. The exceptions are: interbank search cost, additional cost of funding in case of deterioration of solvency in the group of peer banks (step E), solvency threshold and sensitivity of the threshold beyond which funding costs depend on solvency positions (step D). Second, all the directions of sensitivity obtained in the estimation procedure are consistent with the intuition (with a prior, indicated by (-) or (+) in the table) except for the elasticity of the funding cost $\phi$. Conversely, the LGD has a significant and intuitively negative sign unlike in the case of the randomly generated systems (Table 5). The outcomes are informative about the set of parameters the level of which does not materially effect the results of the model and can be set rather freely within a reasonable range. Conversely, the parameters that are significant and drive the results should be carefully calibrated in the simulations. Moreover, the sensitivity analysis helps in reducing the dimensionality of the calibration process and increasing the computational tractability.

# 5 Contagion effects

The dynamics of our model is associated with the contagion mechanism emerging in the modelling system after a financial shock hits some of the agents. The section elaborates on those contagion channels.

To analyse and illustrate contagion channels and the impact of the funding shock size and distribution across agents on the agents' financial standing we conduct some stylised and systematic simulations. In general, a group of randomly selected banks or AMs is assumed to be hit by a shock the contagion spreading is measured by the difference in capital ratios after step F and the initial capital ratios.

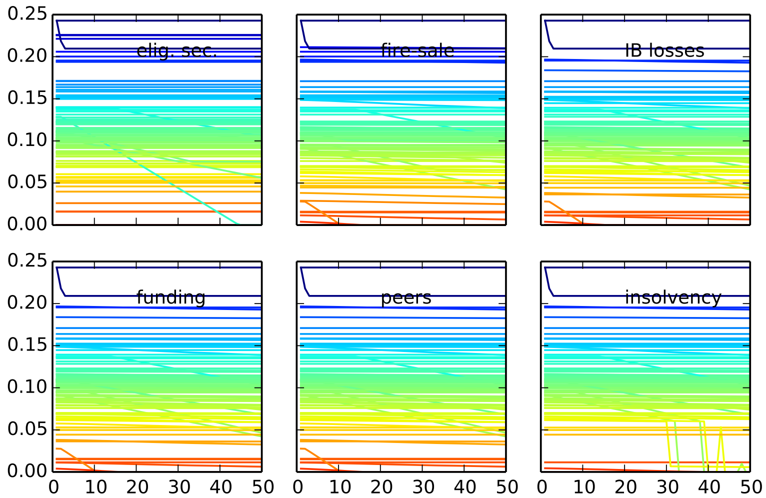
Banks' responses to the shock are heterogenous across the sample. We present a decomposition of the impact of corporate funding and a covered bond shock originated to one country on the outcomes of the six steps, see Figure 4. We consider various outflow parameters ranging from 1% to an extreme case of 50%. A large majority of banks is not significantly affected by the shocks. Their capital positions stay at the initial level independent of the size of the shock (or fall slightly due to a common revaluation caused by the fire sales). However, there is a subset of banks which

Table 6: Panel regression of the model sensitivity to key ABM parameters

| | *Dependent variable:* | |
|---|---|---|
| | Capital Adequacy Ratio | |
| | (pooling) | (within) |
| LGD(-) | $-0.149278^{***}$ | $-0.150359^{***}$ |
| | (*-3.191625*) | (*-3.492218*) |
| $\rho(+)$ | $0.175315^{***}$ | $0.175806^{***}$ |
| | (*2.884893*) | (*3.142706*) |
| $\tau$(-) | $-0.045642$ | $-0.046256$ |
| | (*-1.016730*) | (*-1.119339*) |
| $c^I$(-) | 0.061337 | 0.065069 |
| | (*1.377236*) | (*1.587114*) |
| $\Delta^2 c$(-) | 0.048744 | 0.048613 |
| | (*1.042925*) | (*1.129899*) |
| $\tau^b$(-) | 0.189030 | 0.193883 |
| | (*1.373361*) | (*1.530222*) |
| $\alpha$(-) | $-0.797283^{***}$ | $-0.801831^{***}$ |
| | (*-16.478400*) | (*-18.002710*) |
| $\tau^f_m(+)$ | 0.016142 | 0.025721 |
| | (*0.205451*) | (*0.355623*) |
| $\phi$(-) | $0.415292^{***}$ | $0.415674^{***}$ |
| | (*5.477053*) | (*5.955366*) |
| SCALEAGENTTYPEFACTOR(-) | $-0.298048^{***}$ | $-0.300390^{***}$ |
| | (*-5.828745*) | (*-6.381648*) |
| $R$(-) | $-0.123654^{***}$ | $-0.116623^{***}$ |
| | (*-2.792191*) | (*-2.860657*) |
| ibdeg | $0.191802^{***}$ | 0.066289 |
| | (*4.442354*) | (*1.016535*) |
| owdeg | $1.019886^{***}$ | $-0.125492^{*}$ |
| | (*22.500940*) | (*-1.770902*) |
| Constant | $-1.298209^{***}$ | |
| | (*-5.893750*) | |
| Observations | 70,566 | 70,566 |
| $R^2$ | 0.025709 | 0.006751 |
| Adjusted $R^2$ | 0.025704 | 0.006739 |
| F Statistic | $143.208000^{***}$ (df = 13; 70552) | $36.827910^{***}$ (df = 13; 70439) |
| *Note:* | | $^{*}$p<0.1; $^{**}$p<0.05; $^{***}$p<0.01 |

(-)/(+) indicates priors on direction of sensitivity; t-stats in brackets
estimation using *plm* and *stargazer* for R
([sign]) – expected sign of a coefficient

Figure 4: Sequence of simulations: outflow of EUR-denominated non-bank corporate sight deposits (NON_BANK_CORP_DEP_SIGHT) non-bank corporate term deposits (NON_BANK_CORP_DEP_TERM) and covered bonds (COVER_BONDS) in one selected country



