Leaning against inflation experiences *

Stefan Nagel†
University of Chicago,
NBER, CEPR, and CESifo

March 6, 2024

A large share of secular variation in real interest rates can be understood as the effect of monetary policy that leans against experience-based long-run inflation expectations. Survey microdata suggests that adaptive learning from experienced inflation produces very persistent, slow-moving long-run inflation expectations. Faced with experience-based expectations, monetary policy must be persistently hawkish or dovish such that realized inflation, through agents' belief-updating, eventually pulls long-run expectations towards the inflation target. Consistent with this mechanism, I find that there is a strong positive relationship between experience-based long-run inflation expectations and the level of real interest rates in the U.S., Germany, U.K. and Japan. Because their views about the long-run path of inflation are shaped by experience rather than monetary policy communication, private-sector agents do not anticipate future reversion of inflation towards the central bank's target. As a consequence, consistent with the data, long-term real interest rates move as much with experience-based long-run inflation expectations as short-term real interest rates do. Moreover, secular changes in real interest rates are accompanied by secular patterns in interest-rate forecast errors. Overall, the interaction of monetary policy and adaptive learning from experience generates secular real interest rate variation that is distinct from variation in the natural rate of interest.

*I am grateful for comments to Kilian Huber, Rohan Kekre, Ralph Koijen, Carolin Pflueger, Harald Uhlig, Constantine Yannelis and seminar participants at Chicago, Stanford, and UC San Diego.

†University of Chicago, Booth School of Business, 5807 South Woodlawn Avenue, Chicago, IL 60637, e-mail: stefan.nagel@chicagobooth.edu

I. Introduction

Large secular changes in real interest rates constitute one of the fundamental puzzles in macroeconomics. Researchers have explored explanations including demographic change, productivity, stagnation, inequality, safe asset demand. Many of these explanations focus on secular changes in the natural interest rate r^*—the short-term real interest rate consistent with output at potential and stable inflation.

In this paper, I pursue a different and complementary explanation: A substantial part of secular variation in real rates can be attributed to monetary policy makers' prolonged struggle against extremely persistent deviations of the public's long-run inflation expectations from the level that policy markers are aiming for. This explanation rests on three elements: The public forms expectations of long-run inflation by adaptive learning experienced inflation, monetary policy leans against deviations of these long-run inflation expectations from an inflation target, and a discounted Euler equation partially disconnects real bond yields from expected output growth.

The first element builds on the learning-from-experience framework and the evidence in Malmendier and Nagel (2016) (MN). MN show that time-varying differences between individuals of different age in their reported inflation expectations can be explained well by an adaptive learning model in which people learn about the parameters of the inflation process from their life-time experiences of inflation, with relatively more weight given to more recent experiences. MN further show that the time-series dynamics of the average individual's belief—averaging across individuals of different age—are well approximated by a constant-gain learning algorithm. Differently from standard applications of constant-gain learning, however, the gain parameter that controls the sensitivity of parameter updates in response to inflation surprises is identified from the dynamics of cross-sectional differences in inflation expectations between cohorts. MN's estimates imply that the public's views about the expected long-run inflation rate are changing only slowly in response to inflation observations. Moreover, beliefs are changing in response to actually observed inflation, not in response to central bank communication policy commitments. Individuals who are learning from experience need to be able to see inflation close to target for a prolonged period for their beliefs to move towards the target.

As a first step in my analysis, I update the empirical analysis in MN by showing that the learning-from-experience forecasts of inflation constructed based on their gain parameter estimate explain well the between-cohort differences of inflation expectations in the Michigan Survey of Consumers out-of-sample after the end of MN's sample in 2010. In this out-of-sample period, until the recent burst in realized inflation after the COVID pandemic, older people had higher inflation expectations than younger people, consistent with older people carrying the memories of the high inflation rates of the 1970s. I then use the average across cohorts in the long-run inflation expectations implied by the learning-from-experience model as a proxy for a representative agent's perception of the long-run mean of inflation. These perceptions of long-run inflation change very slowly, with a weight of about 1.6% each quarter to an inflation surprise. After rising from around 2% in the 1960s to close to 6% in the early 1980s, they fell slowly to slightly above 2% before rising very recently back up to almost 3%.

I then show that slow-moving long-run inflation expectations induce secular variation in real interest rates in a variant of the New Keynesian (NK) model. The monetary policy authority in this model is aware that deviations of the public's long-run inflation expectations from target inflation today portend similar deviations in the future. As a consequence, policy makers lean against long-run inflation expectations with a nominal interest-rate rule that responds more than one-for-one to deviations of long-run inflation expectations from the target. Real interest rates therefore rise with long-run inflation expectations. Moreover, as the public perceives the future evolution of its own long-run inflation expectations as a martingale, the public's perception is that this response of real interest rates is extremely persistent. As a consequence, real long-term rates respond as much as real short rates do. At the same time, the response of output to these shifts in the real term structure is muted via a discounted Euler equation as in McKay, Nakamura, and Steinsson (2017).

The empirical evidence is consistent with the predictions of this variant of the NK model. In U.S. data from 1961 to 2023, real interest rates are strongly positively associated with long-run inflation expectations by the learning-from-experience model. How well experience-based long-run inflation expectations can explain secular real interest rate dynamics depends on the gain parameter in the adaptive learning model that I use to construct the long-run inflation expectations. If the gain is too high, long-run inflation expectations vary too quickly to match the persistent secular changes

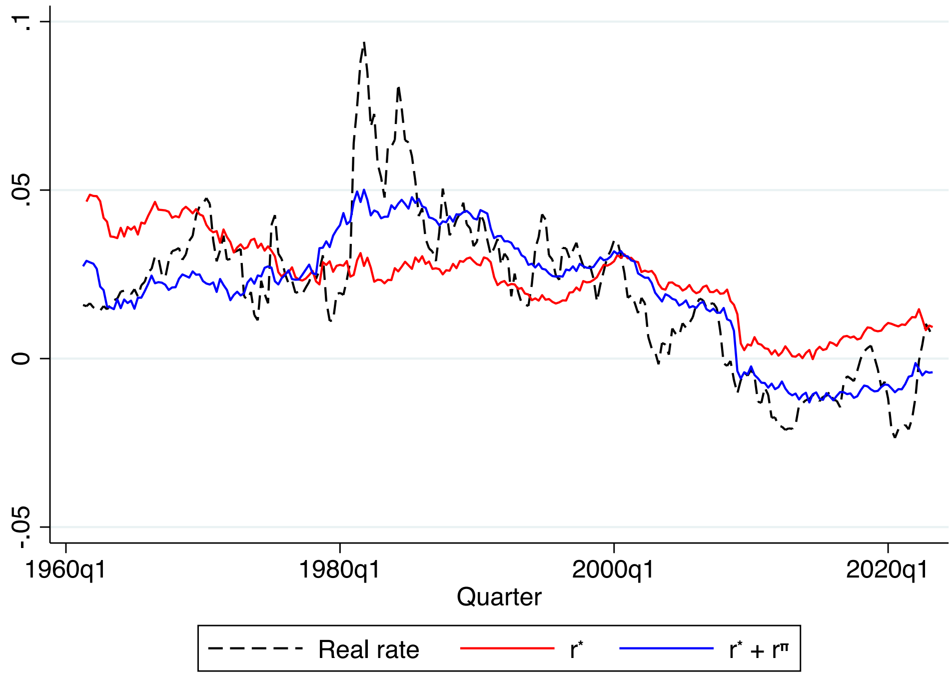


FIGURE I
Real interest rates, r^*, and r^π

The long-run real interest rates is measured as the 5-year zero-coupon yield minus 5-year inflation expectations from the Michigan Survey of Consumers, the r^* estimates are one-sided filtered estimates based on Laubach and Williams (2003) as provided by the FRBNY and adjusted for the average CPI-Core PCE inflation spread. The r^π estimates represent the component of long-horizon real interest rates associated with variation in long-run inflation expectations implied by the learning-from-experience model.

in real interest rates; if the gain is too low, they are too slow-moving. It is therefore important to keep in mind that I do not choose the gain parameter to fit real interest rate dynamics. Instead, the gain parameter is estimated by matching the learning-from-experience model to cross-sectional differences in cohort-level inflation expectations in survey microdata from the Michigan Survey of Consumers.

This strong relationship between real rates and experience-based long-run inflation expectations is present when real rates measured relative to inflation expectations households or professional forecasters, and also in ex-post realized real rates. It holds not only for short-term interest rates, but for maturities across the term structure. A one percentage point experience-based long-run inflation expectations is associated not only with an approximately one percentage point higher ex-ante short-term real rate, but also with similarly-sized increase of real rates at a 7-year forward horizon. This is consistent with the model's implication that the public expects real interest rates to be extremely persistent. For example, in a situation like around 1980, when inflation expectations and real rates are both high at the same time, private-sector agents do not believe that the high real rates will lead to lower inflation. Instead, they expect both inflation and real rate levels to persist. Implicitly, therefore, they believe that the natural rate of interest is high and persistent.

As a consequence, r^{π}, the component of real interest rates that arises from monetary policy makers' leaning against long-run inflation expectations, adds substantial explanatory power over and above r^* in explaining long-horizon real rates. Figure I illustrates this for long-horizon real rates based on 5-year real interest rates measured relative to 5-year inflation expectations from the Michigan Survey of Consumers. Adding r^{π} to r^* helps explain why real rates were lower than r^* in the 1960s and early 1970s, higher than r^* from in the 1980s and 1990s, and lower more until recently. In particular, much of the secular decline of real rates from the early 1980s until around 2020 can be attributed to r^{π}.

I find similar results for Germany, the U.K., and Japan. Ex-post realized short-term and long-term real interest rates in these countries are also strongly positively associated with experience-based long-term inflation forecasts. In this analysis of other advanced economies, I assume that private-sector agents learn from past inflation experiences with the same gain that I estimated for U.S. consumers in the Michigan Survey of Consumers.

My paper connects to a number of earlier works in the literature. Most closely related, Bianchi, Lettau, and Ludvigson (2022) develop and estimate a model that shares with this paper the notion that the interaction of monetary policy and adaptive learning is key for explaining secular real interest rate dynamics. The difference is that in their model, regime-shifts in monetary policy, and the public's perception of these shifts, are the mechanism. A situation such as around 1980 in the U.S., viewed through the lens of their model, is one in which monetary policy switches to a more hawkish regime, and real rates rise as a consequence. But the public is not yet convinced, until many years later, that the hawkish regime is here to stay. In contrast, in my model, there are no regime shifts and monetary policy always leans against deviations of the public's long-run inflation expectations from the inflation target. The public sees high real rates that follow from this policy as permanent and as a manifestation of a high natural rate of interest. As a consequence, long-term real rates move one-for-one with short-term real rates, consistent with the fact that real rates were elevated across the term structure in the early 1980s. That high real rates were a “new normal” may have been plausible at the time. Evidence on professional forecasters’ short-term interest rate forecasts in Cieslak (2018) supports this view. Forecast errors were on average strongly negative during the secular decline of real rates in the subsequent decades. That said, while my analysis shows that much of the secular dynamics of real rates can be understood without regime shifts, some of the real rate movements in the data are left unexplained—for example, the low short-term real rates during much of the 1970s—and the different mechanisms emphasized in the two papers may be complementary. Moreover, my empirical evidence broadly supports the notion in both papers that slow-moving real rate dynamics driven by monetary policy.

Kozicki and Tinsley (2001) attribute much of long-term Treasury bond yield variation to “shifting endpoints,” i.e., changing views about long-run expected short-term interest rates, which are in turn driven by shifting perceptions about the inflation target of the monetary policy authority. The results in this paper also imply a shifting endpoints view of long-term bond yield variation, but with shifting endpoints not only in nominal, but also in the real rates perceived by private-sector agents.[1] These shifts in nominal and real rates across the term structure due to the shift in private sector agents’ perception of long-run inflation also help explain several puzzling properties of bond prices, including the comovement of short-term interest rates with long-term yields (Giglio and Kelly (2018), Hanson and Stein (2012)), the high sensitivity of long-term yields to macroeconomic announcements (Gürkaynak, Sack, and Swanson (2005), Altavilla, Giannone, and Modugno (2017)), and the high unconditional mean excess returns of long-term bonds on macroeconomic announcement days during the secular decline of long-run inflation expectations from the early 1980s to 2020 (Jones, Lamont, and Lumsdaine (1998), Alam (2023)).

1. Using a decade of long-term inflation expectations data from the firm Drexel Burnham Lambert, Kozicki and Tinsley (2001) actually find that nominal rates move significantly more than one-for-one with long-term inflation expectations, but they dismiss the possibility of real rate endpoint shifts.

II. Subjective long-run inflation expectations under learning from experience

I use the learning-from-experience framework of MN as the model of expectations formation of private-sector agents. In this framework, individuals perceive the law of motion of inflation as an AR(1) process

$$\pi_{t+1} = a + \rho\pi_t + \varepsilon_{t+1}, \tag{1}$$

and they use inflation observed during their life-times to estimate the parameters $\boldsymbol{b} \equiv (a, \rho)'$ of this process. An individual in a cohort born at time s uses the recursive updating rule

$$\boldsymbol{b}_{t,s} = \boldsymbol{b}_{t-1,s} + \phi_{t,s}\boldsymbol{R}_{t,s}^{-1}\boldsymbol{x}_{t-1}(\pi_t - \boldsymbol{b}'_{t-1,s}\boldsymbol{x}_{t-1}), \tag{2}$$

$$\boldsymbol{R}_{t,s} = \boldsymbol{R}_{t-1,s} + \phi_{t,s}(\boldsymbol{x}_{t-1}\boldsymbol{x}'_{t-1} - \boldsymbol{R}_{t-1,s}), \tag{3}$$

where $\boldsymbol{x}_t \equiv (1, \pi_t)'$. Different from standard versions of adaptive learning (see, e.g., Evans and Honkapohja (2001)), where the gain $\phi_{t,s}$ is either constant or a decreasing function of just t, here the gain depends also on s,

$$\phi_{t,s} = \begin{cases} \frac{\theta}{t-s} & \text{if } t - s \geq \theta \\ 1 & \text{if } t - s < \theta, \end{cases} \tag{4}$$

The parameter θ determines the shape of the implied function of weights on past inflation experiences. In their baseline specification, MN estimate a value of $\theta = 3.044$ for quarterly data. The recursion starts with $\phi_{t,s} = 1$ for $t - s < \theta$, which implies that data before shortly after birth is ignored. After birth, the gain is decreasing with age. A value of $\theta = 1$ would imply that data experienced after birth is weighted equally, and hence individuals' AR(1) parameter estimates at each point in time are simple OLS estimates on the experienced life-time data. With $\theta = 3.044$, as estimated by MN, memory of past inflation might fades away over time as an individual ages.

