

# Credit and Liquidity in Interbank Rates: A Quadratic Approach

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ECB Workshop on Non-Standard Monetary Policy Measures -  
June 17, 2013

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  - QTSM Models: A General Framework
  - The EURIBOR-OIS Spread Modelling
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  - Decomposition of the Term Structure
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# Introduction

- The interbank market risk is at the heart of the (on-going) financial crisis.
- The IBOR-OIS spreads are some of the most scrutinized indicators of interbank-market risks.

During the crisis, conventional and unconventional actions taken by the central banks include:

- drop in the central bank interest rates,
- new facilities for liquidity providing to financial institutions (e.g. TAF in the US, VLTRO in the Euro-zone).

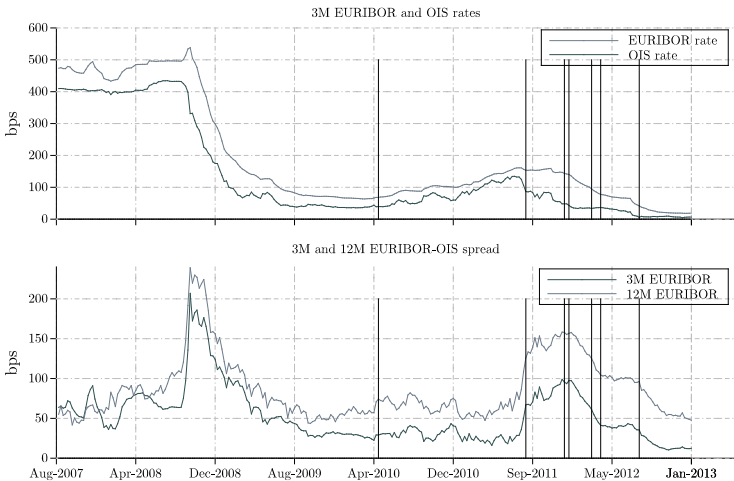
⇒ Have those unconventional actions been effective?

# The Interbank Rates

- *EURIBOR rates*: unsecured interbank rates proxy. It contains:
  - *Credit risk*: default of the borrower before due date.
  - *Liquidity risk*: important liquidity need of the lender before due date  $\implies$  Additional cost.
- *Overnight Indexed Swaps (OIS)*: riskless interbank rates proxy.  
Fixed leg  $\iff$  Floating leg indexed on EONIA.
  - Netting and credit-enhancement mechanisms (margin calls).
  - Nearly no immobilisation of capital.  
 $\implies$  Almost no credit and liquidity risk.

# The Term Structure of Interbank Rates

Weekly data: EURIBOR-OIS spreads for four maturities (3M, 6M, 9M, 12M) from August 31, 2007 to January 4, 2013.



# Motivations

- Separate bank credit risk from liquidity risk in the IBOR-OIS spread.  
→ *Observe the cause of fluctuations.*
- Extract the risk-premia linked to longer-term risk-bearing.  
→ *Necessitate no-arbitrage term structure model.*
- Generate strictly positive spreads under both measures.  
→ *Quadratic specification.*

Double decomposition to analyse monetary policy actions:

- Securities market program (May 2010, Aug. 2011).
- Very long-term refinancing operations (Dec. 2011 → Mar. 2012).
- Outright monetary transactions (late Aug. 2012).

## Related literature

- *Quadratic term structure models*  
Ahn, Dittmar & Gallant (2001), Constantinides (1992),  
Gourieroux & Sufana (2002), Leippold & Wu (2002a, 2002b)
- *Interbank rates modelling*  
Michaud & Upper (2008), Taylor & Williams (2009), Schwarz  
(2009), Filipovic & Trolle (2011), Christensen, Lopez &  
Rudebusch (2009), Angelini *et al.* (2011)
- *Decomposition of interest rates*  
Liu, Longstaff & Mandell (2006), Feldhutter & Lando (2008),  
Longstaff, Mithal & Neis (2008), Monfort & Renne (2012)

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# Pricing the Interbank Risk-Free Rate

We denote:

$r_t$  the short-term risk-free interest rate,

$R_{t,h}^{OIS}$  the OIS rate at time  $t$  of maturity  $h$ .

$$\implies R_{t,1}^{OIS} = r_t.$$

Under the absence of arbitrage opportunities:

- existence of both a historical ( $\mathbb{P}$ ) and a risk-neutral measure ( $\mathbb{Q}$ ).

Pricing formula of secured rates under risk-neutral measure:

$$R_{t,h}^{OIS} = -\frac{1}{h} \log \left( \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left\{ -\sum_{k=0}^{h-1} r_{t+k} \right\} \right] \right)$$

# Pricing the Unsecured Interbank Rates

We denote:

$d_t$  a dummy variable indicating either a default or an illiquidity event.