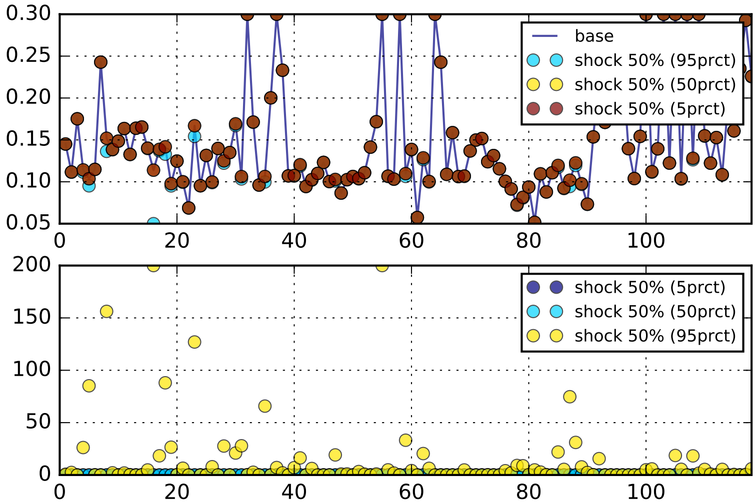

Note: x-axis – outflow (%); y-axis – CAR; each line represents a bank in the sample and colors helps to trace banks that are sensitive to the changes in the severity of the shock.
Source: own calculation based on ECB stress testing data 2014

react quite strongly: their capitalisation deteriorates steadily with the magnitude of the shock. An interesting nonlinearity of the responses can be observed in the last step of the algorithm. There is a threshold level of the shock (above 30% but varying for banks) that drives a few banks into a negative capitalisation. It is a result of some substantial cross-holdings of bank bonds implied by the applied random matching algorithm linking own debt issued (on the liability side) with non-sovereign securities (on the asset side).

The two chosen funding categories – corporate deposits and covered bonds – have a significant share in banks' balance sheets. It is interesting to see that a much less common funding class (ABS) can also be a source of contagion. Nevertheless, a visible decline in capital ratios occurs only for an extreme level of the shock and only for a small subset of banks (Figure 5).

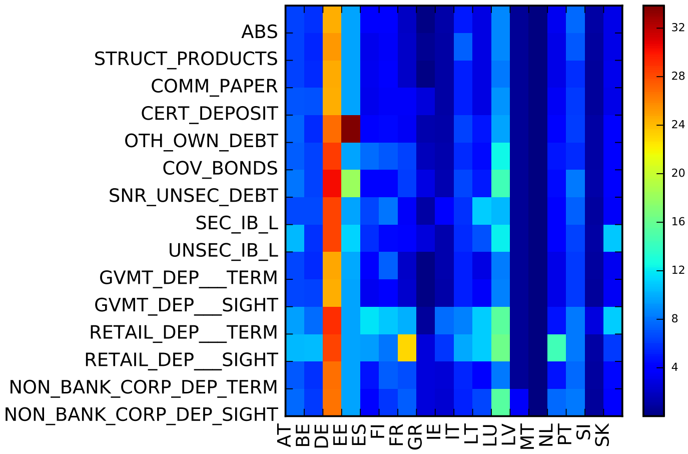
Spreading of a liquidity shock can be contained within national borders or can have a cross-border dimension. In the later case, the effectiveness of any macroprudential policy that tries to reduce the consequences of the liquidity shock is more complicated since it depends on the coordination between jurisdictions. We performed a stylised simulation to assess the potential magnitude of the cross-border impact of a 20% funding outflow. We conducted the simulation for each funding class, randomly selecting a sample of banks. The sample was drawn from a superposition of Poisson and uniform distributions: first, the number of banks $n$ follow a Poisson distribution with mean equal to 2 and then $n$ banks are drawn from a uniform distribution on the whole analysed group of banks. For confidentiality reasons we aggregated the results per country. The effects measured by capital reduction are presented in Figures 6 and 7. In the worst cases of the category-country pairs affected by the initial shock, the overall impact is of a magnitude of 20 bps. Capital is unaffected in 70% of pairs. There is one outstanding country-level banking subsystem that have a much higher potential to spread contagion that the other, due to its size. In general, the cross-border effects are quite pronounced – it explains about half of the overall impact on the average capital ratios of banks in a given country. The magnitude of the cross border effects correlates with the domestic impact, i.e. the more domestic vulnerability to the shock the higher the cross-border spill-over.

Figure 5: Impact of 50% outflow of ABS funding on CAR



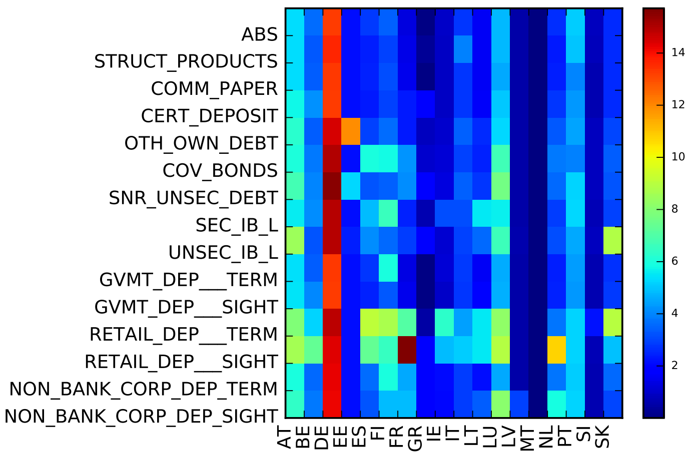

Note: top pane – CAR; bottom pane – ΔCAR (bps); percentiles refer to a random selection of banks affected by the outflow (number of banks drawn from Poisson distribution with mean equal to 2); x-axis: bank n (AMs are not presented since the capital ratio concept is not well-suited); y-axis: top pane – the capital ratio of a bank n ('base' is the starting point capital ratio, 'Xprct' is the X percentile of the capital ratios of the bank n across the outcomes of simulations each assuming 50% shock to the ABM volume of funding of a randomly selected banks in the sample), bottom pane – change in capital ratio of the banks n before the 6 steps and after the contagion ('base' and 'Xprct' as in the top pane).
Source: own calculations based on CAST 2014 data

Figure 6: Heatmap of the shock transmission (20% outflow) aggregated by country (bps)




Note: x-axis – country of a bank shocked by the funding outflow; y-axis – shocked funding category; each cell for [XX] funding class and [YY] country of any subplot corresponds to the decline (in bps) of the average capital ratio of all banks in a sample once a funding outflow shock of 25% for [XX] class hits a bank in country [YY] – the banks in a given country are selected one by one and the decline of the CAR is presented as an average across the simulations
Source: own calculation based on ECB stress testing data 2014

Figure 7: Heatmap of the cross-border spill-overs of a funding shock (20% outflow) aggregated by country




Note: x-axis – country of a bank shocked by the funding outflow; y-axis – shocked funding category; each cell for [XX] funding class and [YY] country of any subplot corresponds to the decline (in bps) of the average capital ratio of all banks in a sample outside country [YY] once a funding outflow shock of 25% for [XX] class hits a bank in country [YY] – the banks in a given country are selected one by one and the decline of the CAR is presented as an average across the simulations.
Source: own calculation based on ECB stress testing data 2014

# 6 Simulations supporting policy

The developed ABM is useful for analysis of policy instruments that can be applied to mitigate contagion risk. The framework is particularly appealing for randomly generated systems.