Before using the subjective inflation expectations based on the MN framework to explain secular real interest rate dynamics, I first provide an update on their estimates to see whether their results still hold in an extended sample that includes the years after 2009 when their sample ended. Table

TABLE I

Learning-from-Experience Model: Update of MN(2016) Estimates

Each cohort born at time s is assumed to recursively estimate an AR(1) model of inflation, with the decreasing gain $\phi_{t,s} = \theta/(t-s)$. The table reports the results of OLS regressions of one-year survey inflation expectations in quarter t (cohort means) on the one-year forecasts implied by the learning-from-experience model using inflation data up to the quarter prior to the survey quarter. Standard errors shown in parentheses are two-way clustered by time and cohort.

	(1) MN sample	(2) Full sample	(3) Full sample	(4) Recent sample
Experience-based forecast	0.67	0.65	0.64	0.48
	(0.08)	(0.07)	(0.07)	(0.08)
#Obs.	8,215	11,015	11,015	2,800
adj. R^2	0.61	0.61	0.61	0.58
θ	3.044	3.044	2.653	2.653
Sample	1953 - 2009	1953 - 2023	1953 - 2023	2010 - 2023
Time FE	Yes	Yes	Yes	Yes

I presents regressions of one-year inflation expectations from the Michigan survey of consumers, aggregated, as in MN, to mean expectations at birth-year cohort level, on the cohort-level forecast implied by the learning-from-experience adaptive learning scheme (see Appendix A for more detail on the data). All regressions include time dummies, which means that identification is based purely on cross-sectional differences between cohorts in their experiences and expectations and the time-variation of these cross-sectional differences over time.

Column (1) reports the estimates from the original MN sample using quarterly inflation measured with the seasonally-adjusted CPI-U and with experience-based forecasts calculated with MN's estimate of $\theta = 3.044$.[2] Column (2) extends the sample all the way to 2023Q2. As in the MN sample, there is a strong positive relationship between the learning-from-experience forecast and inflation expectations. A one percentage point higher learning-from-experience forecast implies a 0.65 percentage point higher inflation expectation. Column (4) uses an updated estimate of θ,

2. The slope coefficient reported here is the same as in MN, but the R^2 is slightly lower (61% instead of 64%). The reason is that I implemented a minor improvement in the treatment of survey responses who provided a categorical response of "up" ("down") about expected inflation, but not a percentage response. MN use a procedure recommended in Curtin (1996) that draws percentage responses from the empirical distribution of percentage responses of those in the same age who gave the same categorical response of "up" ("down") in the same survey period. In MN, the imputed responses were drawn from a sample that include missing responses. Here, I draw only from the sample of responses that excludes missing values.

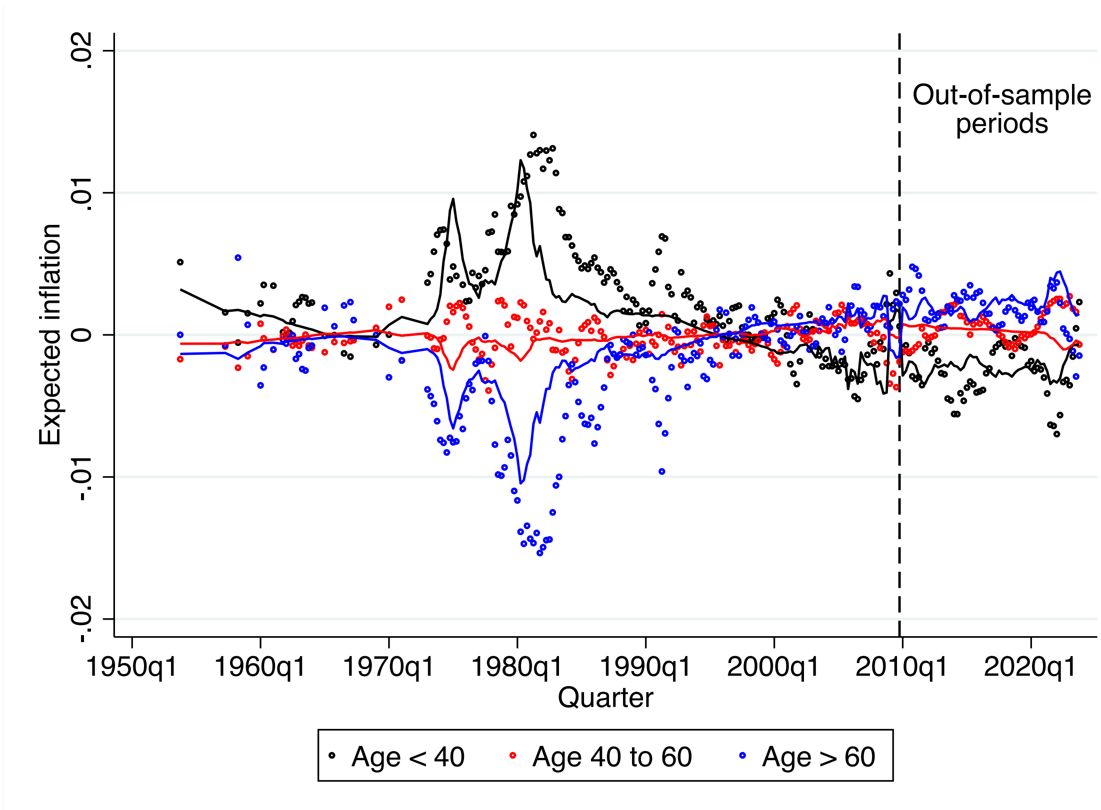


Figure II

Comparison of Learning-from-Experience Forecasts with One-Year Inflation Expectations from the Michigan Survey of Consumers

Four-quarter moving averages for individuals below age of 40, between 40 and 60, and above 60, shown as deviations from the cross-sectional mean expectation. The fitted values are based on estimates in column (3) in Table I. The dashed line indicates the end of the sample in MN's data set.

which is estimated as in MN using nonlinear least squares, but now with the full sample until 2023Q2. The estimate of 2.653 is slightly lower than MN's estimate of 3.044, but the weighting of past data and the experience-based forecasts implied by this new estimate are very similar to those implied by MN's estimate.[3] Column (4) presents an out-of-sample test, using only the data after the end of MN's sample. There is still a strong positive relationship. The slope coefficient estimate of 0.48 is a bit lower than in the original sample, but still of large magnitude.

Figure II illustrates the fit with expectations data from the survey and the fitted values from column (3) in Table I. The series shown in this figure are aggregated by taking means within

3. The standard error of the θ estimate is 0.16 and hence very small relative to the magnitude of the parameter.

TABLE II

Learning-from-Experience Model: Long-horizon inflation expectations

Each cohort born at time s is assumed to recursively estimate an AR(1) model of inflation, with the decreasing gain $\phi t, s = \theta/(t-s)$ with $\theta = 3.044$ and using quarterly annualized inflation rate data up to the end of quarter $t-1$. The table reports the results of OLS regressions of five-year survey inflation expectations in quarter t (cohort means) on the five-year forecasts implied by the learning-from-experience model. Standard errors shown in parentheses are two-way clustered by time and cohort.

	(1)	(2)
Experience-based forecast	0.91	
	(0.14)	
Experience-based long-run mean		0.87
		(0.14)
#Obs.	8,951	8,951
adj. R^2	0.43	0.43
θ	2.653	2.653
Sample	1953 - 2023	1953 - 2023
Time FE	Yes	Yes

three broad age groups, subtracting the overall cross-sectional mean each period, and forming four-quarter moving averages. The figure reveals that the learning-from-experience updating scheme continues to explain well the persistent differences between younger and older individuals in the out-of-sample periods after 2009. Until just before the very end of the sample, when inflation rises sharply, older individuals had higher inflation expectations than younger individuals. The learning-from-experience model explains this with older individuals still carrying memory of the high average inflation rates in the 1970s and early 1980s. Interestingly, at the very end of the sample, following several post-COVID quarters with high inflation, young and old again switched sides. Now the young again have higher inflation expectations than older people. The forecasts implied by the learning-from-experience model have also come close to switching signs. Thus, the learning-from-experience model also explains how expectations of people of different age react to the most recent burst of inflation.

Long-horizon inflation expectations play an important role in my analysis of real interest rates. For this reason, I extend MN's analysis to five-year inflation expectations from the Michigan Survey. Table II shows a regression of the average annual inflation rate over the next five years expected by survey respondents on the five-year forecast implied by the learning-from-experience model. As for short-run expectations, there is a strong positive relationship. Column (1) shows that a one percentage point higher learning-from-experience forecast translates into 0.91 percentage points higher five-year inflation expectation.

At these long horizons, the persistence parameter in the AR(1) perceived law of motion does not play much of a role. Column (2) shows this by replacing the forecast based on the AR(1) with just the long-run mean implied by the AR(1) parameter estimates (i.e., the forecast at horizon $h \to \infty$). As the estimates show, the long-run mean has about the same explanatory power for the observed 5-year expectations. In my analysis of secular dynamics of real interest rates, I will focus on this long-run mean.

Furthermore, in the following analysis of real interest rates, I abstract from cross-sectional heterogeneity and I focus on the dynamics of the average belief across individuals of different age. MN have shown that for individuals who are learning from experience, the dynamics of this average beliefs is very well approximated by a constant-gain learning updating scheme. While each individual learns with decreasing gain from their experiences, the average gain across the age distribution is constant due to generational turnover. MN's estimate of $\theta = 3.044$ implied a gain value in a constant-gain learning scheme for the average belief dynamics of 0.018 for quarterly data; the estimate of $\theta = 2.653$ here implies a gain of 0.016. In this spirit, and with tractability in mind, the model in the next section features a representative private-sector agent who learns with constant gain 0.016 in quarterly data.

III. Subjective long-run inflation expectations and the real rate in a New Keynesian model

Slow-moving expectations of long-run inflation that are updated based on experiences are a challenge for monetary policy. These long-run expectations cannot easily be influenced by monetary policy makers through communication about intended future policy. If the public's long-run inflation expectations have moved away from monetary policy makers' target, then to bring these expectations back to the target, the public has to see evidence in terms of actually realized inflation. To achieve this, real interest rates need to deviate from their steady-state values for a prolonged period. I illustrate this with a minimal version of a log-linearized New Keynesian model. I focus on the extreme case where expectations are formed exclusively based on experiences of realized inflation. It would be more plausible to also allow central bank communication to affect the public's beliefs about future inflation to some extent. But I leave the model intentionally simple and stark to highlight the key mechanism.

Realized inflation, π_t, the nominal interest rate, i_t, and output x_t are all given in terms of deviations from zero-inflation steady-state values. I denote subjective expectations with $\widetilde{\mathbb{E}}_t$ and objective expectations based on the true law of motion with $\mathbb{E}_t$.

To focus on long-run expectations and secular dynamics, I assume that the public perceives inflation as an AR(1) process as in (1), but with $\rho = 0$. The public's subjective inflation expectations are then simply equal to the expected long-run mean,

$$\widetilde{\mathbb{E}}_t \pi_{t+j} = \widetilde{\pi}_{t-1}, \quad j = 1, 2, ..., \infty. \tag{5}$$

where learning about the long-run mean takes place with constant gain

$$\widetilde{\pi}_t = \phi \pi_t + (1 - \phi)\widetilde{\pi}_{t-1}. \tag{6}$$

Constant-gain learning at the representative agent level is an approximation for learning-from-experience at the cohort level. To allow for measurement delays, I assume that realized inflation at time t is not yet observable to the public when they form time-t expectations in (5), hence they form expectations based on $\widetilde{\pi}_{t-1}$, which in turn is based on realized inflation up to $t-1$.[4] Equation (5) jointly with (6) then also implies that $\widetilde{\mathbb{E}}_t \widetilde{\pi}_{t+j} = \widetilde{\pi}_{t-1}$ for $j \geq 0$.

Aggregate supply and demand relations are similar to rational expectations NK models, but they involve subjective expectations of inflation and output, as in Bullard and Mitra (2002).[5] The Phillips curve takes the form

$$\pi_t = \kappa x_t + \beta \widetilde{\pi}_{t-1} + u_t, \tag{7}$$

4. An alternative interpretation, as in Gaspar, Smets, and Vestin (2006), is that private sector agents can observe π_t at t, but it takes another period for them to update parameters of their forecasting model. Here the long-run mean of inflation is the only forecast model parameter, so this is an equivalent interpretation.

5. As Honkapohja, Mitra, and Evans (2013) note, this requires that the law of iterated expectation applies to subjective expectations, which is the case here.

where $0 < \beta < 1$ and $\widetilde{\mathbb{E}}_{t-1} u_t = \mathbb{E}_{t-1} u_t = 0$. The Euler equation under agents' subjective beliefs is

$$x_t = -\zeta\psi(i_t - \widetilde{\pi}_{t-1}) + \alpha\widetilde{\mathbb{E}}_t x_{t+1}. \tag{8}$$

This is a discounted Euler equation as in McKay, Nakamura, and Steinsson (2017) with $0 < \alpha < 1$ and $0 < \zeta < 1$. As McKay et al. show, the discounting with α approximates outcomes in a model with uninsurable income risk and borrowing constraints. Cognitive discounting as in Gabaix (2020) is an alternative microfoundation. Discounting in the Euler equation removes the arguably implausible implication of the model that news about real interest rates at any arbitrarily far horizon has the same effect on current consumption as similarly sized news about near-term real interest rates. By partly decoupling output from long-term real yield movements, discounting generates room for large movements in the level of the real yield curve without huge output effects.