$\lambda_t$  the intensity representing the underlying risks in the economy.

$$\mathbb{P}(d_t = 1 | \underline{d}_{t-1}, \underline{r}_t, \underline{X}_t) = 1 - \exp(-\lambda_t)$$

Pricing formula of EURIBOR rates under risk-neutral measure:

$$R_{t,h}^{EUR} = -\frac{1}{h} \log \left( \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left\{ -\sum_{k=0}^{h-1} r_{t+k} + \lambda_{t+k+1} \right\} \right] \right)$$

# Standard Results in Term Structure Models

We denote:

$X_t$  a vector of factors in the economy.

If for all  $t$ ,  $r_t$  and  $\lambda_t$  are affine functions (resp. quadratic) of  $X_t$ ,

- the secured and unsecured rates are affine functions (resp. quadratic) of  $X_t$ ,
- these functions are available in closed-form for all maturities,
- the factor loadings are computable recursively.

## General pricing formulae for QTSM

$$R_{t,h}^{OIS} = a_h^{OIS} + b_h'^{OIS} X_t + X_t' c_h^{OIS} X_t$$

$$R_{t,h}^{EUR} = a_h^{EUR} + b_h'^{EUR} X_t + X_t' c_h^{EUR} X_t$$

# Modelling the EURIBOR-OIS Spread

- Implicitly, EURIBOR and OIS are considered as zero-coupons rates,
- We assume the short-term rate is independent from the intensity:

## Spread formula

$$\begin{aligned} S(t, h) &= R_{t,h}^{EUR} - R_{t,h}^{OIS} \\ &= -\frac{1}{h} \log \left( \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left\{ -\sum_{k=1}^h \lambda_{t+k} \right\} \right] \right) \end{aligned}$$

$\implies$  No need to express  $r_t$  for the spread modelling.

Remark:  $\lambda_t \geq 0 \implies S(t, h) \geq 0.$

# What We Need

- Definition of factors with
  - $\mathbb{P}$ -dynamics,
  - $\mathbb{Q}$ -dynamics,
- Specification of intensity  $\lambda_t = f(X_t)$ ,
- Identification constraints.

# The Historical Dynamics

- Credit and liquidity latent risk factors:  $X_t = (x_{c,t}, x_{l,t})'$ .
- $x_{c,t}$  and  $x_{l,t}$  are not instantaneously correlated.
- VAR(1) representation with independent idiosyncratic shocks.

$$\begin{pmatrix} x_{c,t} \\ x_{l,t} \end{pmatrix} = \begin{pmatrix} \mu_c \\ \mu_l \end{pmatrix} + \begin{pmatrix} \varphi_{1,1} & \varphi_{1,2} \\ \varphi_{2,1} & \varphi_{2,2} \end{pmatrix} \begin{pmatrix} x_{c,t-1} \\ x_{l,t-1} \end{pmatrix} + \begin{pmatrix} \sigma_c & 0 \\ 0 & \sigma_l \end{pmatrix} \begin{pmatrix} \varepsilon_{c,t} \\ \varepsilon_{l,t} \end{pmatrix}$$

where  $(\varepsilon_{c,t}, \varepsilon_{l,t})' \sim \mathcal{IIN}^{\mathbb{P}}(0, I_2)$ .

- For identification purposes,  $\sigma_c^2 + \sigma_l^2 = 1$ .
- We also define  $x_t = x_{c,t} + x_{l,t}$ .

# Risk-Neutral Specification and Intensity Process

- Also VAR(1) dynamics under  $\mathbb{Q}$ -measure with constraints  
 $\implies$  AR(1)  $\mathbb{Q}$ -dynamics for  $x_t$ .

$$x_t = \mu^* + \varphi^* x_{t-1} + \varepsilon_t^* \quad \text{where} \quad \varepsilon_t^* \sim \mathcal{IIN}^{\mathbb{Q}}(0, 1)$$

- Intensity is one-factor dependent:

$$\begin{aligned} \lambda_t &= \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2 && \text{with:} \\ \lambda_0 &\geq \lambda_1^2 / 4\lambda_2 && \implies \lambda_t \geq 0 \end{aligned}$$

## Reduced-form pricing formulas

$$S(t, h) = \theta_{0,h} + \theta_{1,h} x_t + \theta_{2,h} x_t^2$$

with  $\theta_{i,h}$  functions of  $(\lambda_0, \lambda_1, \lambda_2, \mu^*, \varphi^*)$  computable recursively.