We conducted three experiments on the generated banking systems to understand the sensitivity of the contagion risk to (i) the heterogeneity of the system, (ii) the liquidity conditions and (iii) the solvency conditions. The experiments shed light on the effectiveness of policy instruments directly targeting market structure and its heterogeneity and on macroprudential tools related to liquidity requirements.

(i) We want to see how the heterogeneity of banks' sizes matters for the contagion spreading. Therefore, we computed the impact of the outflow shock for various parameters $\gamma$ of the Gamma distribution. We operated with the following grid: $\gamma \in \{1, 2, \ldots, 10\}$. Figure 9 shows a very weak relationship between the contagion risk and the heterogeneity although the lower the $\gamma$ (and equivalently the bigger the differences in the sizes) the more contagion is generated.

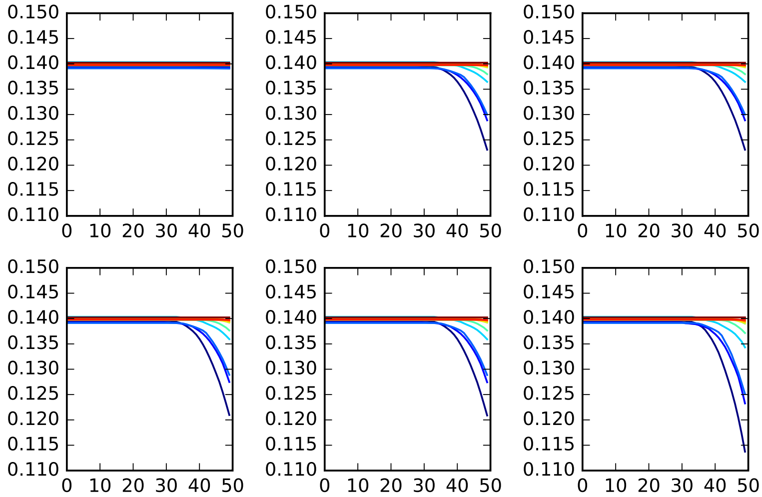
(ii) Contrary to the case (i), the liquidity requirement has a significant impact on the systemic risk. Consequently, this policy measure can be viewed as an effective instrument to curb the risk of contagious defaults. We conducted a simulation of the contagion shock with the liquidity minimum requirements ranging from (the baseline) 100% to 110%. The highest assumed ratio immunizes in the simulation the system to the specific shock of the corporate deposit outflow even in the extreme case of 50% loss of the corporate deposit base. Nevertheless, the conclusions should be treated in a qualitative term, i.e. indicating that liquidity requirement limits can be potentially effective tools in mitigating the contagion risk but a calibration of an exact threshold should be supported by a more thorough analysis.

(iii) Capital does not seem to play the most significant role in the model. It happens this way rather by construction since the chain of events is triggered by a liquidity shock. Therefore, an increase of liquidity requirements reduces probability that a shock surpasses the liquidity counterbalancing capacity.

The policy regarding the contagion risk can be analysed by experimenting with the risk buffers in the model. Specifically, we can work with the real structure of the financial market to test how the contagion spreading depends on the requirements imposed on the maximum utilisation of the eligible assets providing the policy makers with a tool to access how stringent the rules can be to avoid negative externalities of financial contagion. Notably, *repo*ing of these assets increases encumbrance and leaves less buffer to cover future possible shocks. These would create additional pressure on asset prices in the future if banks do not have capacity to restore LCR swiftly and to keep it at a regulatory level also right after the funding shock. Consequently, the less stringent a limit is the more price impact can be expected in case future shocks come. The policy of preserving a minimum level of LCR has direct effects. First, it can stimulate banks to plan restoration of the counterbalancing capacities on a continuous basis. This would help them keep sufficient buffers readily available even for repetitive shocks. However, the second one is related to the activation of the less liquid buffers that can for instance lead to excessive fire sales. We are able to verify whether the second one prevails and at which minimum levels of the limit the contagion risk is sensitive to the policy instrument.

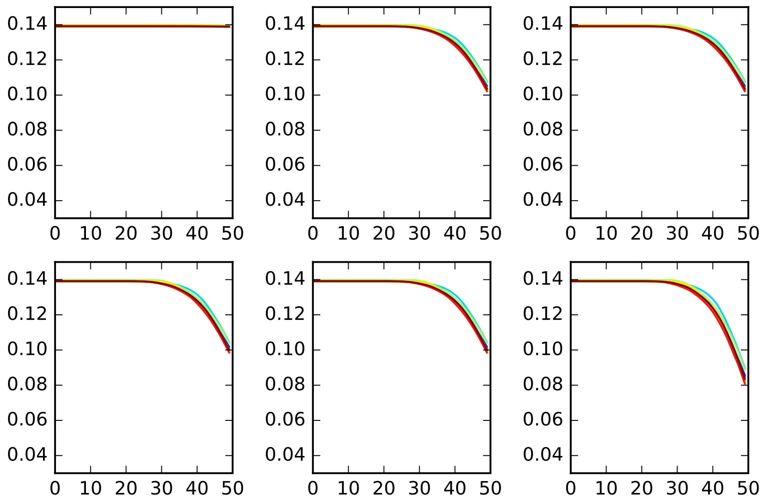
The scenario analysis we conducted confirms that the LCR limit set on a pre-shock system does matter for the contagion losses, both in total but for the cross-border channel as well. For a given limit we ran simulations of 6-step contagion after stressing one funding category of banks in a given country (15 categories times 18 countries). We considered four levels of limits: 0 (no limit), 0.25, 0.50 and 0.75. As presented on Figures 10 and 11. Although maximum contagion losses remain contained in the four scenarios of the LCR limits, the number of affected banks increases reaching almost the whole sample for the case of 0.75 which constraints the utilisation of the liquidity buffer the most.