Finally, the monetary authority as a target of zero for inflation and sets the nominal interest rate according to the following rule

$$i_t = \widetilde{\pi}_{t-1} + \gamma_\pi \widetilde{\pi}_{t-1} + \gamma_u u_t + \gamma_x x_t, \tag{9}$$

where γ_π, γ_u, and γ_x are positive coefficients that are known to private-sector agents. The only shock in this system is the supply shock u_t. I do not include demand shock and monetary policy shocks as the supply shock is sufficient to illustrate the key mechanism.[6]

In this model, private-sector agents do not know that the central bank's inflation target is zero, and they estimate based on experience. As a consequence, their perceived steady state differs from the actual zero-inflation steady state (that the model is log-linearized around). Agents' perceived steady-state inflation, and hence their perception of the central bank's inflation target, is $\widetilde{\pi}_{t-1}$, which exceeds the central bank's actual inflation target by $\widetilde{\pi}_{t-1}$. To have an internally consistent view of interest rates, agents' perceived steady-state real interest rate $\widetilde{r}^*_t$ must then exceed the actual steady-state real interest rate (natural rate of interest) by $\gamma_\pi \widetilde{\pi}_{t-1}$. For example, when

6. Since π_t will turn out to be a linear function of $\widetilde{\pi}_{t-1}$ and u_t, there exist values for γ_π and γ_u such that this specification also maps into the more conventional specification where the rule sets the interest rate as a function of π_t and x_t, but these parameter constellations will typically not be suitable for producing the empirical relationships between long-run inflation expectations and real rates that I find in the empirical analysis.

$\widetilde{\pi}_{t-1} > 0$, real interest rates are high because the central bank leans against these long-run inflation expectations, but agents subjectively perceive the high real interest rate not as a consequence of a low inflation target but rather a high natural rate of interest. Hence, from agents' viewpoint at time t, the nominal interest rate follows

$$i_t = \widetilde{r}_t^* + \widetilde{\pi}_{t-1} + \gamma_\pi(\widetilde{\pi}_{t-1} - \widetilde{\pi}_{t-1}) + \gamma_u u_t + \gamma_x x_t. \tag{10}$$

Comparison with (9) shows that the perceived natural rate of interest then is $\widetilde{r}_t^* = \gamma_\pi \widetilde{\pi}_{t-1}$.

Solving this system of (7), (8), and (9), I obtain current inflation of

$$\pi_t = (\beta - \gamma_\pi \kappa \zeta \psi \eta)\widetilde{\pi}_{t-1} + b_\pi u_t, \qquad \eta = \frac{1}{1 - \alpha + \gamma_x \zeta \psi}, \tag{11}$$

where b_π is a constant that is a function of model parameters (see Appendix B for the solution for this one and other coefficients in the model solution). There are three channels at work. First, long-run inflation expectations feed into current inflation through the Phillips curve (β). Second, monetary policy leans against long-run inflation expectations, which pulls inflation to zero via the Euler equation and then the Phillips curve (κ). Third, supply shocks u_t directly feed into inflation, but the central bank also leans against them in the monetary policy rule.

The ex-post realized real interest rate is

$$i_t - \pi_t = \left[(1-\alpha)\gamma_\pi + (1 - \alpha + \zeta\psi\gamma_x)(1-\beta) + \zeta\psi\kappa\gamma_\pi\right]\eta\widetilde{\pi}_{t-1} + b_r u_t \tag{12}$$

for some constant b_r. The loading on $\widetilde{\pi}_{t-1}$ is positive. The ex-ante real interest rate under subjective expectations, $\tilde{i}_t^{(r)} = i_t - \widetilde{\mathbb{E}}_t \pi_{t+1}$, is

$$\tilde{i}_t^{(r)} = (1-\alpha)\gamma_\pi \eta \widetilde{\pi}_{t-1} + b_s u_t \tag{13}$$

for some constant b_s. As $\eta(1-\alpha) > 0$, this subjective real rate is positively related to $\widetilde{\pi}_{t-1}$. Comparison with (12) shows that the loading of ex-ante rates on $\widetilde{\pi}_{t-1}$ is smaller than the loading of ex-post realized real rates.

Equations (12) and (13) represent the key relationships that I investigate in my empirical analysis. Because the central bank leans against slow-moving experience-based expectations of long-run inflation, real interest rates are subject to secular changes associated with secular dynamics of $\widetilde{\pi}_{t-1}$.

Real forward rates are equal to subjectively expected future real rates.[7] Taking expectations of future $\tilde{i}^{(r)}_{t+j}$ according to (13), using $\widetilde{\mathbb{E}}_t\widetilde{\pi}_{t-1+j} = \widetilde{\pi}_{t-1}$ and $\widetilde{\mathbb{E}}_t u_{t+j} = 0$, the horizon-$n$ subjective real forward rate $\widetilde{f}^{(r)}_{n,t} = \widetilde{\mathbb{E}}_t i_{t+n-1} - \widetilde{\mathbb{E}}_t\pi_{t+n}$ is

$$\widetilde{f}^{(r)}_{n,t} = \eta(1-\alpha)\gamma_\pi\widetilde{\pi}_{t-1} \tag{14}$$

and hence subjective real yields at all horizons have the same loading on $\widetilde{\pi}_{t-1}$. When $\widetilde{\pi}_{t-1}$ changes, the entire real yield curve shifts in parallel.

Due to the discounting in the Euler equation, the effect of such inflation-expectations-induced shifts in the real yield curve on current output are muted. Without discounting in the Euler equation, shifts in the real yield curve would produce huge effects on current output, for the same reasons that give rise to the forward guidance puzzle in rational expectations models (Del Negro, Giannoni, and Patterson 2023).

To an observer with rational expectations (RE) of inflation, under knowledge of the true law of motion, real forward rates look different. The RE observer expects inflation to converge to the monetary policy authority's target of zero in the long-run while agents in the model expect long-run inflation to be at $\widetilde{\pi}_{t-1}$. Therefore, to the RE observer, real forward rates in the long-horizon limit are the subjective forward rates in (14) plus $\widetilde{\pi}_{t-1}$, i.e.,

$$f^{(r)}_{\infty,t} - \mathbb{E}_t\,\pi_{t+\infty} = [1+\eta(1-\alpha)\gamma_\pi]\widetilde{\pi}_{t-1}. \tag{15}$$

At shorter horizons, the wedge between subjective forward rates and RE forward rates will be smaller than in the long-horizon limit, because in the near term, future inflation be kept away from the monetary policy authority's target, via (11), so the wedge between subjective expectations and

7. I assume that the expectations hypothesis holds under subjective beliefs of investors. Recent evidence in Cieslak (2018) and Nagel and Xu (2023) suggests that this is a reasonable assumption. Using professional forecast data, they show that low-frequency time-series variation in yields is associated largely with movements in subjective expectations of future short-term interest rates rather than variation in subjective risk premia.

RE is smaller. Thus, an econometrician running regressions of ex post realized real forward rates $f_{n,t} - \pi_{t+n}$ on $\widetilde{\pi}_{t-1}$ should find that the longer the horizon n, the stronger ex-post realized forward rates co-move with $\widetilde{\pi}_{t-1}$. More precisely, for very long-horizons n, following (15), the loading on $\widetilde{\pi}_{t-1}$ should be one plus the loading of subjective forward rates on $\widetilde{\pi}_{t-1}$ that is predicted by (14). I will examine this prediction in the empirical analysis.

Figure III shows the response of output, inflation, and real interest rates to a sequence of unanticipated supply shocks. I chose the supply shock sequence to produce results that somewhat resemble the 1970s stagflation. This is not meant to be a realistic description of the macroeconomic situation in the 1970s, but just to produce an illustration of how real rates behave in this model following a prolonged period of high and rising inflation. I set $u_t = 0.03$ for 20 quarters, followed by linear decline to zero over the following 20 quarters.

As panel (A) shows, this produces stagflation with high inflation and falling output. As experiences of high inflation accumulate, the public's expectations of long-run inflation rise and they decay only very slowly once the supply shocks have stopped.

Panel (B) shows the response of real interest rates. Ex-ante real interest rates based on subjective expectations rise almost one-for-one with subjective expectations of long-run inflation. This is a consequence of the monetary authority having to lean against the persistent perceptions of high long-run inflation for many periods. Getting these experience-based expectations down towards the target requires many periods of low inflation induced by high real rates. Realized rates follow a similar trajectory once the supply shocks have stopped, but during the earlier stagflation period, due to high realized inflation, realized real rates are actually negative.

IV. Empirical real rate dynamics

IV.A. Data and summary statistics

Table III summarizes the data for the empirical analysis. All numbers are annualized. Data sources are detailed in Appendix A. Zero-coupon bond yields to construct real interest rates are available from 1961Q2 to 2023Q2 and so the table reports summary statistics for this sample period.

As in MN, I use inflation data reaching back to the late 19th century to construct learning-

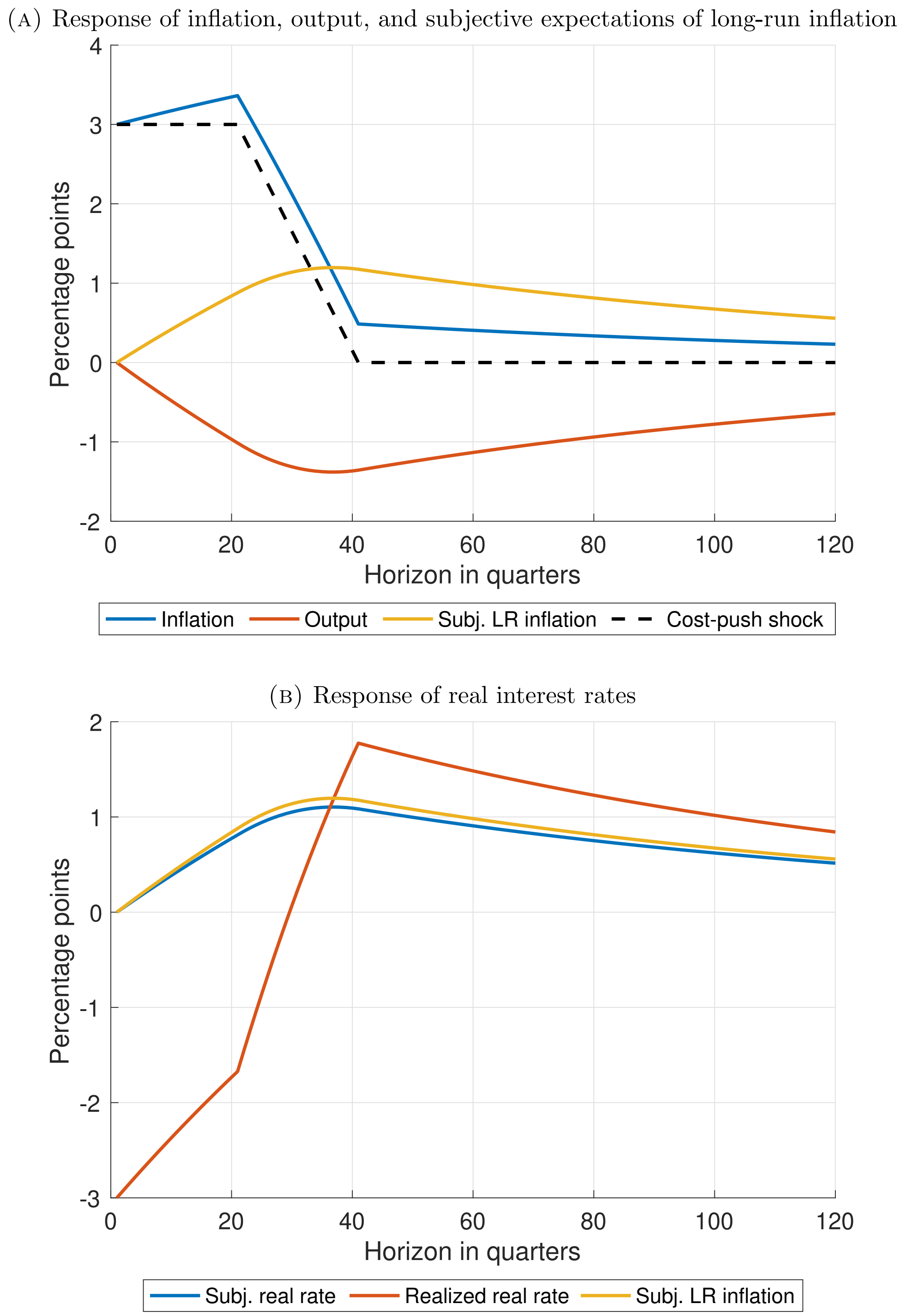


FIGURE III

Response to persistent unanticipated supply shock

Parameter values: $\beta = 0.99$, $\alpha = 0.8$, $\kappa = 0.5$, $\zeta = 0.5$, $\psi = 0.5$, $\gamma_\pi = 1.5$, $\gamma_u = 0$, $\gamma_x = 0.5$, $g = 0.016$.

TABLE III

Summary statistics

This table presents annualized summary statistics for quarterly data from 1961Q2 to 2023Q2. Ex-post realized real rates in Panel B are calculated based on realized CPI inflation. Ex-ante real rates for households in Panel C are based on the Michigan Survey of Consumers. Ex-ante real rates for professional forecasters in Panel D are based on the Survey of Professional Forecasters augmented with the Livingston Survey.