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# Identification Strategy

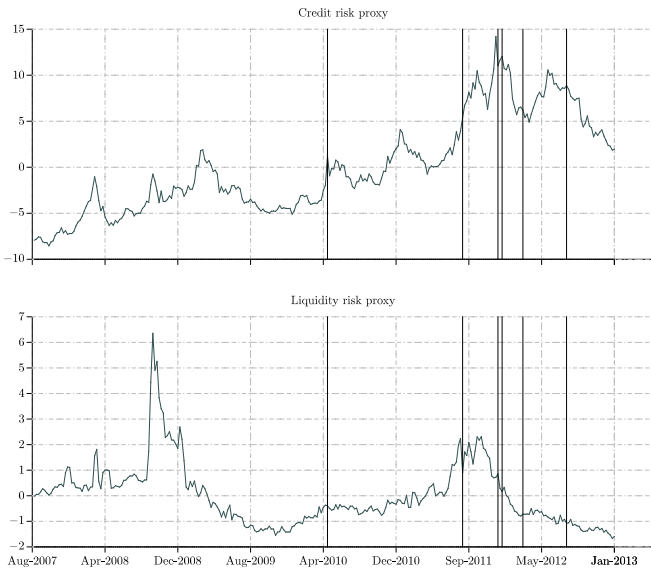
- Proxy for credit risk  $P_{c,t}$  → first PC of 36 Euro-zone bank CDS
- Proxy for liquidity risk  $P_{l,t}$  → first PC of
  - 5Y KfW-Bund spread
  - Spread of 3M general collateral *repo* rate versus 3M German treasury bill
  - Bank Lending Survey data (BLS): percentage of '—' and '— —' answers to the question *over the past three months, how has your bank's liquidity position affected the credit standards as applied to the approval of loans or credit lines to enterprises?*

## Proxies equations

Proxies are assumed quadratic functions of the corresponding factor with measurement errors.

$$\begin{cases} P_{c,t} &= \pi_{c,0} + \pi_{c,1}X_{c,t} + \pi_{c,2}X_{c,t}^2 + \sigma_{\nu_c}\nu_{c,t} \\ P_{l,t} &= \pi_{l,0} + \pi_{l,1}X_{l,t} + \pi_{l,2}X_{l,t}^2 + \sigma_{\nu_l}\nu_{l,t} \end{cases}$$

# Proxies



# The state-space representation

Transition and measurement equations :

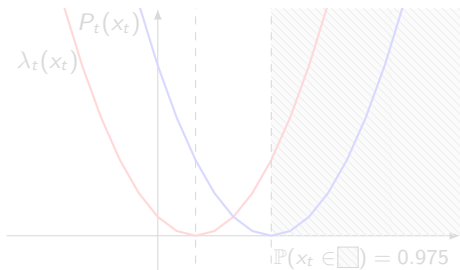
**Transition** Two-factor  $\mathbb{P}$ -dynamics.

**Measurement** Spread pricing formulae and proxies specification.

$\implies$  Maximum likelihood estimation with the Quadratic Kalman Filter (Monfort, Renne, & Roussellet (2013)).

Estimation constraints:

- Intensity and proxies functions are monotonously increasing *in most cases* in both factors.



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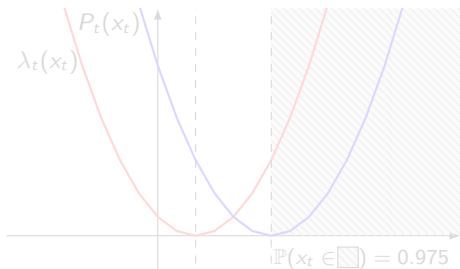
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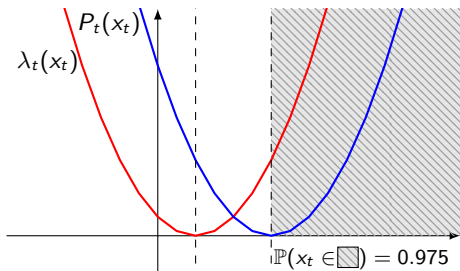
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## Results

Table : Risk-neutral and measurement parameter estimates

Equation		Estimate		Estimate		Estimate
$x_t$	$\mu^*$	0,2627*** (0,0387)	$\varphi^*$	0,9962*** (0,0019)		
$P_{c,t}$	$\pi_{c,0}$	-8,9650*** (0,4296)	$\pi_{c,1}$	-0,000006 (3,0444)	$\pi_{c,2}$	0,4496*** (0,0594)
$P_{l,t}$	$\pi_{l,0}$	-1,3098** (0,7577)	$\pi_{l,1}$	0,1382*** (0,0534)	$\pi_{l,2}$	0,0045*** (0,0006)
$\lambda_t$	$\lambda_0$	0,1015 (0,0666)	$\lambda_1$	0,0003 (0,0261)	$\lambda_2$	0,0023*** (0,0003)
noise	$\sigma_{\nu_c}^2$	0,0081 (0,4206)	$\sigma_{\nu_l}^2$	0,1000 —	$\sigma_{\eta}^2$	0,0106*** (0,0003)