Figure 8: Impact of the liquidity requirements on the contagion spreading in a simulated interbank system (1000 systems)



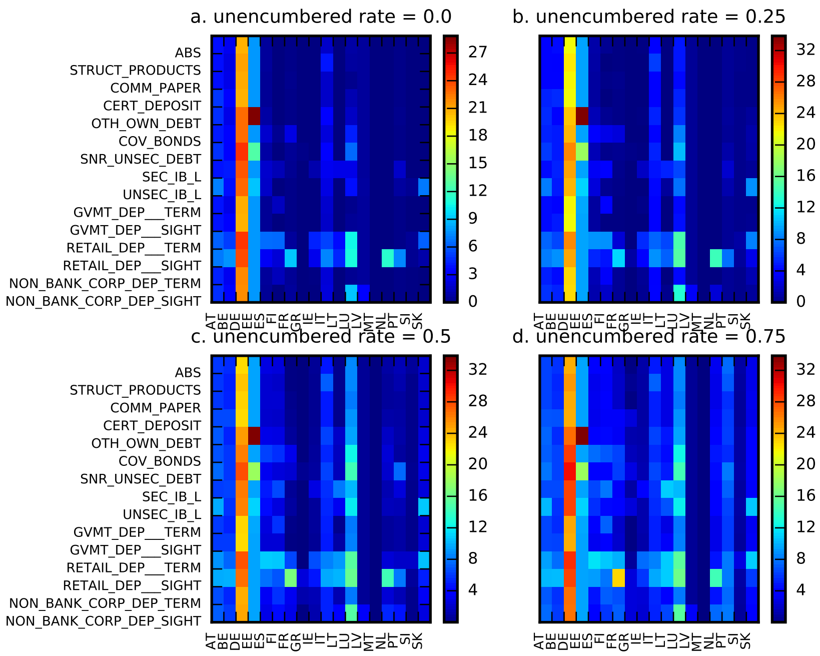

Note: x-axis – outflow, corporate LT debt (%); y-axis – CAR; Subplots ordered from top-left to bottom-right to represent outcomes of each step of the 6-step algorithm; lines correspond to different liquidity requirements ranging from LCR=1.0 (baseline LCR meaning that for each generated banks expected outflows need to be covered in 100% but projected inflows) to LCR=1.1 (stringent LCR meaning that each generated bank the projected inflows should exceed by 10% the expected funding outflows.)
Source:

Figure 9: Impact of the heterogeneity of the simulated interbank system in terms of the size of the banks on the contagion spreading (1000 systems)




Note: x-axis – outflow, corporate LT debt (%); y-axis – CAR; Subplots ordered from top-left to bottom-right to represent outcomes of each step of the 6-step algorithm; lines correspond to different scale parameters of gamma distribution from which total assets of the banks in the system are drawn (lines from red to blue correspond to $\gamma$ ranging from 1 to 10.)
Source: own calculations

Figure 10: Effect of limits to the minimum level of unencumbered assets on the contagion spreading

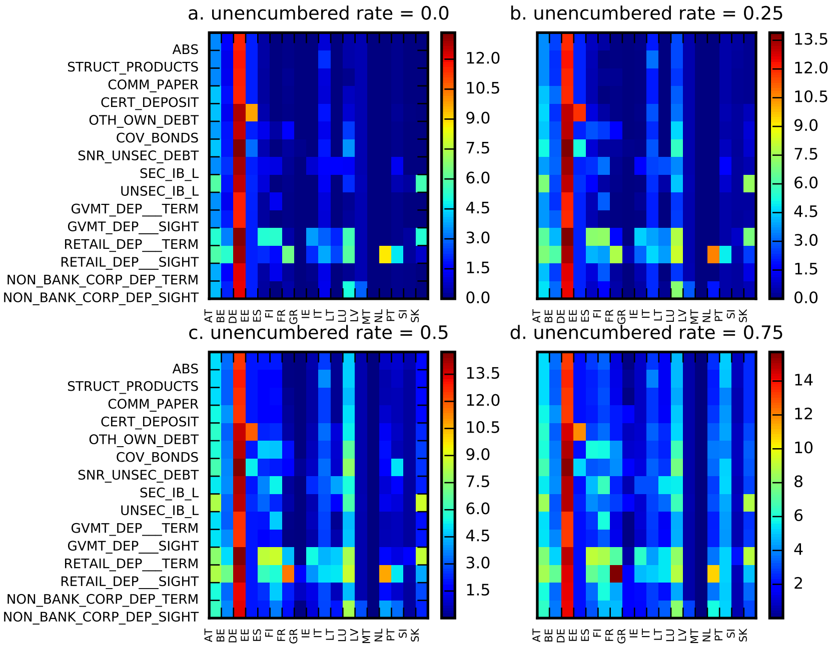



Note: unencumbered rate, expressed in term of the liquidity coverage ratio sets the limit of the usage of the eligible assets in step A of the 6-step algorithm; each cell for [XX] funding class and [YY] country of any subplot corresponds to the decline (in bps) of the average capital ratio of all banks in a sample once a funding outflow shock of 25% for [XX] class hits a bank in country [YY] – the banks in a given country are selected one by one and the decline of the CAR is presented as the average across the simulations; 0.25 on subplot b means that banks are allowed to utilise their counterbalancing capacities until the LCR reaches 0.25, i.e. projected outflows after the stress are covered in 25% by the projected inflows.
Source: own calculations based on the ECB stress test data

The relationship between the minimum required LCR after stress and the magnitude and scope of contagion is nonlinear. For the Liquidity Coverage Ratio below 0.25 the influence of policy instrument on the contagion losses is very limited in total, though more visible for the cross border transmission of the effects (no material difference between heat maps 'a' and 'b' on Figure 10 and a limited one on Figure 11). However, a further increase of the LCR limit creates circumstances for some banks to emerge as more vulnerable to the initial funding shock. Notably, the susceptibility is heterogenous across banks. For some countries contagion is propagated irrespective of the funding class initially stressed (as in Luxemburg). In some countries a significant contagion is instigated only for selected triggers (e.g. Netherlands). It is more pronounced in the cross-border dimension; for the limit as elevated as 0.75 there are a few countries for which a shock to a particular funding category of a bank spread material losses abroad (e.g. sight retail deposits or own debt issued).