	Mean	Median	Std.dev.
Panel A: Inflation and r^*			
$\widetilde{\pi}$	0.036	0.035	0.011
$\pi_{CPI} - \pi_{corePCE}$	0.005	0.005	0.000
r^*	0.028	0.029	0.012
Panel B: Ex-post real rates			
3-month	0.011	0.010	0.032
1-year	0.011	0.010	0.031
7-year forward	0.028	0.035	0.038
Panel C: Ex-ante real rates, households			
1-year	0.013	0.014	0.028
5-year	0.020	0.020	0.023
7-year forward	0.028	0.028	0.020
Panel D: Ex-ante real rates, professional forecasters			
1-year	0.015	0.018	0.021

from experience forecasts at the birth-year cohort level using each cohort's life-time data set of experienced inflation, but here with the updated gain parameter estimate $\theta = 2.653$. Each period t, and for each cohort s, I obtain the cohort's AR(1) parameter estimates from the recursion in (2), and I use these estimates to construct the implied long-run inflation forecast as $a_{t,s}/(1-\rho_{t,s})$. I then average these long-run forecasts across cohorts, from age 25 to 74, to obtain the average long-run inflation forecast $\widetilde{\pi}_t$. Panel A shows that the average of $\widetilde{\pi}_t$ (annualized) is 3.6% during the sample period.

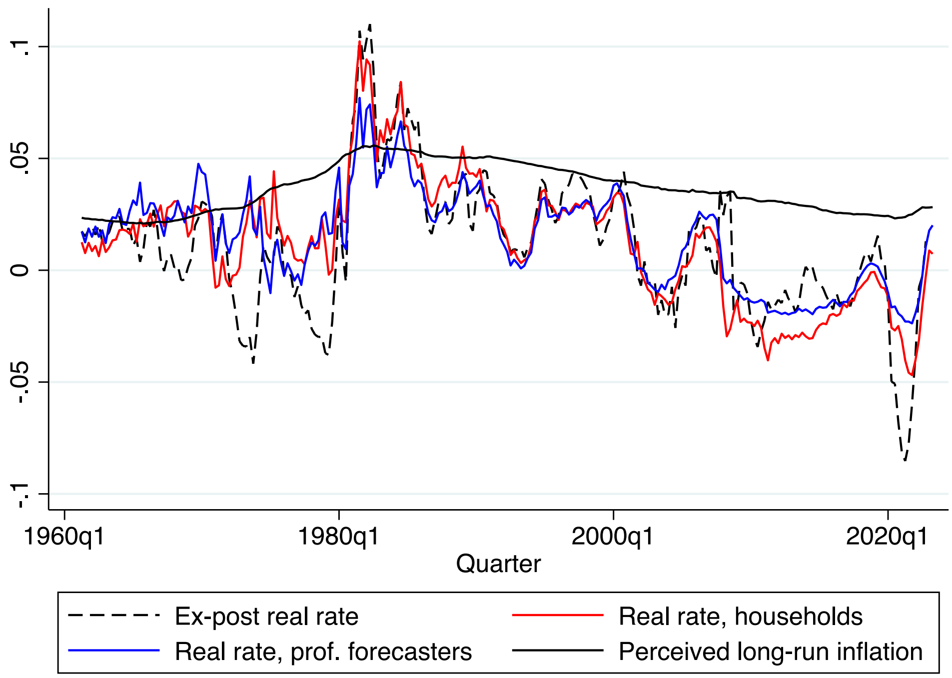
Panel A also shows summary statistics for r^*, an estimate of the natural rate of interest following Laubach and Williams (2003) provided by the Federal Reserve Bank of New York. I use this variable as an additional explanatory variable for real interest rates. These r^* estimates are constructed using core PCE inflation instead of the CPI inflation that I use here. Core PCE inflation is usually lower than CPI inflation during the sample period. For this reason, in Figure I in the introduction, I adjust r^* by subtracting the average spread between CPI and core PCE inflation, which is 0.5 percentage points, as shown in Panel A of the table.

The remaining panels show summary statistics for ex-post realized real interest rates (Panel B), calculated based on realized CPI inflation; ex-ante real rates for households in Panel C based on inflation expectations from the Michigan Survey of Consumers (MSC), and ex-ante real rates for professional forecasters in Panel D based on inflation expectations from the Survey of Professional Forecasters (SPF) augmented with the Livingston Survey in periods when the SPF is not available.

The realized real rates in Panel B at 3-month horizon are calculated as the average daily realized fed funds rate minus the CPI inflation rate during the same quarter, those at 1-year horizon are one-year zero-coupon yields extracted from Treasury bond yields at the end of quarter t minus the inflation rate from the end of quarter t to $t+4$, and the 7-year real forward rate is the instantaneous 7-year forward rate at the end of quarter t minus the realized inflation rate from the end of quarter $t+26$ to $t+30$.

Figure IV plots the main time series that I use in the analysis. Panel (A) shows different version of real rates and the the long-run inflation expectations implied by learning from experience, $\widetilde{\pi}_t$. The plot shows that there is a close alignment of the secular changes in real rates and $\widetilde{\pi}_t$. The figure also already reveals an econometric challenge: the series are highly persistent. Teasing out

(A) Real interest rates at one-year horizon



(B) Term structure of ex-ante real interest rates (households)

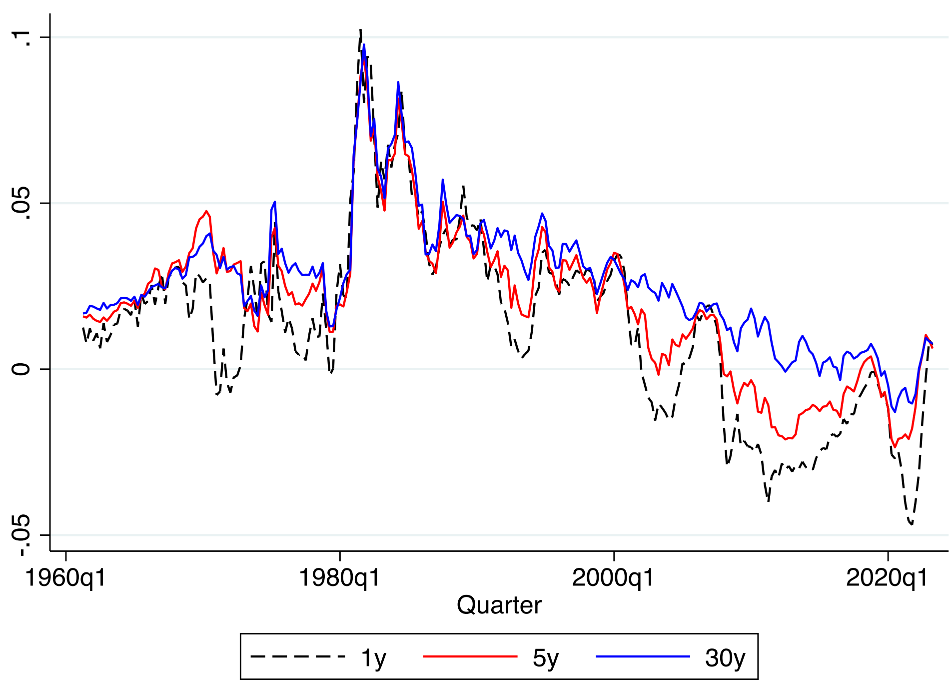


FIGURE IV
Real interest rates and long-run inflation forecasts based on learning from experience

whether there is a relationship in levels between such highly persistent series is difficult. In the econometric analysis, I address this issue in detail.

Panel (B) shows the term structure of ex-ante real rates based on household inflation expectations. The close alignment of $\widetilde{\pi}_t$ and real rates is even more apparent for long-term real rates. Furthermore, the plot shows that the rise and subsequent fall was as pronounced for very long-term real rates as for short-term real rates. This is consistent with the view that these secular changes are driven by perceptions of permanent shifts in real rates—or "shifting endpoints" in the language of Kozicki and Tinsley (2001).

IV.B. Econometric approach

My main regression specification is motivated by the relationship between subjective real rates and long-term inflation expectations in equations (13) and (15). I am therefore interested in relationships of the form

$$y_t = \beta_0 + \boldsymbol{\beta}' \boldsymbol{x}_t + e_t, \tag{16}$$

where y_t is either a realized real rate, $i_t - \pi_t$, an ex-ante real rate under subjective beliefs, $\tilde{\imath}_t^{(r)}$, or an ex-ante forward rate, $\widetilde{f}_{n,t}^{(r)}$, and $\boldsymbol{x}_t = \widetilde{\pi}_t$ or $\boldsymbol{x}_t = (\widetilde{\pi}_t, r_t^*)'$ to account for a possibly time-varying natural rate of interest.

Estimating the coefficient vector $\boldsymbol{\beta}$ is challenging because $\widetilde{\pi}_t$, r_t^*, and possibly also the supply shocks, demand shocks that remain in the residual e_t, are highly persistent and may feature unit roots. A level relationship between y_t and the elements of $\boldsymbol{x}_t$ estimated via OLS could therefore be spurious due to stochastic trends. On the other hand, it is plausible that y_t, $\widetilde{\pi}_t$ and r_t^* are cointegrated and hence the OLS estimator of $\boldsymbol{\beta}$ would be superconsistent. Even so, the asymptotic properties of the OLS estimator in this cointegration case are such that the usual OLS standard errors are not valid.

For descriptive purposes, I report OLS estimates with Newey-West standard errors using optimal bandwidth selection as in Newey and West (1994). But for the main analysis, to address the potential unit root problem, I estimate an error correction model (ECM),

$$\Delta y_t = c - a(y_{t-1} - \boldsymbol{\beta}'\boldsymbol{x}_{t-1}) + \sum_{j=1}^{p-1} \psi_{yj}\Delta y_{t-j} + \boldsymbol{\omega}'\Delta \boldsymbol{x}_t + \sum_{j=1}^{q-1} \boldsymbol{\psi}'_{xj}\Delta \boldsymbol{x}_{t-j} + \xi_t. \tag{17}$$

In the case of cointegration $a > 0$ holds and the deviations from the long-run relationship between y_{t-1} and $\boldsymbol{x}_{t-1}$ tend to revert in the future. In this case, the long-run coefficient vector $\boldsymbol{\beta}$ can be estimated from the ECM model. But if $a = 0$, no long-run relationship exists between y_t and $\boldsymbol{x}_t$. To test for the existence of a long-run relationship, one therefore must reject not only $\boldsymbol{\beta} = 0$ but also $a = 0$. Along with the estimated long-run coefficient vector $\boldsymbol{\beta}$, I therefore report results from the bounds test of Pesaran, Shin, and Smith (2001) (PSS). A rejection in the PSS test means acceptance of $(a > 0) \cap (\boldsymbol{\beta} \neq 0)$ and hence the existence of a levels relationship. I use critical values from Kripfganz and Schneider (2020). I choose optimal lag lengths p and q based on the Bayesian Information Criterion (BIC).

IV.C. Results

Table IV presents the results for ex-post realized real rates at horizons of three months, one year, and seven year forward.

OLS estimates in column (1) for the 3-month horizon show a large coefficient of 1.431, which would imply that a one-percentage point rise in the public's long-run inflation expectations, as implied by the learning-from-experience model, leads to a 1.431 point rise in the real interest rate. As shown at the bottom of the table, the PSS test in this case rejects the null of no level relationship between $\widetilde{\pi}$ and the realized real rate at a 5% level. The coefficient estimate of 1.943 from the ECM in Panel B should therefore be reliable estimate of the long-run relationship between $\widetilde{\pi}$ and the realized real rate and the t-statistics for the long-run coefficients are valid for inference. This magnitude broadly in line with the calibration of the NK model in Figure IIIa. The t-statistic of 3.05 rejects the hypothesis of a zero long-run coefficient with high confidence.

However, the specification in column (1) may suffer from an omitted variable problem. Theory suggests that time-variation in in the natural rate of interest should also influence real interest rates. As it turns out, r^* and $\widetilde{\pi}$ are almost uncorrelated, which is why inclusion of r^* as explanatory

TABLE IV

Ex-post real rate dynamics and long-term inflation expectations implied by learning from experience

The sample runs from 1961Q2 to 2023Q2. Panel A shows OLS estimates of regressions of ex-post realized real rates at various horizons (all annualized) on $\widetilde{\pi}$ with t-statistics in parentheses that are based on Newey-West standard errors with optimal bandwidths shown in the table. Columns (2), (5), and (8) add r^* as an explanatory variable; columns (3), (6), and (9) impose a coefficient of unity on r^* by subtracting it from the dependent variable. Panel B shows the long-run coefficients implied by ECM estimates. The bottom line shows whether the Pesaran-Shin-Smith bounds test rejects the null hypothesis of no relationship in levels between the dependent and explanatory variables. If it rejects at 10%, 5%, or 1% levels, this is reported in the table. Failure to reject at at least 10% is shown as "-".

	(1) 3m	(2) 3m	(3) 3m - r^*	(4) 1y	(5) 1y	(6) 1y - r^*	(7) fwd7y	(8) fwd7y	(9) fwd7y - r^*
Panel A: OLS estimates									
$\widetilde{\pi}$	1.431	1.441	1.441	1.598	1.613	1.616	3.005	2.979	3.213
	(3.44)	(4.56)	(4.60)	(3.21)	(4.73)	(4.91)	(11.92)	(11.08)	(8.45)
r^*		0.979			0.852			-0.125	
		(5.42)			(3.85)			(-0.44)	
Intercept	-0.041	-0.069	-0.070	-0.047	-0.072	-0.077	-0.085	-0.080	-0.123
	(-2.53)	(-5.24)	(-6.36)	(-2.38)	(-5.14)	(-6.41)	(-7.02)	(-5.08)	(-6.60)
adj. R^2	0.22	0.36	0.26	0.32	0.43	0.36	0.71	0.71	0.67
#Obs.	249	249	249	247	247	247	221	221	221
Opt. NW bandwidth	11	25	25	25	25	25	15	9	24
Panel B: ECM estimates, long-run coefficients									
$\widetilde{\pi}$	1.943	1.780	1.534	1.579	1.472	1.476	2.957	2.940	3.091
	(3.05)	(3.56)	(4.44)	(3.87)	(6.05)	(5.96)	(10.29)	(10.21)	(7.21)
r^*		0.673			1.060			-0.101	
		(1.35)			(4.53)			(-0.37)	
PSS test rejection at	5%	10%	1%	1%	1%	1%	1%	1%	1%

variable in column (2) has very little effect on the slope coefficient for $\widetilde{\pi}$.