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# Decomposition method

## Decomposition of the spread

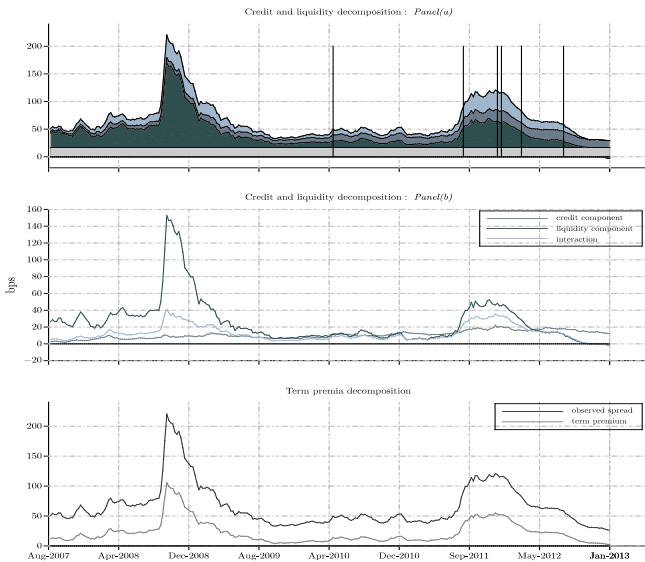
$$\begin{aligned}
 S(t, h) &= \theta_{0,h} + \theta_{1,h}x_t + \theta_{2,h}x_t^2 \\
 &= \underbrace{\theta_{1,h}x_{c,t} + \theta_{2,h}x_{c,t}^2}_{\text{credit spread}} + \underbrace{\theta_{1,h}x_{l,t} + \theta_{2,h}x_{l,t}^2}_{\text{liquidity spread}} + \underbrace{2\theta_{2,h}x_{c,t}x_{l,t}}_{\text{interaction}} + \theta_{0,h}
 \end{aligned}$$

- credit risk part,
- liquidity risk part,
- interaction part: presence and comovement of both risks in the economy,
- constant effect  $\theta_{0,h}$ : not attributable to any of the previous effects.

⇒ Decomposition in credit/liquidity and expected hypothesis component/term premia.

## Decomposition of the Spread

## Decomposition Results: 6M Spread



# Time series decomposition

## Liquidity component:

- High level on average and high-frequency fluctuations,
- represents most of the spread during Lehman crisis
- disappears at the end of the sample.

## Credit component:

- Globally increasing and low-frequency fluctuations,
- represents more than 20 bps at the end of the sample.

## Interaction term:

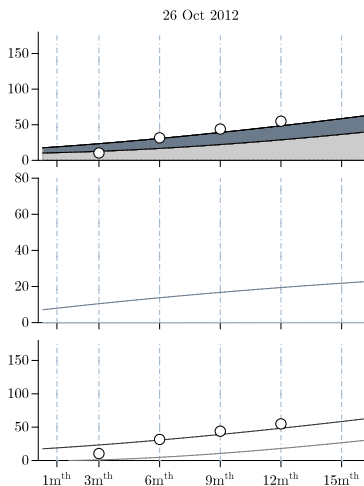
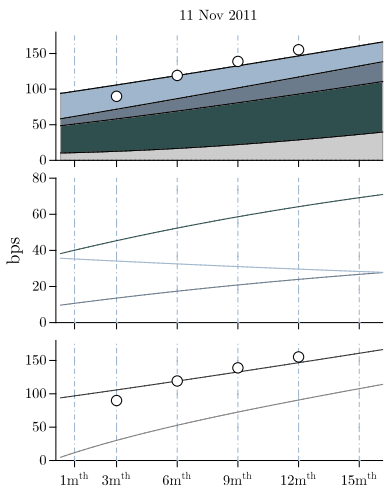
- Represents between 0 and 40 bps for the 6-month spread,
- fades out at the end of the sample.

## Term premia:

- Possess similar features as the observed spread,
- fluctuates between 0 and 60 bps for the 6-month spread.

## Decomposition of the Term Structure

## Decomposition of the Term Structure



# Efficiency of Unconventional Monetary Policies

**SMP** No clear drop or increase in any spread component.  
⇒ No effect.

**VLTRO** Significant drop after the announcement due mostly to liquidity and to a lesser extent to the interaction term.  
The two allotments do not change this trend.  
⇒ Nearly a 50 bps drop in 16 weeks.

**OMT** Disappearing of both liquidity and interaction terms 2 months after Mario Draghi's London Speech.  
⇒ Contributed to erase liquidity risk in the Euro Area.

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# Conclusion

In this paper,

- We use a quadratic no-arbitrage term structure model of EURIBOR-OIS spreads.
- We perform a decomposition of interbank spreads in credit and liquidity components.
- We extract the term premia from the observed spread.
- We show that the SMP program had no significant influence on interbank risk whereas the OMT contributed to erase the liquidity risk for all maturities.