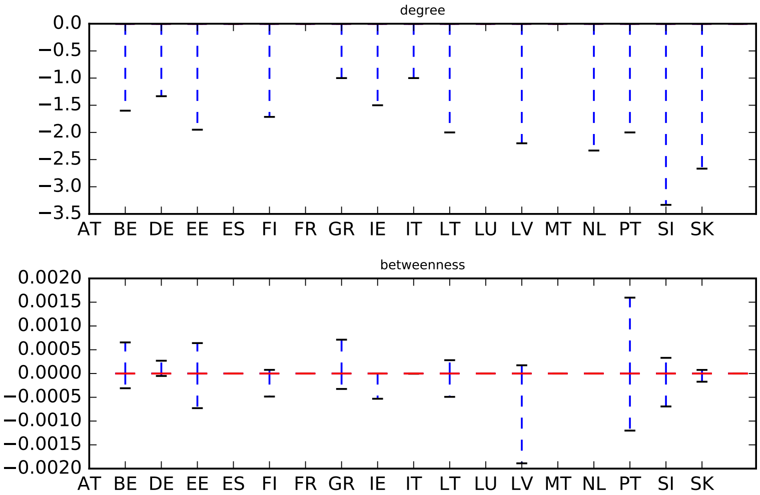
The policy can be analysed also from a market structure perspective, i.e. how the topology of the market may be affected by the liquidity requirements. This topic has been rased in the context of possible externalities of macroprudential policies in general. Fahr and Żochowski (2015) argue that the policies targeting liquidity position of banks may affects the network structure of the system, which is an important factor determining contagion. To examine the influence of the stringent LCR rules we use again the instrument of asset encumbrance limit. We study the topological structure

Figure 11: Effect of limits to the minimum level of unencumbered assets on the contagion spreading across country borders




Note: ‘unencumbered rate’, expressed in term of the liquidity coverage ratio, sets the limit of the usage of the eligible assets in step A of the 6-step algorithm; each cell for [XX] funding class and [YY] country of any subplot corresponds to the decline (in bps) of the average capital ratio of all banks in a sample once a funding outflow shock of 25% for [XX] class hits a bank in country [YY] – the banks in a given country are selected one by one and the decline of the CAR is presented as the average across the simulations; ‘unencumbered rate = 0.25’ on subplot b means that banks are allowed to utilise their counterbalancing capacities until the LCR reaches 0.25, i.e. projected outflows after the stress are covered in 25% by the projected inflows.
Source: own calculations based on the ECB stress test data

Figure 12: Variation in topological structure of interbank lending network following contagion spreading




Note: boxplots present distribution of differences in topological measures of the interbank network as an outcome of the 6-step algorithm in two regimes of asset encumbrance limit: 0.5 versus 0.0 (no limit); 'degree' is the average in- and out-degree of banks in a given country after 6-step algorithm propagates contagion across the system. Analogously, 'betweenness' is the average betweeness centrality in a country (betweenness centrality of a given node (bank) measures the number of shortest paths from all nodes to all others that pass through that given node, normalised by all possible paths); box plots – red bar indicates mean, whiskers correspond to 10th and 90th percentiles.
Source: own calculations based on ECB stress test data

of the interbank networks after the chain of steps unwinds under two regimes of the encumbrance limits: (i) 0 meaning that the whole pool of eligible assets is used as a buffer and the volume of the eligible assets can be equal to 0, (ii) 0.5 meaning that the level of the eligible assets after the funding shock can not be lower than half of its initial stock. We used a range of shock scenarios, each of them being a 20% outflow of funding for a given bank and a given funding category and we iterated for all banks and all funding categories. We collected the output in boxplots on Figure 12. They present the distribution of differences between the two liquidity regimes of the topological measures for interbank networks modified by the 6-step contagion mechanism. For most of the scenarios we do not observe any material change of the structure of the interbank network. However, for some of the scenarios some of the nodes become more central as indicated by the betweenness centrality gauging the size of risk that a shock can easily percolate across the market. Therefore, the market structure is not immune to the stringency of liquidity management rules.

The final policy-relevant remark is about the design of an ABM to correctly capture market mechanisms that stir propagation of the shocks across the system. The sensitivity analysis of our model shows a significant role of the asset managers in fueling the fire sales contagion channel. This helps to justify that a comprehensive set of types of agents needs to be integrated to adequately measure the magnitude of contagion. The result supports the recent emergence of a concept of a system-wide stress test (e.g. Demekas (2015); Constancio (2015); Dent et al. (2016); ECB (2017)). In general, agent-based models resembling the one presented in our paper can be very helpful to start operationalising this idea of system-wide exercises measuring impact of an adverse scenario on the financial system.

## 7 Conclusions

We have built an analytical framework to analyse systemic implications of funding shocks in the financial system following an agent-based modelling approach and bringing the model to the data of the real banking system. We showed how to use the framework to analyse the channels of contagion spreading and the effectiveness of policies mitigating the risk and magnitude of contagion. The framework can be applied as a system-wide stress test with multiple types of interacting financial institutions.

There are at least two possible avenues to follow to extend the analysis. First, agents' reactions to shocks could be modelled in a more behavioural fashion assuming that banks and asset managers would adopt optimally and dynamically to changing parameters of their balance sheets and changing general market conditions.[12] They could change counterparties based on their credit worthiness. They could adjust the structure of their balance sheets trying to either maximise profitability or survival probability (or a mixture of both criteria). Second, the model can be extended to a multi-period setup to study the importance of the timing of the stress on the situation of the banking system. Notably, both potential extensions would bring significant additional complexity to the system of assumptions we use, for instance on the objective functions of banks or mean reversion of the dynamics of the market parameters.

## References

G. Afonso, A. Kovner, and A. Schoar. Stressed, not frozen: The federal funds market in the financial crisis. *Journal of Finance*, 66(4):1109–1139, 2011.

D. Aikman, P. Alessandri, B. Eklund, P. Gai, S. Kapadia, E. Martin, N. Mora, G. Sterne, and M. Willison. Funding liquidity risk in a quantitative model of systemic stability. Working Paper 372, Bank of England, 2009.

K. Anand, M. Paddrik, and L. Valderrama. Agent based models (abms). BIS Working Paper 29, in *Making supervisory stress tests more macroprudential: Considering liquidity and solvency interactions and systemic risk*, 2011.

N. Arinaminpathy, S. Kapadia, and R. May. Size and complexity in model financial systems. *Proceedings of the National Academy of Sciences*, 109(45):18338–18343, 2012.

Ch. Aymanns, C. Caceres, Ch. Daniel, and L.B. Schumacher. Bank solvency and funding cost. IMF Working Papers 16/64, International Monetary Fund, 2016.

R. Babihuga and M. Spaltro. Bank Funding Costs for International Banks. IMF Working Papers 14/71, International Monetary Fund, April 2014.

Deutsche Bank. Risk report. Annual report, 2015.