Column (3) imposes the restriction implied by theory that the coefficient on r^* should be equal to unity (assuming perfect measurement of r^*) and subtracts r^* from the dependent variable. Imposing this restriction lowers somewhat the ECM coefficient on $\widetilde{\pi}$ to 1.534. The PSS test now rejects at a 1% level.

Columns (4) to (6) repeat the same analysis with ex-post realized real rates at a one-year horizon. The results are similar, and the PSS tests rejects the null of no level relationship in all three cases.

Columns (7) to (9) look at ex-post realized real rates on 7-year forwards. In this case, r^* has little explanatory power. In contrast, $\widetilde{\pi}$ is strongly related to these long-term realized real rates, as can be seen by the large long-run coefficients around 3.0 obtained from the ECM and the very high R^2 above 60% in the OLS regressions. Importantly, the PSS test again rejects in all three cases so there is little reason to believe that the high R^2 is a consequence of spurious regression.

Table V shows results for subjective ex-ante real rates based on household inflation expectations from the Michigan Survey of Consumers at three horizons: one year, five years, and seven years forward.

Recall from the discussion following equation (14) that the model predicts that ex-ante subjective real yields at all maturities should move with $\widetilde{\pi}$ by the same magnitude. Table V shows that the estimates are close to this prediction. Focusing on the estimates in columns (3), (6), and (9) which subtract r^* from the dependent variable, the estimated long-run coefficients for $\widetilde{\pi}$ are 1.469, 1.229, and 1.324 for 1-year, 5-year, and 7-year forward horizons, respectively. For all of these specifications, the PSS tests rejects the null of no level relationship at a 1% level.

Consistent with the model in Section III, the loadings of ex-ante real rates on $\widetilde{\pi}$ are all smaller in magnitude than the loadings of realized real rates on $\widetilde{\pi}$ in Table IV. Moreover, consistent with the model, the wedge gets bigger at longer horizons. It is actually a little bit too big for 7-year forward rates. The model predicts that the wedge between ex-post and ex-ante real rate loadings should be equal to unity at horizon infinity, but for 7-year forward rates it is $3.091 - 1.324 = 1.767$.

I use the ECM long-run coefficient estimate in column (6) to construct r^{π}, the component of real interest rates associated with variation in long-term inflation expectations implied by the

TABLE V

Subjective real rate dynamics and long-term inflation expectations implied by learning from experience: Household expectations

The sample runs from 1961Q2 to 2023Q2. Panel A shows OLS estimates of regressions of subjective real rates (all annualized) based on household inflation expectations from the Michigan survey of Consumers at various horizons on $\widetilde{\pi}$ with t-statistics in parentheses that are based on Newey-West standard errors with optimal bandwidths shown in the table. Columns (2), (5), and (8) add r^* as an explanatory variable; columns (3), (6), and (9) impose a coefficient of unity on r^* by subtracting it from the dependent variable. Panel B shows the long-run coefficients implied by ECM estimates. The bottom line shows whether the Pesaran-Shin-Smith bounds test rejects the null hypothesis of no relationship in levels between the dependent and explanatory variables. If it rejects at 10%, 5%, or 1% levels, this is reported in the table. Failure to reject at at least 10% is shown as "-".

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	1y	1y	1y - r^*	5y	5y	5y - r^*	fwd7y	fwd7y	fwd7y - r^*
Panel A: OLS estimates									
$\widetilde{\pi}$	1.616	1.630	1.626	1.287	1.299	1.297	1.304	1.312	1.315
	(3.02)	(6.24)	(5.51)	(2.83)	(6.38)	(5.95)	(4.12)	(7.78)	(8.43)
r^*		1.316			1.155			0.726	
		(9.17)			(9.58)			(5.48)	
Intercept	-0.046	-0.084	-0.075	-0.027	-0.060	-0.056	-0.019	-0.040	-0.048
	(-2.05)	(-7.83)	(-7.08)	(-1.36)	(-8.33)	(-7.13)	(-1.44)	(-5.70)	(-8.27)
adj. R^2	0.39	0.72	0.57	0.37	0.75	0.60	0.51	0.71	0.62
#Obs.	249	249	249	249	249	249	249	249	249
Opt. NW bandwidth	25	25	25	25	25	25	19	25	25
Panel B: ECM estimates, long-run coefficients									
$\widetilde{\pi}$	1.475	1.546	1.469	1.270	1.252	1.229	1.497	1.323	1.324
	(2.24)	(5.30)	(4.47)	(2.63)	(6.99)	(4.57)	(3.40)	(8.06)	(6.50)
r^*		1.288			1.068			0.801	
		(4.84)			(6.14)			(5.10)	
PSS test rejection at	-	1%	1%	5%	1%	1%	-	1%	1%

learning-from-experience model, shown in Figure I.

$$r^{\pi} = a + 1.229 \times \widetilde{\pi}_t \tag{18}$$

I choose the constant a such that the mean of $r^* + r^{\pi}$ is equal to the mean ex-ante 5-year real interest rate. I also adjust r^* downward by the average difference between CPI inflation (that I use to construct $\widetilde{\pi}_t$) and PCE inflation (which the Federal Reserve focuses on and which is used in the estimation of r^*) from Table III. Figure I in the introduction shows the resulting time series of $r^* + r^{\pi}$.

Overall, the results show that a substantial part of secular variation in real interest rates is associated with long-run inflation expectations as implied by the learning-from-experience model. As Figure I shows, the r^{π} component explains about as much of the decline in real interest rates since the early 1980s as r^* does.

Table VI shows results for one-year subjective ex-ante real rates based on professional forecaster inflation expectations. The results are very similar to the estimates for household expectations. With r^* subtracted from the dependent variable in column (3), the estimated ECM long-run coefficients indicate that a one percentage point rise in $\widetilde{\pi}$ is associated with a 0.908 percentage point rise in the subjective real rate. For the specification in column (3) the PSS test rejects at a 1% level. In column (2) it rejects at a level of 5%. Only in column (1) I cannot reject the hypothesis of no level relationship, but in this specification r^* is an omitted variable. Overall, the results are similar to those with real rates based on household expectations.

IV.D. Robustness checks

Could $\widetilde{\pi}$ in the earlier regressions perhaps simply capture an effect of recent lagged inflation or does it really matter than $\widetilde{\pi}$ is a slow-moving function of many lags of past inflation? In conventional Taylor rule estimation, researchers often use the inflation rate over the past year to explain the path of interest rates. Table VII shows that $\widetilde{\pi}$ captures variation that is distinct from the variation in interest rates associated with recent lagged inflation. I repeat the regressions from earlier tables with ex-post realized real rates and ex-ante real rates using household and professional forecaster

TABLE VI

Subjective real rate dynamics and long-term inflation expectations implied by learning from experience: Professional forecaster expectations

The sample runs from 1961Q2 to 2022Q3. Panel A shows OLS estimates of regressions of subjective real rates (all annualized) based on one-year inflation expectations from the Survey of Professional Forecasters supplemented with the Livingston Survey on $\widetilde{\pi}$ with t-statistics in parentheses that are based on Newey-West standard errors with optimal bandwidths shown in the table. Column (2) adds r^* as an explanatory variable; column (3) imposes a coefficient of unity on r^* by subtracting it from the dependent variable. Panel B shows the long-run coefficients implied by ECM estimates. The bottom line shows whether the Pesaran-Shin-Smith bounds test rejects the null hypothesis of no relationship in levels between the dependent and explanatory variables. If it rejects at 10%, 5%, or 1% levels, this is reported in the table. Failure to reject at at least 10% is shown as "-".

	(1) 1y	(2) 1y	(3) 1y - r^*
Panel A: OLS estimates			
$\widetilde{\pi}$	0.992	1.004	1.003
	(2.21)	(5.22)	(4.87)
r^*		1.129	
		(12.66)	
Intercept	-0.021	-0.054	-0.050
	(-1.11)	(-7.80)	(-7.22)
adj. R^2	0.25	0.66	0.42
#Obs.	249	249	249
Opt. NW bandwidth	25	25	25
Panel B: ECM estimates, long-run coefficients			
$\widetilde{\pi}$	0.852	0.906	0.908
	(1.40)	(3.10)	(3.53)
r^*		1.065	
		(3.98)	
PSS test rejection at	-	5%	1%

TABLE VII

Real rate dynamics and long-term inflation expectations implied by learning from experience: Controlling for lagged inflation

The sample runs from 1961Q2 to 2023Q2. Panel A shows OLS estimates of regressions of real rates (all annualized) on $\widetilde{\pi}$, r^*, and four-quarter inflation based on the change in CPI from the end of quarter $t-5$ to the end of quarter $t-1$, with t-statistics in parentheses that are based on Newey-West standard errors with optimal bandwidths shown in the table. In this table, all real rates are measured at a one-year horizon. Panel B shows the long-run coefficients implied by ECM estimates. The bottom line shows whether the Pesaran-Shin-Smith bounds test rejects the null hypothesis of no relationship in levels between the dependent and explanatory variables. If it rejects at 10%, 5%, or 1% levels, this is reported in the table. Failure to reject at at least 10% is shown as "-".

	(1) Ex-post	(2) Ex-ante households	(3) Ex-ante professionals
Panel A: OLS estimates			
$\widetilde{\pi}$	1.781	1.618	1.021
	(5.51)	(6.09)	(5.05)
r^*	0.952	1.309	1.139
	(4.99)	(8.86)	(11.92)
$\pi_{t-5:t-1}$	-0.182	0.012	-0.018
	(-1.75)	(0.21)	(-0.26)
Intercept	-0.074	-0.084	-0.054
	(-5.47)	(-7.84)	(-7.68)
adj. R^2	0.45	0.72	0.65
#Obs.	247	249	249
Opt. NW bandwidth	25	25	25
Panel B: ECM estimates: Long-run coefficients			
$\widetilde{\pi}$	1.578	1.199	0.824
	(6.11)	(2.80)	(2.56)
r^*	0.966	1.036	1.005
	(4.27)	(2.75)	(3.49)
$\pi_{t-5:t-1}$	-0.237	0.391	0.096
	(-1.85)	(1.86)	(0.70)
PSS test rejection at	1%	10%	10%

expectations, but now with lagged four-quarter inflation included as an additional explanatory variable. As the results in the table show, after controlling for lagged inflation, the loading of real rates on $\widetilde{\pi}$ is still about the same magnitude as in the earlier specifications without lagged inflation.

V. International evidence

In this section, I examine whether the relationship between experience-based long-run inflation expectations and real interest rates in other advanced economies (Germany, U.K., Japan) is similar to what I find in U.S. data. Survey data on inflation expectations is not available for a sufficiently long periods in these countries. For this reason, I focus on an analysis of ex-post realized real interest rates. For all three countries, I calculate learning-from-experience forecasts using the local CPI index and the same gain parameter value $\theta = 2.653$ that I estimated in U.S. microdata.

V.A. Germany

One challenge with calculation of experience-based forecasts in Germany is the hyperinflation in the 1920s. The inflation rates during the hyperinflation are so high that they would completely dominate any weighted average that puts even a small weight on the hyperinflation years. Hyperinflation may induce some long-run experience effects, perhaps also intergenerationally, but it is not plausible that individuals would regard the hyperinflation observations as drawn from the same data-generating process as inflation observations during the post-hyperinflation decades. As argued in Sargent (1983), endings of hyperinflations are associated with resetting of beliefs about the fiscal and monetary regime. While hyperinflation memories may have some lingering effects in subsequent decades, beliefs must have been mostly reset in this way, otherwise it would be impossible to make sense of the moderate level of nominal interest rates in post-hyperinflation decades. Therefore, to avoid this dominance of hyperinflation observations in the calculation of experience-based forecasts, I winsorize the inflation rates at a quarterly 10% rate. The results are not sensitive to choosing a different threshold in a wide range around 10%.

I use the natural interest rate for the Euro area from Holston, Laubach, and Williams (2023) as provided, with updates, by the Federal Reserve Bank of New York. This natural interest rate series goes back to 1972. As in the U.S. case, these are one-sided (filtered) estimates.

TABLE VIII

Ex-post real rate dynamics in Germany

The sample runs from 1972Q2 to 2023Q1. Panel A shows OLS estimates of regressions of ex-post realized real rates at various horizons (all annualized) on $\widetilde{\pi}$ with t-statistics in parentheses that are based on Newey-West standard errors with optimal bandwidths shown in the table. Columns (2), (5), and (8) add r^* as an explanatory variable; columns (3), (6) and (9) impose a coefficient of unity on r^* by subtracting it from the dependent variable. Panel B shows the long-run coefficients implied by ECM estimates. The bottom line shows whether the Pesaran-Shin-Smith bounds test rejects the null hypothesis of no relationship in levels between the dependent and explanatory variables. If it rejects at 10%, 5%, or 1% levels, this is reported in the table. Failure to reject at at least 10% is shown as "-".