S. Bansal, B.T. Grenfell, and L.A. Meyers. When individual behaviour matters: homogeneous and network models in epidemiology. 4(16):879–891, 2007.

R. Baptista, J.D. Farmer, M. Hinterschweiger, K. Low, D. Tang, and A. Uluc. Macroprudential policy in an agent-based model of the uk housing market. Staff Working Paper 619, Bank of England, 2016.

A. Beatty and S. Liao. Financial accounting in the banking industry: A review of the empirical literature. *Journal of Accounting and Economics*, 58(2–3):339–383, 2014. 2013 Conference IssueMassachusetts Institute of Technology.

[12]However, as Sinitskaya and Tesfatsion (2015) emphasise that the actions of agents in the ABMs should "be locally constructive, unsupported by externally imposed coordination and optimality restrictions". In other words, optimality should emerge as an outcome of interactions between agents driven by simple behavioural rules.

E. Beau, J. Hill, T. Hussain, and D. Nixon. Bank funding costs: what are they, what determines them and why do they matter? Quarterly bulletin, Bank of England, 2014.

I. Ben-David, F. Franzoni, and R. Moussawi. Hedge fund stock trading in the financial crisis of 2007-2009. *The Review of Financial Studies*, 25(1):1–54, 2012.

J.B. Berk and R.C. Green. Mutual fund flows and performance in rational markets. *Journal of Political Economy*, 112(6):1269–1295, 2004.

J.B. Berk and I. Tonks. Return Persistence and Fund Flows in the Worst Performing Mutual Funds. NBER Working Papers 13042, National Bureau of Economic Research, Inc, April 2007.

U. Bindseil. Central bank collateral, asset fire sales, regulation and liquidity. Working Paper 1610, ECB, 2013.

P. Bonaldi, A. Hortaşu, and J. Kastl. An empirical analysis of funding costs spillovers in the euro-zone with application to systemic risk. Working Paper 21462, National Bureau of Economic Research, August 2015.

R. Bookstaber and M. Paddrik. An agent-based model for crisis liquidity dynamics. Working Paper 15-18, Office of Financial Research, 2015.

R. Bookstaber, M. Paddrik, and B. Tivnan. An agent-based model for financial vulnerability. *Journal of Economic Interaction and Coordination*, Feb 2017.

F. Bräuning and F. Fecht. Relationship lending in the interbank market and the price of liquidity. *Review of Finance*, 21(1):33–75, 2017.

BSBC and IOSCO. Margin requirements for non-centrally cleared derivatives. Technical report, March 2015.

S. Calimani, G. Hałaj, and S. Żochowski. Simulating fire-sales in a banking and shadow banking system. Working Paper Series 46, ESRB, 2017.

G. Cappolletti and P.E. Mistrulli. Multiple lending, credit lines and financial contagion. Working Paper 2089, ECB, 2017.

J. Cetina, B. Loudis, and C. Taylor. Capital buffers and the future of bank stress tests. Brief Series 17-02, OFR, February 2017.

CGFS. Asset encumbrance, financial reform and the demand for collateral assets. CGFS Papers 49, BIS, May 2013.

S. Chernenko and A. Sunderam. Liquidity transformation in asset management: Evidence from the cash holdings of mutual funds. Working Paper 23, ESRB, 2004.

S. Claessens, S.R. Ghosh, and R. Mihet. Macro-prudential policies to mitigate financial system vulnerabilities. *Journal of International Money and Finance*, 39(C):153–185, 2013.

V. Constancio. The role of stress testing in supervision and macroprudential policy. Technical report, 2015. URL `www.ecb.europa.eu/press/key/date/2015/html/sp151029.en.html`.

L.B. de Oliveira and E. Camponogara. Multi-agent model predictive control of signaling split in urban traffic networks. *Transportation Research Part C: Emerging Technologies*, 18(1):120 – 139, 2010.

D. Demekas. Designing effective macroprudential stress tests: Progress so far and the way forward. Working Paper 146, International Monetary Fund, 2015.

K. Dent, B. Westwood, and M. Segoviano. Stress testing of banks: an introduction. Quarterly Bulletin, topical paper 56(3), Bank of England, Q3 2016.

G. Dosi, G. Fagiolo, and A. Roventini. Schumpeter meeting Keynes: A policy-friendly model of endogenous growth and business cycles. *Journal of Economic Dynamics and Control*, 34(9): 1748–1767, September 2010.

ECB. Stamp€: Stress-test analytics for macroprudential purposes in the euro area. ed. S. Dees, J. Henry and R. Martin, European Central Bank, February 2017.

A. Ellul, C. Jotikasthira, and C.T. Lundblad. Regulatory pressure and fire sales in the corporate bond market. *Journal of Financial Economics*, 101(3):596 – 620, 2011.

H. Elsinger, A. Lehar, and M. Summer. Using Market Information for Banking System Risk Assessment. *International Journal of Central Banking*, 2(1), March 2006.

F. Eser and B. Schwaab. Evaluating the impact of unconventional monetary policy measures: Empirical evidence from the ECB's Securities Markets Programme. *Journal of Financial Economics*, 119(1):147 – 167, 2016.

European Banking Authority. *CRD IV - CRR / Basel III monitoring exercise*. March 2015.

European Banking Authority. *Risk Dashboard Annex*. Q4 2016.

S. Fahr and D. Żochowski. A framework for analysing and assessing cross-border spillovers from macroprudential policies. Special Feature, Financial Stability Review A, ECB, 2015.

J.D Farmer and D. Foley. The economy needs agent-based modelling. *Nature*, 460:685–686, 2009.

P. Feldhütter. The same bond at different prices: Identifying search frictions and selling pressures. *The Review of Financial Studies*, 25(4):1155–1206, 2012.

M.A. Ferreira, A. Keswani, A.F. Miguel, and S.B. Ramos. The flow-performance relationship around the world. *Journal of Banking & Finance*, 36(6):1759 – 1780, 2012.

P. Gai, A. Haldane, and S. Kapadia. Complexity, concentration and contagion. *Journal of Monetary Economics*, 58(5):453–470, 2011.

D. Delli Gatti, M. Gallegati, B. Greenwald, A. Russo, and J.E. Stiglitz. Business fluctuations in a credit-network economy. *Physica A: Statistical Mechanics and its Applications*, 370(1):68–74, 2006.

S. Giansante, C. Chiarella, S. Sordi, and A. Vercelli. Structural contagion and vulnerability to unexpected liquidity shortfalls. *Journal of Economic Behavior & Organization*, 83(3):558 – 569, 2012.