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	3m	3m	3m - r^*	1y	1y	1y - r^*	fwd7y	fwd7y	fwd7y - r^*
Panel A: OLS estimates									
$\widetilde{\pi}$	2.649	1.794	1.850	3.127	2.001	2.343	3.084	1.834	2.513
	(4.61)	(3.04)	(3.45)	(5.19)	(3.25)	(4.25)	(4.55)	(4.53)	(3.33)
r^*		1.070			1.435			2.188	
		(1.69)			(2.70)			(3.47)	
Intercept	-0.064	-0.061	-0.061	-0.072	-0.069	-0.070	-0.050	-0.059	-0.054
	(-3.89)	(-4.43)	(-4.26)	(-4.16)	(-4.74)	(-4.63)	(-2.36)	(-2.72)	(-2.29)
adj. R^2	0.34	0.38	0.21	0.56	0.64	0.46	0.44	0.62	0.40
#Obs.	204	204	204	200	200	200	176	176	176
Opt. NW bandwidth	24	24	24	24	24	24	4	15	20
Panel B: ECM estimates: Long-run coefficients									
$\widetilde{\pi}$	2.894	1.598	2.069	3.489	1.980	2.692	4.103	1.834	3.164
	(5.66)	(2.61)	(4.52)	(4.74)	(2.55)	(4.25)	(2.78)	(2.11)	(3.21)
r^*		1.578			1.827			3.110	
		(2.91)			(2.57)			(3.47)	
PSS test rejection at	1%	1%	1%	5%	1%	5%	-	10%	-

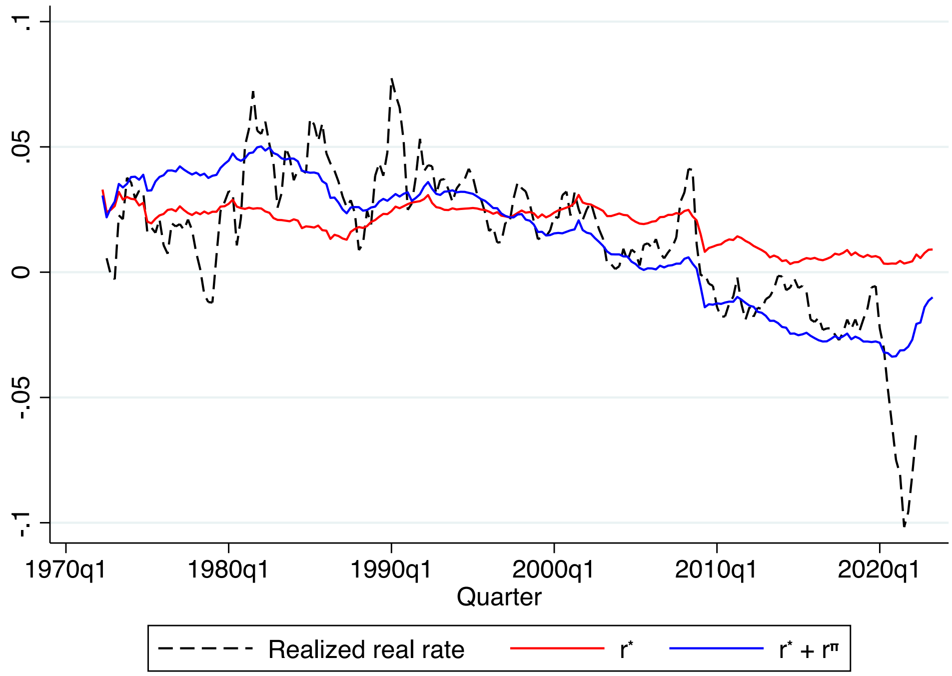


Figure V
Ex-post realized real interest rates, r^*, and r^π in Germany

Real interest rates are measured as 1-year zero coupon yields minus inflation over the next four quarters, the r^* estimates are one-sided filtered estimates based on Holston, Laubach, and Williams (2023) for the Euro area as provided by the FRBNY. The r^π estimates represent the component of real interest rates associated with variation in long-run inflation expectations implied by the learning-from-experience model.

Table VIII presents the results. They are broadly similar to the results for the U.S. in Table IV. The ECM coefficients on $\widetilde{\pi}$ in Panel B are larger than unity, and they are larger for long-term real rates than for the short-term real rate.

Figure V shows the realized real rates at 1-year horizon together with r^* and $r^* + r^\pi$ implied by the ECM estimates in column (6) of Table VIII. Unlike in the ex-ante real rates in Figure I for U.S. data, here the real rates are ex-post realized real rates and hence noisier. Nevertheless, it is clearly apparent that r^π makes a substantial contribution to explain secular dynamics of real rates in Germany.

TABLE IX

Ex-post real rate dynamics in the U.K.

The sample runs from 1961Q2 to 2019Q3 for 3-month real rates and from 1970Q1 to 2019Q3 to 2023Q1 for longer maturities. Panel A shows OLS estimates of regressions of ex-post realized real rates at various horizons (all annualized) on $\widetilde{\pi}$ with t-statistics in parentheses that are based on Newey-West standard errors with optimal bandwidths shown in the table. Columns (2), (5), and (8) add r^* as an explanatory variable; columns (3), (6), and (9) impose a coefficient of unity on r^* by subtracting it from the dependent variable. Panel B shows the long-run coefficients implied by ECM estimates. The bottom line shows whether the Pesaran-Shin-Smith bounds test rejects the null hypothesis of no relationship in levels between the dependent and explanatory variables. If it rejects at 10%, 5%, or 1% levels, this is reported in the table. Failure to reject at at least 10% is shown as "-".

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	3m	3m	3m - r^*	1y	1y	1y - r^*	fwd7y	fwd7y	fwd7y - r^*
Panel A: OLS estimates									
$\widetilde{\pi}$	0.993	0.945	1.066	1.518	1.493	1.533	2.094	2.222	2.182
	(2.83)	(2.31)	(3.15)	(5.62)	(5.25)	(4.97)	(6.57)	(6.56)	(6.75)
r^*		-0.676			-1.717			1.457	
		(-0.47)			(-1.49)			(1.63)	
Intercept	-0.042	-0.023	-0.070	-0.078	-0.036	-0.102	-0.086	-0.129	-0.116
	(-2.20)	(-0.49)	(-4.06)	(-5.02)	(-1.17)	(-5.54)	(-4.24)	(-3.83)	(-5.39)
adj. R^2	0.17	0.17	0.18	0.36	0.42	0.34	0.58	0.61	0.62
#Obs.	234	234	234	199	199	199	183	183	183
Opt. NW bandwidth	25	25	25	24	24	24	19	23	23
Panel B: ECM estimates: Long-run coefficients									
$\widetilde{\pi}$	1.193	1.215	1.315	1.596	1.596	1.526	2.173	2.392	2.239
	(3.12)	(3.04)	(3.19)	(7.11)	(7.31)	(8.85)	(5.69)	(6.33)	(6.53)
r^*		0.260			0.636			2.791	
		(0.22)			(0.78)			(2.25)	
PSS test rejection at	1%	1%	1%	1%	1%	1%	5%	5%	1%

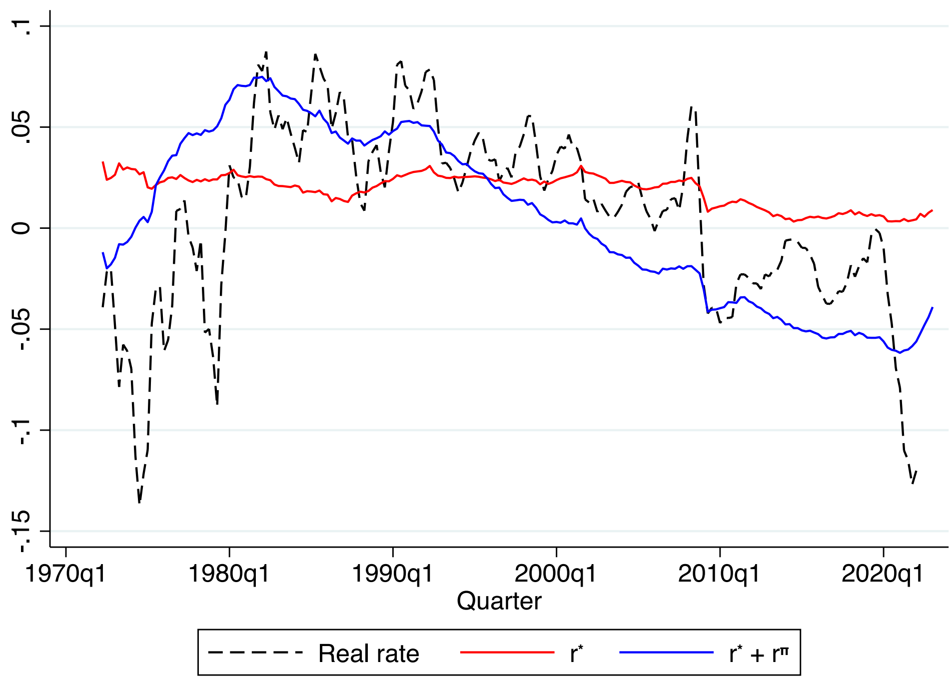


FIGURE VI
Ex-post realized real interest rates, r^*, and r^π in the U.K.

Real interest rates are measured as 1-year zero coupon yields minus inflation over the next four quarters, the r^* estimates are one-sided filtered estimates based on Holston, Laubach, and Williams (2017). The r^π estimates represent the component of real interest rates associated with variation in long-run inflation expectations implied by the learning-from-experience model.

V.B. United Kingdom

For the U.K., I obtain one-sided (filtered) estimates of the natural interest rate using the code in the replication package of Holston, Laubach, and Williams (2017) provided by the FRBNY for data up to 2019Q2. The results in Table IX show the broadly similar patterns as in the U.S. and Germany. ECM coefficients on $\widetilde{\pi}$ are all greater than unity for real rates at different maturities. There is again a tendency for coefficients to be larger for longer maturities.

Figure VI shows that the role of r^π in explaining real interest rates is substantially more pronounced in the U.K. than in the U.S. or Germany. Inflation rates were higher in the U.K. in the 1970s than in the U.S. or Germany, which lead, subsequently, to higher experience-based long-run inflation expectations. As the Figure shows, this greater variation in experience-based long-run inflation expectations was also accompanied by greater variation in 1-year horizon realized real interest rates.

V.C. Japan

Japan is an interesting case, because real interest rates reached low levels much earlier than in other advanced economies. While real interest rates of U.S., Germany, and U.K. had somewhat similar secular dynamics, real interest rates in Japan took a somewhat different path. A drop close to zero already happened in the mid-1990s. Japan therefore provides a useful check: is this different path of real interest rates also well explained by experience-based long-run inflation expectations?

Unfortunately, the data set is shorter for Japan than for the other countries. Throughout much of the 1970s, money market interest rates in Japan were highly regulated, Treasury Bills did not exist, and secondary market trading of government bonds was rare. I use data starting in 1979 when the interbank call money market was liberalized (Eken 1984). For bond yields, I use a 3-year maturity as this is the only series that is already available in 1979.

The natural interest rate series for Japan is from Wynne and Zhang (2018).[8] They extend the Laubach and Williams (2003) model to a two-country setting and apply it to Japan. As in Germany, there is a hyperinflation problem in the Japanese CPI data. In Japan's case, the hyperinflation occurred in the years following the end of the second world war. As in the German data, I winsorize the inflation rates at a quarterly 10% rate. The results are not sensitive to choosing a different threshold in a wide range around 10%.

Table X presents the results. For 3-month real rates, the estimated ECM coefficients on $\widetilde{\pi}$ in Panel B are similar to those in the other countries. For 1-year real rates, the coefficients are a little lower than in the other countries. Figure VII shows that the estimated ECM provides a good account of the secular change in real interest rates in Japan: realized real rates are quite close to $r^* + r^{\pi}$.

Overall, the evidence from Germany, U.K, and Japan broadly confirms the picture seen in U.S. data. In all four countries, secular dynamics of real interest rates are closely associated with slow-moving changes in experience-based long-run inflation expectations.

8. I thank Ren Zhang for providing the data on his website.

TABLE X

Ex-post real rate dynamics in Japan

The sample runs from 1979Q1 to 2022Q4. Panel A shows OLS estimates of regressions of ex-post realized real rates at various horizons (all annualized) on $\widetilde{\pi}$ with t-statistics in parentheses that are based on Newey-West standard errors with optimal bandwidths shown in the table. Columns (2) and (5) add r^* as an explanatory variable; columns (3) and (6) impose a coefficient of unity on r^* by subtracting it from the dependent variable. Panel B shows the long-run coefficients implied by ECM estimates. The bottom line shows whether the Pesaran-Shin-Smith bounds test rejects the null hypothesis of no relationship in levels between the dependent and explanatory variables. If it rejects at 10%, 5%, or 1% levels, this is reported in the table. Failure to reject at at least 10% is shown as "-".

	(1) 3m	(2) 3m	(3) 3m - r^*	(4) 1y	(5) 1y	(6) 1y - r^*
Panel A: OLS estimates						
$\widetilde{\pi}$	0.782	0.862	0.445	0.984	1.033	0.647
	(5.66)	(7.06)	(2.78)	(11.70)	(10.49)	(4.29)
r^*		-0.237			-0.144	
		(-0.72)			(-0.63)	
Intercept	-0.010	-0.014	0.006	-0.014	-0.016	0.002
	(-2.39)	(-2.49)	(0.95)	(-4.54)	(-3.58)	(0.36)
adj. R^2	0.33	0.33	0.12	0.77	0.77	0.47
#Obs.	154	154	154	154	154	154
Opt. NW bandwidth	13	18	23	23	23	23
Panel B: ECM estimates: Long-run coefficients						
$\widetilde{\pi}$	1.284	1.472	1.094	0.982	0.896	0.762
	(5.49)	(4.85)	(3.10)	(7.96)	(4.93)	(2.99)
r^*		-0.521			0.258	
		(-1.23)			(0.69)	
PSS test rejection at	5%	5%	10%	5%	5%	-

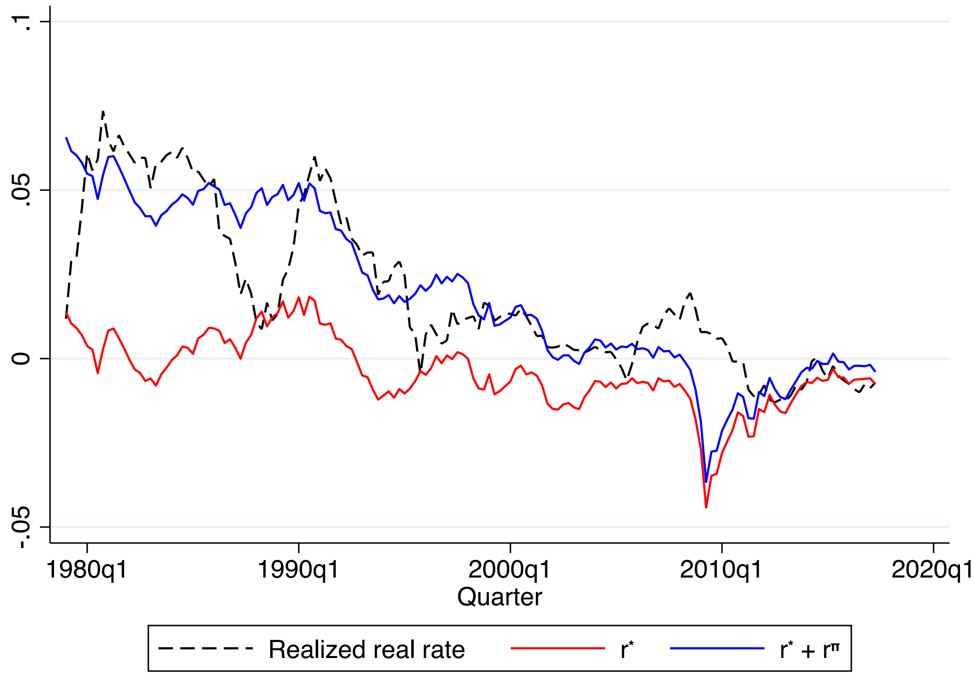


Figure VII

Ex-post realized real interest rates, r^*, and r^π in Japan

Real interest rates are measured as 3-year government bond yields minus realized average inflation over the next three years, the r^* estimates are one-sided filtered estimates based from Wynne and Zhang (2018). The r^π estimates represent the component of real interest rates associated with variation in long-run inflation expectations implied by the learning-from-experience model.

VI. Implications for long-term bond yields and interest-rate forecasts at higher frequency

In addition to the low-frequency time-series evidence, the learning-against-inflation-experiences model can also explain a number of empirical puzzles concerning the higher-frequency dynamics of long-term bond yields and interest-rate forecasts.

VI.A. Comovement of short-term interest rates, long-term bond yields, and long-horizon forecasts of interest rates

Empirical studies have found that nominal and real long-term bond yields positively comove with short-term interest rates to a degree that is puzzling for standard models. Giglio and Kelly (2018) show that nominal long-term bond yields covary more strongly with short-term nominal interest rates than standard term-structure models can explain, given typical modeling assumptions and the empirical behavior of short-term interest rates. Hanson and Stein (2012) find that daily changes in long-term real forward rates at horizons between 5 to 20 years covary strongly with daily changes in nominal two-year yields.

As Kozicki and Tinsley (2001) argue, the comovement of long-term nominal rates with short-term nominal rates can be explained by shifting inflation targets of the central bank. The comovement of short-term nominal rates with long-term real rates is more puzzling, but it follows naturally from the model developed here. In the model in Section III, a change in subjective long-term inflation expectations by one percentage point moves long-term nominal rates by $1 + \eta(1 - \alpha)\gamma_\pi$ percentage points and long-term real rates by $\eta(1 - \alpha)\gamma_\pi$. Time-varying subjective long-term inflation expectations induce a common component in long-term nominal and real rates because the monetary policy authority is leaning against the deviation of long-run inflation expectations from their inflation target.

The reason why long-term rates move with short-term rates in my model is an expectations hypothesis channel. Long-term yields reflect expectations of future short-term yields, and expectations of future short-term yields reflect expectations of long-run inflation. Consistent with this expectations hypothesis channel, Crump, Eusepi, Moench, and Preston (2023) find a similar comovement at short and long horizons in professional forecasts of inflation and interest rates. Specifically, forecasts of inflation and T-bill yields at 7 to 11 years ahead comove strongly with 3-year-ahead forecasts during the four decades since the early 1980s.

VI.B. Sensitivity of long-term yields to macroeconomic announcements

Another puzzle in the literature is that long-horizon bond yields move a lot in response to surprise information contained in macroeconomic announcements. Using daily data, Gürkaynak, Sack, and Swanson (2005) find that long-horizon forward rates out to horizons of 10-15 years react strongly to macroeconomic announcements in ways that are quantitatively only somewhat weaker than the reaction of one-year yields. More specifically, inflation news in terms of surprises of producer price index (core), employment cost index, and, to a lesser extent, CPI (core) are positively related to long-horizon forward rates. Altavilla, Giannone, and Modugno (2017) aggregate macroeconomic surprises from announcements within longer time intervals and find that they explain much of low-frequency variation in long-term bond yields. As Gürkaynak, Sack, and Swanson (2005) discuss, the sensitivity of long-horizon forward rates to macroeconomic news is not explained by standard macroeconomic models in which long-term inflation expectations are well anchored.

In my model, the drift of perceived inflation targets based on learning from experience, and monetary policy makers' leaning against inflation experiences provide a channel for nominal rates across the whole term structure to move with inflation news.

VI.C. Unconditional mean excess returns of long-term bonds on macroeconomic announcement days

Jones, Lamont, and Lumsdaine (1998) find that most of the excess return earned by long-term U.S. Treasury bonds during their sample from 1979 to 1995 was earned on days of macroeconomic (employment or PPI) announcements. Less than 10% of trading days with such announcements account for about two-thirds of the cumulative excess return of long-term bonds. In data from more recent decades, Alam (2023) finds that the entire secular decline of long-term yields from 1994 to 2019 occurs on macroeconomic announcement days. This means that long-term bonds earned most of their returns over this multi-decade time span on macroeconomic announcement days.[9]

Jones, Lamont, and Lumsdaine (1998) interpret the high average excess return on macro announcement days as a risk premium. There is, however, an alternative explanation that these high excess returns reflect the fact that investors were repeatedly surprised by the decline of inflation. Interpreted through the lens of my model, the secular decline in nominal interest rates since the early 1980s until around 2020 is explained by the secular decline in long-term inflation expectations. Starting with a high level of experience-based subjective inflation expectations in the early 1980s, investors were confronted with a long sequence of repeatedly negative surprises about inflation, which lead to a string of downward revision in their subjective long-term inflation expectations

9. Hillenbrand (2023) finds that the entire secular decline of long-term yields from the early 1980s until 2021 was concentrated in declines during three-day windows around FOMC announcements, but Alam (2023) shows that many FOMC announcements are preceded, on the same day or the day before, by macroeconomic announcements and that these days with macroeconomic announcement account for the cumulative yield decline in these three-day windows around FOMC announcements.

and in long-term bond yields, which in turn resulted in a string of high average excess return realizations for long-term bonds. In other words, these high excess returns are the consequences of forecast errors that have a mean very different from zero due to the secular changes in inflation expectations and interest-rate expectations that took place during the past decades. Consistent with this explanation, Cieslak (2018) finds that the forecast errors of both survey forecasts of the federal funds rate as well as expectations implied by federal funds futures prices were strongly negative on average during the decades following the peak of interest rates in the early 1980s.

VII. Conclusion

This paper presents evidence that experience-based formation of inflation expectations plays an important role in explaining secular dynamics of real interest rates. By leaning against deviations of experience-based long-run inflation expectations from an explicit or implicit inflation target, monetary policy has a substantial, very persistent effect on real interest rates. Secular dynamics of real interest rates across the maturity spectrum are closely tied to time-variation in experience-based long-run inflation expectations. This is true not only in the U.S., but also in Germany, the U.K., and Japan.

Once experience-based long-run inflation expectations are elevated—as in the early 1980s—bringing them down requires extended period of low inflation induced by high real interest rates. That monetary policy can move the public's long-run inflation expectations only by engineering low realized inflation rates rather than through pronouncements about policy targets by central bankers is at odds with the rational expectations view that credible policy commitments could induce expectations to jump to policy makers' target. But there is not necessarily a contradiction. The crux is credibility. In his study of the end of four hyperinflations, Sargent (1983) points out that in each case a dramatic change in the fiscal policy regime was associated with the resetting of beliefs that ended the hyperinflation. It is not clear that any such drastic changes in policy regime occurred in post-WW II U.S. history.

In the most recent inflationary episode after the COVID pandemic, experience-based long-run inflation expectations have not changed much so far. Compared to the 1970s inflation, the recent inflationary episode was not long enough to substantially change people's inflation experiences.

This means that central banks should be able to deal with the inflation problem with less pain than in the 1980s.

References

Alam, Zohair, 2023, "Learning About Fed Policy from Macro Announcements: A Tale of Two FOMC Days," Working paper, University of Toronto.

Altavilla, Carlo, Domenico Giannone, and Michele Modugno, 2017, "Low Frequency Effects of Macroeconomic News on Government Bond Yields," *Journal of Monetary Economics* 92, 31–46.

Bianchi, Francesco, Martin Lettau, and Sydney C Ludvigson, 2022, "Monetary Policy and Asset Valuation," *Journal of Finance* 77, 967–1017.

Bullard, James, and Kaushik Mitra, 2002, "Learning about Monetary Policy Rules," *Journal of Monetary Economics* 49, 1105–1129.

Cieslak, Anna, 2018, "Short-rate Expectations and Unexpected Returns in Treasury Bonds," *Review of Financial Studies* 31, 3265–3306.

Crump, Richard K, Stefano Eusepi, Emanuel Moench, and Bruce Preston, "The Term Structure of Expectations," in *Handbook of Economic Expectations*, pp. 507–540, 2023).

Curtin, Richard, 1996, "Procedure to Estimate Price Expectations," Working paper, University of Michigan Survey Research Center.

Curtin, Richard T., 1982, "Indicators of Consumer Behavior: The University of Michigan Surveys of Consumers," *Public Opinion Quarterly* 46, 340–352.

Del Negro, Marco, Marc P Giannoni, and Christina Patterson, 2023, "The Forward Guidance Puzzle," *Journal of Political Economy Macroeconomics* 1, 43–79.

Eken, Sena, 1984, "Integration of Domestic and International Financial Markets: The Japanese Experience," Working paper, IMF.

Evans, George W., and Seppo Honkapohja. *Learning and Expectations in Macroeconomics* (Princeton University Press, Princeton, NJ, 2001).

Gabaix, Xavier, 2020, "A Behavioral New Keynesian Model," *American Economic Review* 110, 2271–2327.

Gaspar, Vitor, Frank Smets, and David Vestin, 2006, "Adaptive Learning, Persistence, and Optimal Monetary Policy," *Journal of the European Economic Association* 4, 376–385.

Giglio, Stefano, and Bryan Kelly, 2018, "Excess Volatility: Beyond Discount Rates," *Quarterly Journal of Economics* 133, 71–127.

Gürkaynak, Refet S, Brian Sack, and Eric Swanson, 2005, "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic models," *American Economic Review* 95, 425–436.

Gürkaynak, Refet S., Brian Sack, and Jonathan H. Wright, 2007, "The U.S. Treasury Yield Curve: 1961 to the Present," *Journal of Monetary Economics* 54, 2291 – 2304.

Hanson, Samuel, and Jeremy Stein, 2012, "Monetary Policy and Long-Term Real Rates," *Journal of Financial Economics* 115, 429–448.

Hillenbrand, Sebastian, 2023, "The Fed and the Secular Decline in Interest Rates," Working paper, Harvard Business School.

Holston, Kathryn, Thomas Laubach, and John C Williams, 2017, "Measuring the Natural Rate of Interest: International Trends and Determinants," *Journal of International Economics* 108, S59–S75.

Holston, Kathryn, Thomas Laubach, and John C Williams, 2023, "Measuring the Natural Rate of Interest after COVID-19," Working paper, Federal Reserve Bank of New York.

Honkapohja, Seppo, Kaushik Mitra, and George W Evans, "Notes on Agents' Behavioral Rules under Adaptive Learning and Studies of Monetary Policy," in *Macroeconomics at the Service of Public Policy*, Thomas J. Sargent and Juoko Vilmunen, eds., pp. 63–79, 2013).

Jones, Charles M, Owen Lamont, and Robin L Lumsdaine, 1998, "Macroeconomic news and bond market volatility," *Journal of financial economics* 47, 315–337.

Kozicki, Sharon, and Peter A Tinsley, 2001, "Shifting Endpoints in the Term Structure of Interest Rates," *Journal of Monetary Economics* 47, 613–652.

Kripfganz, Sebastian, and Daniel C Schneider, 2020, "Response Surface Regressions for Critical Value Bounds and Approximate p-values in Equilibrium Correction Models," *Oxford Bulletin of Economics and Statistics* 82, 1456–1481.

Laubach, Thomas, and John C Williams, 2003, "Measuring the Natural rate of Interest," *Review of Economics and Statistics* 85, 1063–1070.

Malmendier, Ulrike, and Stefan Nagel, 2016, "Learning from Inflation Experiences," *Quarterly Journal of Economics* 131, 53–87.

McKay, Alisdair, Emi Nakamura, and Jón Steinsson, 2017, "The Discounted Euler Equation: A Note," *Economica* 84, 820–831.

Nagel, Stefan, and Zhengyang Xu, 2023, "Dynamics of Subjective Risk Premia," *Journal of Financial Economics* 150, 103713.

Newey, Whitney K, and Kenneth D West, 1994, "Automatic lag selection in covariance matrix estimation," *Review of Economic Studies* 61, 631–653.

Pesaran, M Hashem, Yongcheol Shin, and Richard J Smith, 2001, "Bounds Testing Approaches to the Analysis of Level Relationships," *Journal of Applied Econometrics* 16, 289–326.

Sargent, Thomas, "The Ends of Four Big Inflations," in *Inflation: Causes and Effects*, Robert E. Hall, ed., pp. 40–109, 1983).

Shiller, Robert J. *Irrational Exuberance* (second edition). (Princeton University Press, Princeton, NJ, 2005).

Wynne, Mark A, and Ren Zhang, 2018, "Estimating the Natural Rate of Interest in an Open Economy," *Empirical Economics* 55, 1291–1318.