R. Greenwood, A. Landier, and D. Thesmar. Vulnerable banks. *Journal of Financial Economics*, 115(3):471 – 485, 2015.

G. Hałaj. Optimal asset structure of a bank – bank reactions to stressful market conditions. ECB Working Paper Series 1533, European Central Bank, April 2013a.

G. Hałaj. Box 3: Literature on financial contagion. Occasional Paper 152, A Macro Stress Testing Framework for Assessing Systemic Risk in Banking Sector, Ed. Henry, J and Kok, Ch., ECB, 2013b.

G. Hałaj. Dynamic balance sheet mmodel with liquidity risk. *International Journal of Theoretical and Applied Finance (IJTAF)*, 19(07):1–37, 2016.

G. Hałaj and Ch. Kok. Assessing interbank contagion using simulated networks. *Computational Management Science*, 10(2-3):157–186, 2013.

G. Hałaj and D. Laliotis. A top-down liquidity stress test framework. Chapter in STAMP€: Stress-Test Analytics for Macroprudential Purposes in the euro area, ed. S. Dees, J. Henry and R. Martin, European Central Bank, February 2017.

J. Helwege and G. Zhang. Financial Firm Bankruptcy and Contagion. *Review of Finance*, 20(4): 1321–1362, 2016.

T.R. Hurd, D. Cellai, S. Melnik, and Q. Shao. Illiquidity and Insolvency: A Double Cascade Model of Financial Crises. Technical report, 2014. URL `https://ssrn.com/abstract=2424877`.

M.O. Jackson and A. Wolinsky. A strategic model of social and economic networks. *Journal of Economic Theory*, 71(1):44–74, October 1996.

P. Klimek, S. Poledna, J.D. Farmer, and S. Thurner. To bail-out or to bail-in? answers from an agent-based model. *Journal of Economic Dynamics and Control*, 50:144 – 154, 2015. Crises and ComplexityComplexity Research Initiative for Systemic InstabilitieS (CRISIS) Workshop 2013.

S. Korsgaard. Incorporating funding costs in top-down stress tests. Working papers 110, Danmarks Nationalbank, 2017.

J. Lakonishok, A. Shleifer, and R.W. Vishny. The impact of institutional trading on stock prices. *Journal of Financial Economics*, 32(1):23 – 43, 1992.

Ch.-Ch. Lee, Ch.-Ch. Lee, J.-H. Zeng, and Y.-L. Hsu. Peer bank behavior, economic policy uncertainty, and leverage decision of financial institutions. *Journal of Financial Stability*, 30:79 – 91, 2017.

A. Liu, M. Paddrik, S.Y. Yang, and X. Zhang. Interbank contagion: An agent-based model approach to endogenously formed networks. *Journal of Banking & Finance*, 2017. doi: http://dx.doi.org/10.1016/j.jbankfin.2017.08.008.

T. Lux. Emergence of a core-periphery structure in a simple dynamic model of the interbank market. *Journal of Economic Dynamics and Control*, 52:A11–A23, 2015.

T. Lux and R. Zwinkels. Empirical validation of agent-based models. mimeo, March 2017. URL `https://ssrn.com/abstract=2926442`.

C.B. Merrill, T. Nadauld, R.M. Stulz, and S.M. Sherlund. Were there fire sales in the rmbs market? Working paper series, Ohio State University, Charles A. Dice Center for Research in Financial Economics, 2014. URL `http://EconPapers.repec.org/RePEc:ecl:ohidic:2014-09`.

M. Mitchell and T. Pulvino. Arbitrage crashes and the speed of capital. *Journal of Financial Economics*, 104(3):469 – 490, 2012.

S. Morris, I. Shim, and H.S. Shin. Redemption risk and cash hoarding by asset managers. *Journal of Monetary Economics*, 89(C):71–87, 2017.

D. Pierret. Systemic risk and the solvency-liquidity nexus of banks. *International Journal of Central Banking*, 11(3):193–227, 2015.

R. Roengpitya, N. Tarashev, and K. Tsatsaronis. Bank business models. BIS Quarterly Review, December 2014.

S.W. Schmitz, M. Sigmund, and L. Valderrama. Bank Solvency and Funding Cost: New Data and New Results. IMF Working Papers 17/116, International Monetary Fund, May 2017.

E. Sinitskaya and L. Tesfatsion. Macroeconomies as constructively rational games. *Journal of Economic Dynamics and Control*, 61(C):152–182, 2015.

E.R. Sirri and P. Tufano. Costly search and mutual fund flows. *The Journal of Finance*, 53(5): 1589–1622, 1998.

A. Turrell. Agent-based models: understanding the economy from the bottom up. Quarterly Bulletin (Topical articles) Q4, Bank of England, 2016.

R.B. Zadeh and A. Goel. Dimension independent similarity computation. *J. Mach. Learn. Res.*, 14 (1):1605–1626, January 2013.

# A Cascade of defaults

To assess contagion risk stemming from the interconnectedness through the interbank deposit network and the cross-holding of bank debt we operate with a generic notion of exposures between banks that form a particular network. An exposure means that there is a credit obligation between the banks and the network created by the relationship is the first element of the cascade procedure. In accounting term the same exposure is recognised on the asset sice of the creditor and on the liability side of the debtor. The second one is a structure of external shocks impacting agents' ability to pay back their debts we employ a cascade procedure. Let $S$ be a vector of real numbers representing a shock structure to the exposures and $L$ – a matrix of exposures. In the implementation the generic $L$ would be replaced by specific $I$ or $B$ depending on the interbank lending contagion or the contagion transmitted by the network of the cross-holding of bank debt. It is assumed to affect the ability to meet the obligations (again, either of the interbank lending or on the debt securities market) and is translated to losses reducing banks' capital buffers. Let $\lambda$ be a vector of loss-given-default (LGD) parameters where its first $K$ entries correspond to the interbank exposures. Let $\mathcal{DS}^0$ be a set of defaulted banks; i.e. those for which the capital ratio falls below a certain threshold $\tau$. The initial shock to banks' assets is calculated as:

$$\Delta^{e^b,0} = \sum_{j=1}^{N} S_j \lambda_j L_{ji}$$

and capital is reduced respectively:

$$e^{b,0} = e^b + \Delta^{e^b,0} \tag{7}$$

The new capital ratio – a ratio of risk weighted assets $\Omega$ and capital $e$ – is calculated as

$$\mathrm{CR}_i^0 = e^{b,0}/(\Omega^b - \Delta^{e^b,0})$$

For the banks with the capital ratio falling below $\tau^b$ we assume that the banks default on their obligations against other banks, implying that the set of defaulted banks is updated as follows

$$\mathcal{DS}^1 = \mathcal{DS}^0 \cup \{b \in \bar{K} | CR_i^0 < \tau^b\}$$

Consequently, the cascade is initiated in a sequential way:

Step 1 Let us suppose that a set $\mathcal{D}^k$, capital position vector $e^k$ and risk-weighted assets vector $R^k$ are known in a certain round $k$ of the cascade

Step 2 Let a set of new defaults be defined as $\mathcal{D}^{\text{new},k+1} = \{i \in \bar{N} | CR_i^k < \tau\}/\mathcal{D}^k$.

- If $\mathcal{D}^{\text{new},k+1} = \emptyset$ then cascade stops.

- If $\mathcal{D}^{\text{new},k+1} \neq \emptyset$ then $\mathcal{D}^{k+1} = \mathcal{D}^k \cup \mathcal{D}^{\text{new},k+1}$ and for all $i \in \bar{N}$

  $$\Delta_i^{e,k+1} = - \sum_{j \in \mathcal{D}^{\text{new},k+1}} \lambda_j L_{ji}$$

  and

  $$e_i^{k+1} = e_i^{k+1} + \Delta_i^{e,k+1}, \qquad \mathrm{CR}_i^0 = e_i^{k+1} / (R_i - \sum_{m=1}^{k+1} \Delta_i^{e,m})$$

  and the cascade returns to the beginning of the step 2 for the next round of default calculations ($k \colon = k + 1$).

Ultimately, the contagion effects are measured by differences between the terminal capital ratio $\mathrm{CR}^{b,\infty}$ (after the cascades is unwound) and the starting capital ratio $\mathrm{CR}_{b,0}$, ie:

$$\Delta\mathrm{CR}^b = \mathrm{CR}^{b,0} - \mathrm{CR}^{b,\infty}$$

# B Notation

Table 7: Key variables

| | |
|---|---|
| $\mathcal{B}$, $\mathcal{A}$ | sets of asset managers and banks respectively |
| $a_n^x$ | asset class $n$ of agent $x$ |
| $l_n^x$ | liability class $n$ of agent $x$ |
| $e^x$ | capital (for banks) or participations (for AMs) of agent $x$ |
| $e^{is,x}$ | capital of an agent $x$ impacted by the interbank search cost of step B |
| $e^{fs*,x}$ | capital of an agent $x$ impacted by the interbank search cost of step B and fire sales |
| $e^{Inc,b}$ | capital of a banks impacted by the interbank search cost, fire sales and net interest income |
| $e^{IncS,b}$ | capital of a banks impacted by the interbank search cost, fire sales, net interest income and the relationship of funding costs and solvency |
| $e^{Peer,b}$ | capital of a banks impacted by the interbank search cost, fire sales, net interest income, the relationship of funding costs and its and peers' solvency |
| $s^b$ | vector of funding shocks |
| $T$ | liquidity horizon |
| $\mathcal{E}$ | a set of eligible asset classes |
| $f_m^O$ | outflow rate for liability class $m$ |
| $\tau^\lambda$ | liquidity threshold |
| $\lambda^A$ | liquidity weight for assets (expected inflow, positive) |
| $\lambda^L$ | liquidity weight for liabilities (expected outflow, negative) |
| $\omega_n$ | risk weight for asset class $n$ |
| $\Omega^b$ | risk-weighted assets of banks $b$ |
| $h_n$ | haircut on asset class $n$, impacting P&L and capital |
| $\alpha_n$ | fire sales elasticity, i.e. sensitivity of asset $n$ price to the volume sold on the market |
| $\tau^b$ | minimum capital level for bank $b$ |
| $\mu^m$ | maturity profile of funding category $m$ |
| $\mathcal{T}_m$ | interval of the changes in the solvency ratio for which the funding cost of category $m$ does not change |
| $\phi_m$ | sensitivity of the cost of funding of category $m$ to the changes of the solvency ration outside $\mathcal{T}_m$ |
| $\Delta^\tau$ | significant reduction of solvency |
| $\mathcal{ES}$ | a group of banks that exhibit a significant change of their solvency ratio (given a shock) greater than $\Delta^\tau$ |
| $\mathcal{NC}$ | non-cash assets held by asset managers |
| $\mathcal{DL}$ | set of banks defaulting due to liquidity problems |
| $I_{k_1k_2}$ | Direct interbank exposure (via interbank deposit) of bank $k_2$ to bank $k_1$ |
| $B_{k_1k_2}$ | Exposure of bank $k_2$ to bank $k_1$ via a cross-holding of debt issued |
| $\rho^a$ | threshold of the AMs' NAV beyond which there is a behavioral redemption happening |
| $R^a$ | rate of the behavioral redemption |
| $\tau$ | similarity tolerance, i.e. banks are similar iff a cosine of vectors of their balance sheets $> 1 - \tau$ |
| $\Delta^2 c_m$ | additional funding cost spread for peers of banks affected directly by a significant drop of CAR |

Source: own specification

**Acknowledgements**

The authors would like to thank Sylvain Barde, Co-Pierre Georg, Sylvain Leduc and Nathan Palmer for interesting discussions and Mateusz Gatkowski for an ongoing cooperation on extensions and parametrisations. Authors are also indebted to participants of 2016 GRI-Fields Conference and Workshop on the Stability of Financial Systems (Toronto), 2016 FSB workshop on system-wide stress testing, Conference on Financial Networks and Sustainability (Zurich), Bank of Canada 2017 invited talk (Ottawa) and 2017 INET Oxford/ UCT Conference (Oxford), ESRB ATC-ASC JEGI workshop 2017 (Frankfurt), Society for Computational Economics 2017 (New York) and Office of Financial Research 2017 invited talk (Washington) for helpful questions and comments. Last but not least, comments of the anonymous referee helped to substantially improve the paper.

**Grzegorz Hałaj** (corresponding author)
European Central Bank, Frankfurt am Main, Germany; email: grzegorz.halaj@ecb.europa.eu



| | | | |
|---|---|---|---|
| ISSN | 1725-2806 (pdf) | DOI | 10.2866/182473 (pdf) |
| ISBN | 978-92-899-3226-4 (pdf) | EU catalogue No | QB-AR-18-001-EN-N (pdf) |