Appendix

A. Data

A.1. Michigan Survey of Consumers

The survey data on inflation expectations from the Survey Research Center at the University of Michigan is from Malmendier and Nagel (2016). Survey data is available since 1953, initially three times per year, then quarterly (1960-1977), and monthly since 1978 (see Curtin (1982)). The files up to 1977 are available from the Inter-University Consortium for Political and Social Research ($ICPSR$) at the University of Michigan. As in MN, the sample is restricted to respondents aged 25 to 74.

MN follow Curtin (1996) in making several adjustments to the raw data to correct known deficiencies. These adjustments are similar to the those used by the Survey Research Center in processing the data to construct its indices (e.g., the consumer sentiment index). Two of these adjustments only concern the early part of the data: imputing missing percentage responses for a small number of respondents expecting prices to decline (in data prior to February 1980) and adjustment of responses that prices will stay the same that reflected misunderstanding (in data prior to March 1982). The third adjustment involves imputing percentage responses when respondents provided only a categorical response of "up" ("down") by drawing from the empirical distribution of percentage responses of those with the same birth year who gave the same categorical response of "up" ("down") in the same survey period (Curtin's method draws from the empirical distribution in the same period, without breaking up the sample by age). In MN, the imputed responses were drawn from a sample that include the missing percentage responses. Here, I draw, with replacement, only from the sample of responses that excludes missing values.

To construct ex-ante real interest rate based on one-year and five-year household inflation expectations, I use the weighted interpolated median of individual respondent's reported expectations (the survey weights provided by the Michigan Survey are used as weights).

In some of the survey waves prior to 1978, the survey only asks about the expected direction of future price changes ("up," "same," or "down"), but not about the expected percentage of price changes. MN impute average percentage expectations at the cohort-level by estimating the relationship between average percentage responses and the proportion of "up" and "down" responses in periods when both categorical and percentage expectations are available. Importantly, this imputation procedure is only designed to impute cross-sectional differences in expected inflation between cohort, not the cross-sectional average of expectations each period. For this reason, I use the imputed data only in the cohort-level analysis in Section II, but not in the calculation of the time series of the average inflation expectation across cohorts in subsequent sections.

For the cohort-level analysis in Section II I follow MN and aggregate the inflation expectations by taking the averages, weighted using survey weights, within birth-year cohort each month. To get quarterly data, I then average monthly data within each quarter.

A.2. Professional forecasts

I obtain one-year CPI inflation forecasts from the Survey of Professional Forecasters, maintained at the Federal Reserve Bank of Philadelphia. The data set provides the median forecast of the professional forecaster participants every quarter, starting in 1981Q3. Prior to this date, I use median inflation forecasts for the GDP deflator, available since 1970Q2. Before 1970Q2 I use data from the Livingston Survey, also available from the Federal Reserve Bank of Philadelphia. The Livingston survey is conducted semi-annually. I construct implied annual inflation forecasts by taking twice the difference of the log of the median forecasts for the CPI 12 months and six months after the survey date.

A.3. Imputation of missing inflation expectations

For the inflation expectations series of households, there are quarters without available survey data in the years prior to 1978. In the professional forecaster data, the part covered by the Livingston Survey has only semi-annual data. To be able to use a common sample period for all types of ex-ante subjective real rates, I impute missing values with the help of the experience-based experience-based one-year inflation forecasts that I constructed in Section II. Let τ_t denote the average, across cohorts, of the cohort-level experience-based one-year forecasts.

When there is a gap of one or more quarters in the survey data series, then I fill this gap by adding changes of the learning-from-experience forecast, averaged across cohorts as discussed in Section II, in the missing periods. I refer to this approach as difference imputation. For example, if the last available observation in the survey data is in quarters $t-2$, while observations in $t-1$ and t are missing, and the survey data resumes in $t+1$, then I impute observations in $t-1$ by adding to the $t-2$ observation the change in the learning-from-experience forecast from $t-2$ to $t-1$. Next, I impute the observation in t by adding the change in the learning-from-experience forecast from $t-1$ to t to the imputed observation in $t-1$. The light gray areas in Figure A.1 show, for all three series of inflation expectations that I use in the construction of subjective real interest rates, the periods in which I impute observations in this way. The figure shows that the imputation produces inflation forecasts that fit very well with the survey data observations before and after the imputed observations. There is no indication of any sharp discrepancies between imputed and actual observations.

Percentage inflation expectations in the Michigan Survey are not available in the first half of the 1960s for 1-year expectations and until the early 1970s for 5-year expectations. In these early parts of the sample period, I impute the one-year expectations in quarter t with the average experience-based one-year inflation forecast τ_t. To impute 5-year expectations I use the average, across cohorts, of the 5-year inflation forecast implied by the learning-from-experience model. I refer to this approach as level imputation.

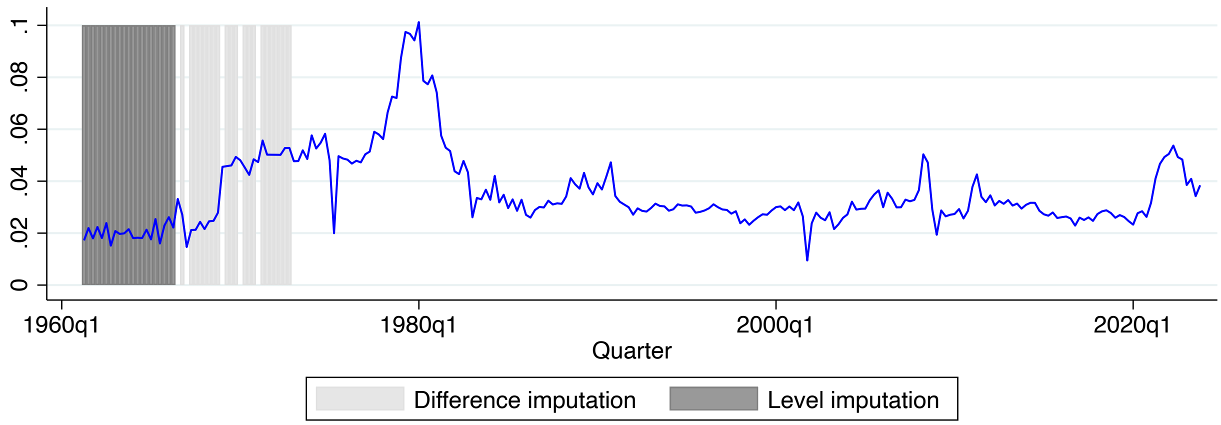
A.4. Inflation data

Fort the U.S., I obtain monthly CPI-U price index data from the FRED database at the Federal Reserve Bank of St. Louis. From 1947Q2 onwards, I quarterly log inflation rates based on the seasonally adjusted CPI-U. Before this data, only unadjusted data is available. Data from 1913Q2 to 1947Q2 is from FRED; inflation rates from 1872Q1 to say 1913Q1 are from Robert Shiller's website (see Shiller (2005)).

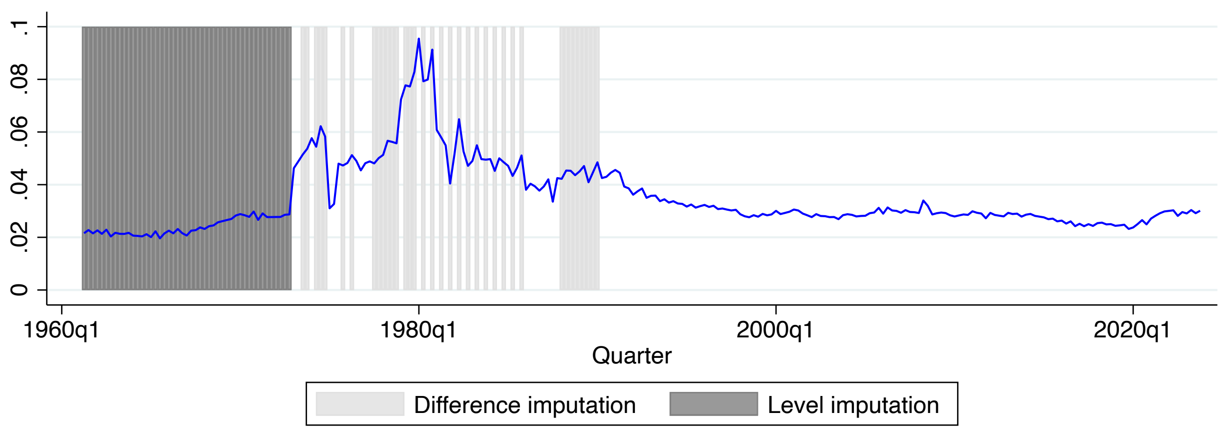
To calculate an adjustment of the natural real interest rate estimate from core PCE to CPI inflation, I obtain personal consumption expenditures excluding food and energy price index from FRED.

For Germany, U.K., and Japan, I obtain CPI series from Global Financial Data. These series are not seasonally adjusted. Seasonality in inflation rates has virtually no effect on the long-run inflation forecasts constructed with the learning-from-experience updating scheme because this long-run forecast is simply a slow-moving weighted average of past inflation rates. But seasonality

(A) Imputation of one-year household expectations



(B) Imputation of five-year household expectations



(C) Imputation of professional forecaster expectations

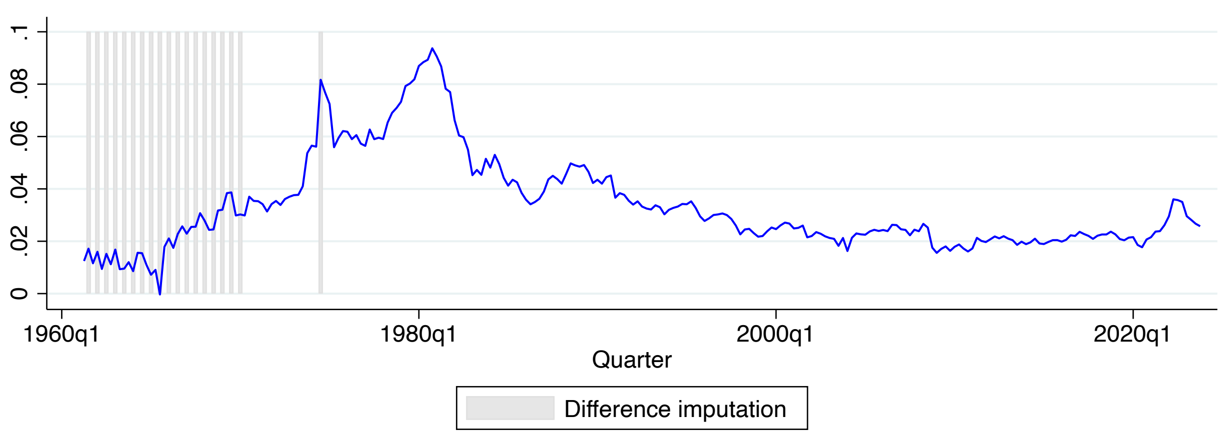


FIGURE A.1
Periods with imputation of missing inflation expectations data

induces some distortions to 3-month ex-post realized real interest rates. For this reason, I apply a seasonal adjustment to inflation data from 1960Q1 onwards in these countries. I use the X-13ARIMA-SEATS program provided by the U.S. Census Bureau.

A.5. Interest rates and bond yields

Continuously compounded zero-coupon yields at 1- and 5-year maturity and instantaneous 7-year forward rates extracted from U.S. Treasury coupon yields are from the updated data set of Gürkaynak, Sack, and Wright (2007) provided by the Federal Reserve Board. The series start in June 1961. The 7-year forward rate is the longest-maturity forward rate that is available already at the start of the data set. I take quarterly averages of the daily yields in this data set. To construct short-term realized real interest rates, I also use daily series of the effective federal funds rate, obtained form FRED. I take quarterly averages of the daily series.

For Germany, I use zero-coupon yields and forward rates provided by the Bundesbank and 3-month T-bill yields from Global Financial Data. For the U.K, I use zero-coupon yields and forward rates provided by the Bank of England and 3-month T-bill yields from Global Financial Data. For Japan, I obtain three-year government bond yields and 3-month TIBOR rates from Global Financial Data.

B. Model solution

Solving equations (7), (8), and (9) I obtain solutions for π_t, x_t, and i_t that depend on $\widetilde{\mathbb{E}}_t x_{t+1}$. In particular,

$$x_t = -\frac{\gamma_\pi \zeta \psi}{1+\gamma_x \zeta \psi}\widetilde{\pi}_{t-1} + \frac{\alpha}{1+\gamma_x \zeta \psi}\widetilde{\mathbb{E}}_t x_{t+1} - \frac{\gamma_u \zeta \psi}{1+\gamma_x \zeta \psi}\widetilde{\pi}_{t-1} \tag{A.1}$$

(An alternative way of solving the system is to express the monetary policy rule as perceived by agents as reflecting the perceived natural interest rate rather than deviations from an inflation target, i.e., $i_t = \widetilde{\pi}_t + \widetilde{r}^*_t + \gamma_u u_t + \gamma_x x_t$. This leads to an identical solution.) Iterating forward, using $\widetilde{\mathbb{E}}_t \widetilde{\pi}_{t+j} = \widetilde{\pi}_{t-1}$, $j \geq 0$, yields

$$x_t = -\frac{\gamma_\pi \zeta \psi}{1-\alpha+\gamma_x \zeta \psi}\widetilde{\pi}_{t-1} - \frac{\gamma_u \zeta \psi}{1+\gamma_x \zeta \psi}u_t. \tag{A.2}$$

I can use this result to substitute out $\widetilde{\mathbb{E}}_t x_{t+1}$ from the earlier solutions of π_t and i_t. This yields equations (11) and (12) in the main text, with

$$b_\pi = \frac{1+\zeta(\gamma_x - \gamma_u \kappa)\psi}{1+\gamma_x \zeta \psi} \tag{A.3}$$

and

$$b_i = \frac{\gamma_u}{1+\gamma_x \zeta \psi} \tag{A.4}$$

and $b_r = b_i - b_\pi$.